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Inchworm locomotion gait for snakelike robot

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Abstract: To establish a universal and easily controlled gait for practical use of snakelike robot movement, an inchworm locomotion gait model based on a serpenoid curve is presented. By analyzing the relations of two adjacent waves in the process of locomotion and doing an approximation of the serpenoid curve, the motion function of relative angles between two adjacent links and the absolute angles between each link and the baseline on the traveling curve are built. Two efficiency criterions of the gait are given as the energy loss function f and the unit displacement in one cycle d_{unit}. Three parameters of the criterions affecting the efficiency of the gait (the number of links that form the traveling wave n, the included angle between two adjacent links α, and the phase difference of adjacent included angles β) are discussed by simulations and experiments. The results show that f is insensitive to n; raising n increases d_{unit} significantly; the maximum wave amplitude of α is a decreasing function of n; and increasing α reduces the displacement influence of f when n is determined. The gait model is suitable for different inchworm locomotions of a snakelike robot whose traveling waves are formed by different numbers of identical links. A wave formed by more links or a greater relative angle between two adjacent links both lead to greater velocity of the movement.

Key words: snakelike robot; multilink; inchworm locomotion gait; efficiency criterion

The most recent studies of snakelike robots have mainly focused on the serpentine locomotion and some successful realizations have been reported in Refs. [1 – 4], etc. However, the inchworm locomotion is also widely used in snakelike robot movement. Refs. [5 – 6] studied the locomotion over uneven solid terrain with a continuous backbone curve mode, but the presented analysis and algorithms do not dwell on the mechanical structures and actuators required to implement the mechanism deformation, so they cannot be used by a real snakelike robot. Ref. [7] successfully realized the inchworm locomotion on a plane with the traveling wave formed by three moving links, but the traveling wave given is not only lacking in versatility but it is also difficult to be controlled because of large speed variation ranges and large velocity ratios among articulations.

To establish a universal and easily controlled gait for practical use of snakelike robot movement, an inchworm locomotion gait model based on a serpenoid curve is presented. The efficiency criterions of the gait are given. Parameters of the criterions and their influence on the efficiency of the gait are discussed by simulations and experiments.

1 Inchworm Locomotion

It is considered that the inchworm locomotion gait is based on stationary waves and traveling waves of mechanism deformation on the plane. We simplify the snakelike robot as a multilink system consisting of N identical links, and each link is an absolutely rigid straight rod whose length is l. For simplicity, we here assume the number of links that form the traveling wave $n = 4$.

Fig. 1 shows the inchworm locomotion process. At the beginning, all the links lie on the baseline as in state a. Assume that the baseline is the x-axis and the positive direction is from point P_0 to point P_N. One propulsion step of a traveling wave is the process from state b to state d. In consequence of the entire cycle of

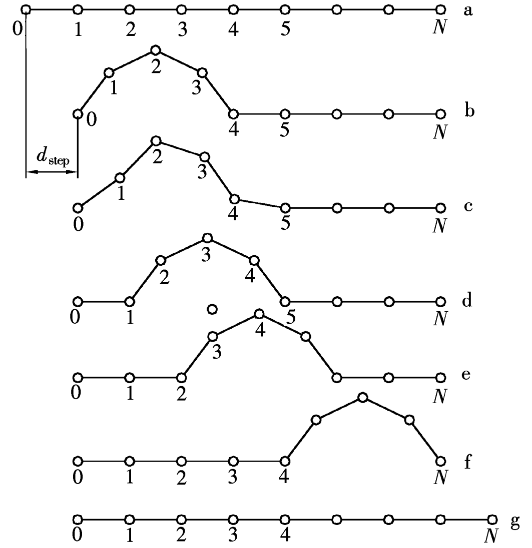

Fig. 1 Inchworm locomotion process of snakelike robot

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movements, the system advances along the x-axis for a distance d_{step}, which is equal to the displacement of the point P_0 from state a to state b.

2 Analysis of Inchworm Locomotion

A snakelike robot is a hyper-redundant system. The inchworm locomotion in the plane vertical to the ground is a coordinated movement of multi articulations. How to coordinate the movement of the multi articulations and make the traveling wave advance more smoothly is the emphasis. There are two restrictions in a propulsion step: one is to keep the distance $P_iP_{i+(n+1)}$ unchanged during the propulsion process; the other is to keep the wave advance smoothly by reducing the velocity fluctuation and the velocity ratio of articulations.

Fig. 2 shows the relationships of two adjacent waves during the propulsion process of a traveling wave. Curve 1 is a state of the snakelike robot at some time; curve 2 is the state that the snakelike robot will arrive after one propulsion step. The shapes of curve 1 and curve 2 are fully the same. Assume that the wave is made of n links. θ_i is denoted as the angle of the i-th link that is measured from the x-axis in a counterclockwise manner; ϕ_i is the relative angle between the ($i+1$)-th link and the i-th link. They have the following relations:

$$\theta_{i+1} = \theta_i + \varphi_i \qquad 1 \leqslant i \leqslant n-1 \tag{1}$$

Since the initial point and the end point of the traveling wave are both on the x-axis, so

$$\sum_{i=1}^{n} l_i \sin\theta_i = 0 \tag{2}$$

In one circle of motion, the displacement of the snakelike robot along the x-axis is

$$d_{\text{step}} = l\left(n - \sum_{i=1}^{n}\cos\theta_i\right)$$

Comparing curve 1 and curve 2 of Fig. 2, we have

$$\left.\begin{aligned} &\theta'_1 = 0 \\ &\theta'_i = \theta_{i-1} \qquad 1 < i \leqslant n+1 \end{aligned}\right\} \tag{3}$$

where θ'_i denotes the absolute angles of the i-th link on curve 2.

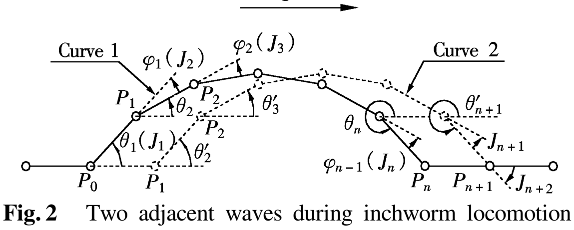

Fig. 2 Two adjacent waves during inchworm locomotion process

The relative angles vary monotonically during the propulsion process from curve 1 to curve 2. If the relative angles of curve 1 and curve 2 were known, the maximum turn angle of every articulation on the curve can be calculated by Eqs. (1) and (3),

$$\left.\begin{aligned} &r_1^{\max} = \theta'_1 - \theta_1 = -\theta_1 \\ &r_2^{\max} = \theta'_2 - \varphi_1 = \theta_1 - \varphi_1 \\ &r_i^{\max} = \varphi'_i - \varphi_i = \varphi_{i-1} - \varphi_i \qquad 3 \leqslant i \leqslant n \\ &r_{n+1}^{\max} = \theta'_{i+1} + \varphi_i = \theta_n + \varphi_{n-1} \\ &r_{n+2}^{\max} = \theta_n \end{aligned}\right\} \tag{4}$$

Eq. (4) shows that the maximum turn angles are related to the waveform parameters θ_1, θ_n and ϕ_i. Here, we assume that the traveling wave is a serpenoid curve which is the most widely used in snakelike robot locomotion studies.

3 Motion Equations

3.1 Serpenoid curve

Based on the serpenoid curve defined in Refs. [8 − 10], an approximation of the serpenoid curve in terms of successively connected n segments is given. Each segment is a straight line with length $1/n$.

$$x_i = \sum_{k=1}^{i} \frac{1}{n}\cos\left[a\cos\left(\frac{kb}{n}\right) + \frac{kc}{n}\right]$$

$$y_i = \sum_{k=1}^{i} \frac{1}{n}\sin\left[a\cos\left(\frac{kb}{n}\right) + \frac{kc}{n}\right]$$

where the parameter a determines the degree of the undulation, b determines the number of periods in a unit length, and c is the macroscopic circular shape.

Denote the angle of the i-th segment measured from the x-axis in a counterclockwise manner by θ_i. Then it is given by

$$\tan(\theta_i) = \frac{y_i - y_{i-1}}{x_i - x_{i-1}} = \frac{\sin\left[a\cos\left(\frac{ib}{n}\right) + \frac{ic}{n}\right]}{\cos\left[a\cos\left(\frac{ib}{n}\right) + \frac{ic}{n}\right]}$$

and

$$\theta_i = a\cos\frac{ib}{n} + \frac{ic}{n}$$

The relative angles that determine the shape of the discrete serpenoid curve are given by

$$\varphi_i = \theta_i - \theta_{i-1} = -2a\sin\frac{\beta}{2}\sin\left(i\beta - \frac{\beta}{2}\right) - \frac{c}{n}$$

where β is the phase difference and $\beta = b/n$.

3.2 Function of the relative angles

When the snakelike robot advances on the horizontal plane with the inchworm locomotion gait, the parameter c is zero. So the inchworm locomotion can be imitated by changing the relative angles in the following manner:

$$\varphi_i(t) = -\alpha\sin(-\omega t + i\beta) + \gamma \qquad 0 \leqslant t \leqslant \frac{T}{N} \tag{5}$$

where $1 \leqslant i \leqslant n-1$; α and β are the parameters that determine the shape of serpenoid curve r; w specifies how fast the serpentine wave propagates along the ground.

Let $t = 0$, the basic shape of the traveling wave and its initial relative angles are

$$\varphi_i^0 = -\alpha\sin(i\beta) \tag{6}$$

The wave is symmetrical in Fig. 2. Taking Eqs. (1), (2) and (6) into account, the initial absolute angles of the wave θ_1^0 and θ_n^0 are

$$\theta_1^0 = -\theta_n^0 = -\frac{1}{2}\sum_{i=1}^{n-1}\varphi_i^0 \tag{7}$$

Assume that the changing of relative angles at points P_1, P_2, P_n and P_{n+1} follows the same serpenoid law as the relative angle ϕ_i during the propelling process from curve 1 to curve 2. The functional equation of the relative angles on the curve is given as follows. This function satisfies the limitations of Eq. (4).

$$\left.\begin{aligned} &J_1(t) = -\theta_1^0 \sin(\omega_2 t) \\ &J_2(t) = (\theta_1^0 - \varphi_1^0)\sin(\omega_2 t) \\ &J_i(t) = -\varphi_i(t) = \alpha\sin(-\omega_1 t + (i-1)\beta) \quad 3 \leqslant i \leqslant n \\ &J_{n+1}(t) = (\theta_n^0 + \varphi_{n-1}^0)\sin(\omega_2 t) \\ &J_{n+2}(t) = -\theta_n^0 \sin(\omega_2 t) \end{aligned}\right\} \tag{8}$$

where $0 \leqslant \omega_1 t \leqslant \beta$, $0 \leqslant \omega_2 t \leqslant \pi/2$, $\omega_1 = 2\beta\omega_2/\pi$. The functions of the absolute angles between link l_i and the x-axis on the traveling curve are given by

$$\left.\begin{aligned} &\theta_1(t) = \theta_1^0 + J_1(t) \\ &\theta_2(t) = \theta_1(t) - \varphi_1^0 + J_2(t) \\ &\theta_i(t) = \theta_{i-1}(t) + J_i(t) \qquad 3 \leqslant i \leqslant n \\ &\theta_{n+1}(t) = \theta_{n-1}(t) - \theta_n^0 + J_n(t) \end{aligned}\right\} \tag{9}$$

Suppose that l is the unit length. Then the length of link $l_i = 1$. Let $n = 6$, $\beta = 30°$, $\alpha = 30°$, and simulate the motion process of the traveling wave from curve 1 to curve 2. The simulation process is shown in Fig. 3.

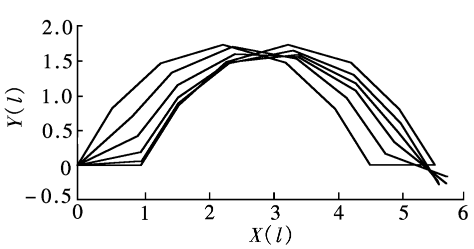

Fig. 3 Motion process from curve 1 to curve 2

Fig. 3 shows that the angular function given by Eq. (8) can make the wave propel along the x-axis smoothly. It also shows that the end point deviates from the perfect position during the propulsion process from curve 1 to curve 2. But at the end of the process it returns back to the perfect position. Define the change in the distance $P_iP_{i+(n+1)}$ as follows:

$$\left.\begin{aligned} X_{\text{error}} &= \sum_{i=1}^{n+1} l_i\cos\theta_i(t) - \sum_{l=1}^{n+1} l_i\cos\theta_i^0 \\ Y_{\text{error}} &= \sum_{i=1}^{n+1} l_i\sin\theta_i(t) \end{aligned}\right\} \tag{10}$$

where X_{error} denotes the bias error of the end point P_{n+1} with its perfect position in the x-axis direction; and Y_{error} denotes the bias error of the end point with its perfect position in the y-axis direction.

It is calculated that the maximum of X_{error} and Y_{error} are $0.21l$ and $-0.3l$, about 1/5 and 3/10 of the length of the link.

The existence of X_{error} and Y_{error} consumes a lot of power and decreases the step pace. It is necessary to reduce the maximum values of X_{error} and Y_{error}.

4 Parameter Choice of Traveling Curve

Eq. (6) shows that α, β and n determine the shape of the traveling curve. The most efficient choices of the three parameters will be given by simulations in this section.

Since the energy loss is a direct ratio to the variation range of distance P_0P_{n+1} during the propulsion process of the snakelike robot, energy loss function f can be expressed by the position deviation as

$$f = \max\left(\sqrt{\left(\frac{X_{\text{error}}}{l}\right)^2 + \left(\frac{Y_{\text{error}}}{l}\right)^2}\right) \tag{11}$$

Suppose that l is the unit length, then the unit displacement of the snakelike robot in one cycle can be written as

$$d_{\text{unit}} = \frac{d_{\text{step}}}{l} = n - \sum_{i=1}^{n}\cos\theta_i^0 \tag{12}$$

Since the snakelike robot advances on the plane surface, the traveling curve must be symmetric and can not exceed half a cyclic curve. The curve must satisfy

$$\varphi_i^0 = \varphi_{n-i}^0, \ 0° < n\beta \leqslant 180°$$

Here let

$$\beta = \frac{180°}{n} \tag{13}$$

From the definition of the serpenoid curve ($c = 0$), there is $\theta_1^0 < 90°$. Substituting Eq. (6) into Eq. (7) and taking the inequality $\theta_1^{\circ} < 90°$, there is

$$\alpha < \frac{180°}{\sum_{i=1}^{n-1}\sin(i\beta)} \tag{14}$$

The inchworm locomotion gait planed with Eq. (8) should have a greater unit displacement d_{unit} and less energy loss f. So f and d_{unit} could be regarded as the efficiency criterions of the gait. While taking the locomotion model into consideration controlling a snakelike robot, the choice of some parameters such as α, β, and n should satisfy the demands of the efficiency criterions.

The following is the discussion of the relationships between objective functions f and d_{unit} and their parameters α and n. Let $n=6$, the quantities f and d_{unit} are computed via simulations for parameter α and plotted in Fig. 4. As shown in the figure, energy loss function f and parameter α are roughly proportional to each other, while the curve of the unit displacement d_{unit} is more like a secondary curve. With the increase in α, f and d_{unit} increase simultaneously; but d_{unit} increases more quickly. When α approaches the maximum value 48. 2°, d_{unit} and f also obtain their own maximum values 3. 35l and 0. 52l.

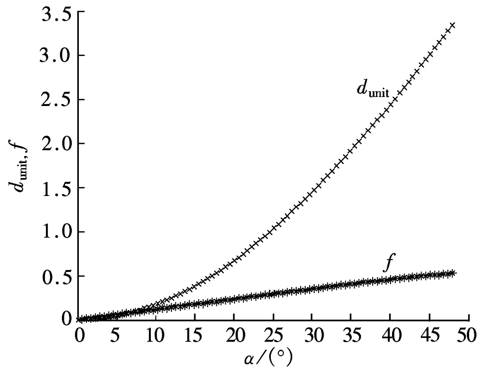

Fig. 4 d_{unit} and f for different values of α ($n=6$)

The same simulations were run with different numbers of links n. For each value of $n=4, 5, \ldots, 11$, the simulation figures are all similar to Fig. 4. Tab. 1 lists the maximum values of f and d_{unit} corresponding to different numbers of links n. It shows that the energy loss function f is insensitive to n; unit displacement d_{unit} is an increasing function of n; whereas the maximum wave amplitude α is a decreasing function of n.

Tab. 1 The maximum values of α, d_{unit} and f corresponding to parameter n

Paramenters	n							
	4	5	6	7	8	9	10	11
$\alpha_{max}/(°)$	74. 6	58. 5	48. 2	41. 1	35. 8	31. 7	28. 5	25. 9
f_{max}	0. 53	0. 52	0. 52	0. 52	0. 53	0. 53	0. 53	0. 54
d_{unit}	2. 40	2. 87	3. 35	3. 79	4. 31	4. 82	5. 25	5. 87

According to the above analysis, there are three ways to improve the inchworm locomotion efficiency of a snakelike robot: ① Reducing the length of link l; ② Choosing the suitable wave amplitude α; ③ Increasing the number of links n which form the serpenoid curve.

5 Experiment

Fig. 5 is the picture of the snakelike robot prototype. The snakelike robot has eight sections. Each section weighs 220 g and its length l is 84 mm.

Fig. 5 Picture of snakelike robot prototype

The snakelike robot can travel forward and backward along the ground. Tab. 2 lists the measured experimental data when it advanced on the wooden floor with different parameters (α, n). In the table, the displacement Δx is the average value of 10 periods of motion. The displacement in theory is calculated by $d=10ld_{unit}$.

Tab. 2 Experimental data

n	$\alpha/(°)$	f	d_{unit}	Δx/mm	d/mm	$(d-\Delta x)$/mm	$\frac{d-\Delta x}{d}$/%
4	30	0. 218	0. 455	332. 5	382. 2	49. 7	13
	45	0. 333	0. 956	711. 7	803. 0	91. 3	11
	58	0. 430	1. 567	1 197. 8	1 316. 3	118. 5	9
5	30	0. 283	0. 858	634. 2	720. 7	86. 5	12
	45	0. 424	1. 824	1 379. 0	1 532. 2	153. 2	10
	58	0. 519	2. 831	2 211. 5	2 378. 0	166. 5	7

Tab. 2 shows that the displacement Δx increases with the value of d_{unit} and the loss of displacement ($d-\Delta x$) also increases with the value of f. The energy loss function f has some influence on the displacement $t\Delta x$, but the influence becomes weaker while increasing α. Besides, the number of links n can obviously increase the displacement Δx. The experimental results are consistent with the simulation results.

6 Conclusion

A new kind of inchworm locomotion gait for the snakelike robot based on a serpenoid curve is introduced. The gait model is suitable for different inchworm locomotions whose traveling waves are built-up by different numbers of identical links n.

In the process of snakelike robot gait control, to get faster movement, n should be at its maximum in the allowable range of variation and α should be as large as it can be in the allowable range of structure.

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蛇形机器人的一种蠕动步态

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摘要：为了建立一个可实际应用于蛇形机器人运动的通用且易于控制的步态，提出了一种基于 serpenoid 曲线的蠕动步态模型. 通过对蠕动运动过程中相邻 2 个波形的分析和对 serpenoid 曲线的近似，建立了运动波形上各相邻连杆间的相对角度运动方程和各连杆与基线之间的绝对角度运动方程. 并给出该步态的 2 个效率判据：能量损失函数 f 和一个周期的单位步长 d_{unit}. 通过仿真和实验讨论了相关的 3 个参数（组成运动波形的连杆数 n、相邻连杆的夹角 α 和相邻夹角的相位差 β）对步态效率的影响. 结果表明：f 基本上不受 n 的影响，增加 n 可显著提高 d_{unit}，α 的最大振幅随着 n 的增大而减小，对于确定的 n，f 对整个位移的影响随着 α 的增大而减弱. 该步态模型可适用于运动波形由不同连杆数组成的蛇形机器人的蠕动，波形连杆较多或连杆夹角较大均可获得较大的运动速度.

关键词：蛇形机器人；多连杆；蠕动步态；效率判据

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