

Statistical condition assessment of existing structures using virtual work error estimator

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Abstract: A statistical damage detection and condition assessment scheme for existing structures is developed. First a virtual work error estimator is defined to express the discrepancy between a real structure and its analytical model, with which a system identification algorithm is derived by using the improved Newton method. In order to investigate its properties in the face of measurement errors, the Monte Carlo method is introduced to simulate the measured data. Based on the identified results, their statistical distributions can be assumed, the status of an existing structure can be statistically evaluated by hypothesis tests. A 5-story, two-bay steel frame is used to carry out numerical simulation studies in detail, and the proposed scheme is proved to be effective.

Key words: virtual work; constitutive parameter; parameter estimation; hypothesis test

The research on structural damage assessment based on system identification (SI) has been expanded recently^[1]. It is classified into two major categories according to the properties of input data: dynamic and static. Both of them have their features. For the dynamic one, the system identification process usually does not occur at the element level, so it is difficult to exactly determine the damage location. Now researchers pay more and more attention to developing damage assessment techniques based on the results of the static system identification^[1-3] which take advantage of static system identification in the assessment of an existing structure's condition at the global and element levels.

The literature on static system identification are relatively fewer than those on the dynamic one. There are two kinds of error definitions to express the discrepancies between the real structure and the analytical model: the force error estimator and the displacement error estimator, with which several methods of structural identification have been proposed. Sanayei et al.^[4] proposed the method on the assumption that the displacements should be measured at the same locations where the external loads were applied. In the paper of Banan and Hjelmstad^[5-6], the unknowns comprised both constitutive parameters and unmeasured displacements. Therefore, the number of unknown variables increased and the stability of the calculating process de-

creased. Sanayei and Onipede^[4] proposed an algorithm in which the unmeasured displacements were condensed, but its limitation was that the degrees of freedom of measured displacements were fixed in all load cases. Although those methods are capable of identifying the parameters of structures, the instability problem in the face of measurement errors still remains.

This paper focuses on improving the identifiability of the SI algorithm in the face of measurement errors. First, a virtual work error estimator (VWEE) is defined to express the difference between the real structure and its analytical model, which can theoretically dissolve some influence of measurement errors itself. The adaptive parameter grouping method is introduced to deal with the measurement sparse problem. Then the recursive quadratic programming algorithm is developed by using the improved Newton method. Finally in the numerical simulation study of a 5-story and 2-bay frame structure, the relationships of the input-output errors are investigated in detail and the structural status is evaluated statistically.

1 System Identification

1.1 Virtual work of error estimators

Consider a linear structure that is variously subjected to a static load set, which consists of n_a load cases. Each case of forces should be neither equal to any other case nor a linear combination of the previous cases of applied forces. Under the k -th load case f_k , the measured displacement vector is Δ_k whose dimension is n_k . If we assume that there be a virtual force vector δf_{1k}

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applied along the directions of measured displacements, their corresponding virtual work V_{1k} can be expressed as

$$V_{1k} = \delta f_{1k}^T \mathbf{\Delta}_k \quad k = 1, 2, \dots, n_a \quad (1)$$

By the finite element method, the structure is parameterized to be an analytical model with n degrees of freedom. The relationship between \mathbf{f}_k and its corresponding displacement \mathbf{u}_k can be described as

$$\mathbf{f}_k = \mathbf{K}(\mathbf{p}) \mathbf{u}_k \quad k = 1, 2, \dots, n_a \quad (2)$$

where the dimensions of \mathbf{f}_k and \mathbf{u}_k are n and $\mathbf{K}(\mathbf{p})$ is the structural stiffness matrix, which can be formulated by the constitutive parameters and the constant matrices of each element,

$$\mathbf{K}(\mathbf{p}) = \sum_{m=1}^{N_m} \sum_{\mu=1}^{M_m} P_{\mu m} \boldsymbol{\lambda}_m \mathbf{B}_m^T \mathbf{D}_{\mu m} \mathbf{B}_m \quad (3)$$

where N_m is the number of the elements, others are messages about element m : M_m is the total number of parameters, $P_{\mu m}$ is the constitutive parameter, $\mathbf{D}_{\mu m}$ is the kernel matrix, $\boldsymbol{\lambda}_m$ is the location matrix, and \mathbf{B}_m is the translative matrix. $\mathbf{B}_m^T \mathbf{D}_{\mu m} \mathbf{B}_m$ is a constant matrix depending on element geometry only. The virtual work of the analytical model V_{2k} is

$$V_{2k} = \delta f_{2k}^T \mathbf{K}(\mathbf{p})^{-1} \mathbf{f}_k \quad k = 1, 2, \dots, n_a \quad (4)$$

where δf_{2k} is the nodal force vector of δf_{1k} .

The discrepancy of the virtual work between the real structure and its analytical model is defined as the index to examine the fitness of the estimated results.

$$E_k(\mathbf{p}) = V_{2k} - V_{1k} = \delta f_{2k}^T \mathbf{K}(\mathbf{p})^{-1} \mathbf{f}_k - \delta f_{1k}^T \mathbf{\Delta}_k \quad k = 1, 2, \dots, n_a \quad (5)$$

If $\mathbf{K}(\mathbf{p})$ exactly captures the properties of the system and the measured data are free from errors, then Eq. (5) will be zero.

1.2 Parameter estimation algorithm

The square of error value is adopted as a criterion of judgment.

$$J(\mathbf{p}) = \sum_{k=1}^{n_a} E_k(\mathbf{p})^2 \quad (6)$$

Now the smaller the $J(\mathbf{p})$, the better accuracy of fitting we obtain. It can be stated as

$$\begin{aligned} & \text{to find } \{p_i, i = 1, 2, \dots, n_b\} \\ & \text{so as to make } J(\mathbf{p}) \rightarrow \min \quad (7) \\ & \text{subject to } \{x_{i1} \leq p_i \leq x_{i2}, i = 1, 2, \dots, n_b\} \end{aligned}$$

It is a nonlinear problem, whose n_b is the number of unknown parameters and x_{i1} and x_{i2} are the lower and upper bounds of the unknown parameters, respectively. They are assumed to be zero and three times the true values, which ensures that the estimated results do not become negative or too large.

To solve this nonlinear optimal problem, we use the improved Newton method^[7] to develop a recursive

quadratic programming algorithm, which requires the gradient (Jacobi vector) and the Hessian matrix of $J(\mathbf{p})$. They are as follows:

Jacobi vector

$$\mathbf{G} = \frac{\partial J(\mathbf{p})}{\partial \mathbf{p}} \quad (8)$$

where the dimension of \mathbf{G} is n_b and its i -th component is expressed as

$$G_i = \frac{\partial J(\mathbf{p})}{\partial p_i} = \sum_{k=1}^{n_a} J_k \quad i = 1, 2, \dots, n_b$$

And

$$J_k = 2(\delta f_{2k}^T \mathbf{K}(\mathbf{p})^{-1} \mathbf{f}_k - \delta f_{1k}^T \mathbf{\Delta}_k) \delta f_{2k}^T \mathbf{K}(\mathbf{p})^{-1} \frac{\partial \mathbf{K}(\mathbf{p})}{\partial p_i} .$$

$$\mathbf{K}(\mathbf{p})^{-1} \mathbf{f}_k = 2E_k(\mathbf{p}) \delta f_{2k}^T \mathbf{K}(\mathbf{p})^{-1} \frac{\partial \mathbf{K}(\mathbf{p})}{\partial p_i} \mathbf{K}(\mathbf{p})^{-1} \mathbf{f}_k \quad (9)$$

Hessian matrix

$$\mathbf{H} = \frac{\partial^2 J(\mathbf{p})}{\partial \mathbf{p} \partial \mathbf{p}} \quad (10)$$

where the dimension of \mathbf{H} is $n_b \times n_b$ and its j -th component in the i -th line is expressed as

$$H_{ij} = \frac{\partial^2 J(\mathbf{p})}{\partial p_i \partial p_j} = \sum_{k=1}^{n_a} H_k \quad i = 1, 2, \dots, n_b; j = 1, 2, \dots, n_b$$

And

$$H_k = 2\delta f_{2k}^T \mathbf{K}(\mathbf{p})^{-1} \frac{\partial \mathbf{K}(\mathbf{p})}{\partial p_j} \mathbf{K}(\mathbf{p})^{-1} \mathbf{f}_k .$$

$$\delta f_{2k}^T \mathbf{K}(\mathbf{p})^{-1} \frac{\partial \mathbf{K}(\mathbf{p})}{\partial p_i} \mathbf{K}(\mathbf{p})^{-1} \mathbf{f}_k + 4E_k(\mathbf{p}) .$$

$$\delta f_{2k}^T \mathbf{K}(\mathbf{p})^{-1} \frac{\partial \mathbf{K}(\mathbf{p})}{\partial p_j} \mathbf{K}(\mathbf{p})^{-1} \frac{\partial \mathbf{K}(\mathbf{p})}{\partial p_i} \mathbf{K}(\mathbf{p})^{-1} \mathbf{f}_k \quad (11)$$

Now the recursive procedure can be set up,

$$\Delta \mathbf{p} = -\mathbf{H}^{-1} \mathbf{G} \quad (12)$$

$$\mathbf{p}^{i+1} = \mathbf{p}^i + \alpha_i \Delta \mathbf{p} \quad (13)$$

where i is the iteration number, and α_i is a damping coefficient matrix to assure that $J(\mathbf{p}^{i+1})$ is smaller than $J(\mathbf{p}^i)$. In order to control the desired accuracy in the identified parameters, two criteria are chosen to check the algorithm for convergence. The first one is the changes of error function $J(\mathbf{p})$, and the second one is the changes in the parameters, p_j^{i+1}/p_j^i , where j is the order number of parameters. When any of the limits are reached, the algorithm is considered to have converged.

1.3 Adaptive parameter grouping algorithm

In order to deal with the measurement sparse problem, the adaptive parameter group subdivision algorithm proposed by Hjelmstad and Shin^[8] is introduced. The calculation starts from a baseline grouping. The main idea of the scheme is to isolate damaged parts in the finite element model by sequentially subdividing parameter groups. Each subdivision stage corre-

sponds to a specific parameter grouping. So after each subdivision stage, a new set of parameter groups is established and the group parameters are estimated. By subdividing a suspicious parameter group, parameters become more sensitive and more representative of the real values. In this process, the parameterized stiffness matrix, shown in Eq. (3), is rewritten as

$$\mathbf{K}(\mathbf{p}) = \sum_{n=1}^N \sum_{m \in \Omega_n} \sum_{\mu=1}^{M_m} p_{\mu m} \boldsymbol{\lambda}_m \mathbf{B}_m^T \mathbf{D}_{\mu m} \mathbf{B}_m \quad (14)$$

where N is the number of parameter groups and the index set Ω_n contains all of the element numbers associated with parameter group n . The subdivision of parameter groups implies that the number of groups and, hence, the number of parameters are variables in the parameterization model.

2 Data Perturbation Method

If the measured data were free of error, a single cycle of calculation by the above algorithm would be enough to track damage out. Measurements are, unfortunately, never free from error. There are also model errors between the real structure and the analytical model, such as manufacturing inconsistencies, residual or thermal stresses, or material flaws, which are not the topic here. We only deal with the errors of measurements, which cause the result errors. Even though the experiment is repeated under identical conditions, the measured data are random variables with a certain distribution, so the estimated parameters based on them are also considered as random variables. Here, we use the Monte Carlo method to simulate the input data and investigate the properties of the above proposed algorithm.

2.1 Modeling of input error

The input data consist of force vectors and displacement vectors. In one load case, let only one freedom be applied by a force and all other freedoms be zero. So the force vector can be assumed to consist of no errors and the displacement vector contains noise. We can simulate the measured data $\boldsymbol{\Delta}$ by adding an error vector \mathbf{n} with the assumed distribution in the calculated displacement values^[3].

$$\boldsymbol{\Delta}_k = \mathbf{Q}\mathbf{K}(\mathbf{p})^{-1}\mathbf{f}_k + \mathbf{n} \quad (15)$$

where $\mathbf{K}(\mathbf{p})$ is the true stiffness matrix of the existing structure and \mathbf{Q} is the Boolean matrix that extracts the measured responses from the complete displacement vector.

The most commonly used distribution of \mathbf{n} is a normal one with zero mean and standard deviation σ . If we set the confidence interval at 95%, the level of in-

put error I_e is equal to 1.96σ . That is

$$\sigma^2 = \frac{I_e^2}{1.96^2} \approx \frac{I_e^2}{4} \quad (16)$$

In this way, the error function Eq. (5) can be described as

$$E_k(\mathbf{p}) = E_0(\mathbf{p}) - \sum_{i=1}^{n_k} n_i \quad (17)$$

where $E_0(\mathbf{p})$ is the error function with no measurement errors, n_i is the i -th component of the noisy vector. It is considered that with the increase of n_k , the influence of those errors decreases because the mean of \mathbf{n} is zero.

2.2 Statistical indices

For the simulated measured data $\{\boldsymbol{\Delta}_k, k = 1, 2, \dots, n_c\}$, the estimated results produce a sample $\{p_{j,k}, k = 1, 2, \dots, n_c; j = 1, 2, \dots, n_b\}$, where n_c is the number of observations and $p_{j,k}$ is the j -th parameter of the results from the k -th observation. So the sample size of every variable is n_c . The statistical indices are used to characterize our results. Taking p_{ej} as the intact value of the j -th parameter, the intact rate of the j -th parameter in the k -th observation is

$$E_{j,k} = \frac{p_{j,k}}{p_{ej}} \quad (18)$$

The mean of the precise rate M_j and the standard deviation D_j of the j -th parameter are

$$M_j = \frac{1}{n_c} \sum_{k=1}^{n_c} E_{j,k} \quad (19)$$

$$D_j = 100 \sqrt{\frac{1}{n_c} \sum_{k=1}^{n_c} (E_{j,k} - M_j)^2} \quad (20)$$

The whole sample size of estimated parameters is $n_b \times n_c$. It is desirable to reduce this large number to single grand mean G_M and single standard deviation G_{SD} .

$$G_M = \frac{1}{n_b} \sum_{j=1}^{n_b} M_j \quad (21)$$

$$G_{SD} = \sqrt{\frac{1}{n_b} \sum_{j=1}^{n_b} D_j^2} \quad (22)$$

Now it is possible to establish an input-output error relationship with a given I_e , from which single values of G_M and G_{SD} are obtained.

2.3 Damage assessment using hypothesis test

With the sample $\{p_{j,k}, k = 1, 2, \dots, n_c; j = 1, 2, \dots, n_b\}$, the statistical distribution of $p_{j,k}$ can be determined by the method of maximum likelihood^[3]. Suppose that the measurements are obtained under the same conditions for both the current structure and its perfect one, so its statistical distributions $N_b(1, \sigma^2)$ of system parameters can be reasonably assumed to be the same as

those of the current structure. It is defined as the baseline distribution and the mean $E_{j,k}$ is equal to 1, which represents its intact status. Those distributions can be taken as the properties of the proposed identification algorithm in the face of measurement errors. If we set the significance level at α , the hypothesis test definition can be defined, which is the same as the one in Ref. [3]. The damage status of a member in the current structure can be evaluated with $(1 - \alpha) \times 100\%$ confidence. The severity of damage S_D can be calculated^[3].

3 Numerical Simulation Study

The purpose of this simulation study is twofold. First, it clearly shows the meaning of some of the quantities that have been defined. Secondly, by simulating the sample of measured data, the performance of the algorithm can be investigated with the measurement errors taken into consideration.

Consider that a 5-story, two-bay steel frame shown in Fig. 1 is used as an example. The frame is divided into 25 frame elements. Each joint node has three degrees of freedom. Elements 1 to 15 make up the columns, and elements 16 to 25 make up the beams of the frame. The cross-sectional areas and the moment of inertia of elements are listed in Tab. 1 and the elastic modulus of every element of the structure is assumed as follows:

- 1) Undamaged structure
Young's modulus for all elements is 206.8 GPa.
- 2) Current structure (or real structure)

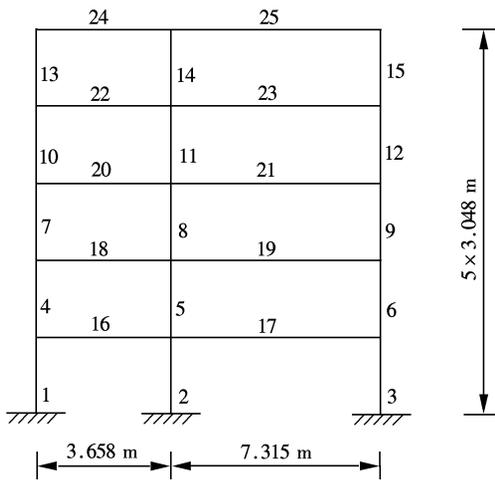


Fig. 1 A 5-story, two-bay steel frame

Tab. 1 Cross sectional properties

Member	Area/cm ²	Moment of inertia/cm ⁴
1 to 15	1 606	442 246
16 to 21	1 606	442 246
22 to 25	1 406	422 246

Damage in the structure is assumed as a reduction in the Young's modulus of element, details of which will be stated for different cases of the study.

The applied load set and the corresponding measured situations are shown in Fig. 2. There is only one load case in this study. In order to examine the results affected by the number of measurements, two cases of measured situations are used. We set $n_c = 30$. The number of parameter groups is set as not greater than 6.

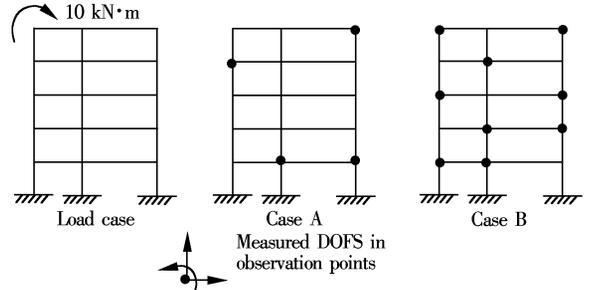


Fig. 2 Applied load and two cases of measurement state

3.1 One damaged member

The element 13 is damaged with a 60% reduction in Young's modulus. All of the other elements are considered to be intact. The input error I_e is from 1% to 10% and the measured data are generated by Eq. (16). Fig. 3 and Fig. 4 show the relationship between the I_e and the single grand mean and standard deviation. From the two figures it can be clearly seen that the results from case B are better than the ones from case A.

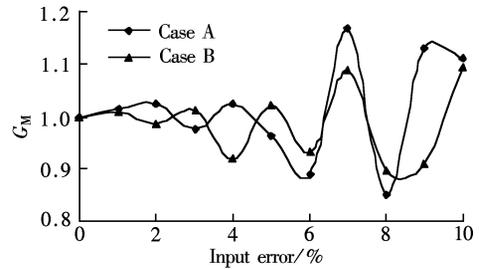


Fig. 3 Variations of single grand mean with input error

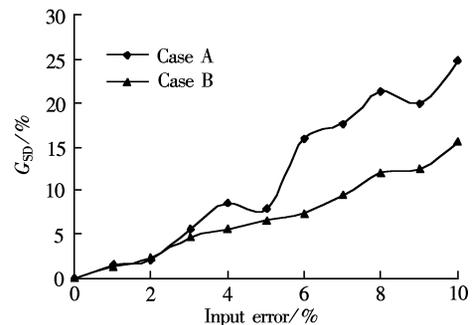


Fig. 4 Variations of single grand standard deviation with input error

If there is only one set of measured data in a practical application, only one sample of estimated results can be obtained. Based on the results as shown in Fig. 4, it is assumed that $\sigma = 8\%$ and 7% in the baseline

distribution $N_b(1, \sigma^2)$ in the face of 5% measurement error for case A and case B, respectively. Then, if we set $\alpha = 5\%$, we obtain $K_\alpha = 1.65$ and $C = 0.87, 0.89$, respectively. One sample is taken from 30 estimated samples. By hypothesis tests, the damaged member is identified as damaged and two undamaged members are assessed as damaged members. Therefore, the status of the current structure of all members is evaluated and the results are plotted in Fig. 5. Since the damage severities of other undamaged members are small compared with those of the damaged member, it is concluded that there are no damage in those members. Those results have 95% confidence.

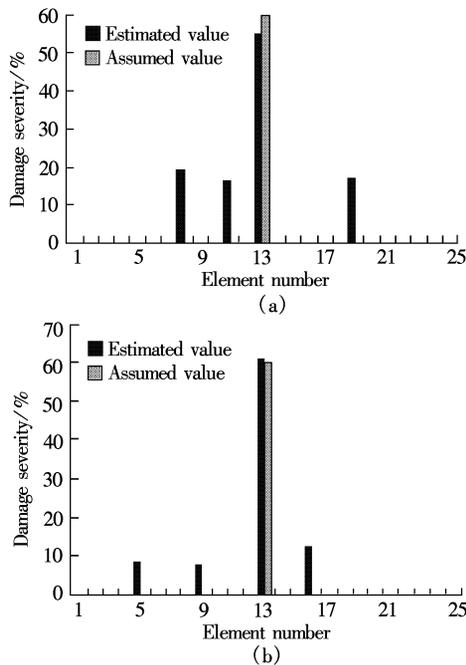


Fig. 5 Statistical status evaluation. (a) Case A; (b) Case B

3.2 Multiple damaged members

Let us consider the situation of the existing structure in which three members are damaged to different degrees. The stiffness deterioration is 50%, 70%, 25% in members 5, 10 and 21, respectively. All of other elements are considered to be intact. The estimated results of all elements, averaged over 30 Monte Carlo trials and 25 elements, the single grand mean and the single grand standard deviation can be calculated in the face of measurement errors from 1% to 10%. To examine the input-output error relationship, it is desirable to plot the G_M and G_{SD} values against the I_e values, which are shown in Fig. 6 and Fig. 7, respectively. They are used to estimate the output error for a given input error, and they can also be used to determine the allowable I_e by limiting output error for the experiment design. The

measurement noise tolerance is expected to vary from structure to structure based on the locations of measurements and the topology of the structure. Even if the results depend on cases, we can determine the stability of the algorithm approximately based on Fig. 6 and Fig. 7. From them it can be considered that the results from case B are better than the results from case A.

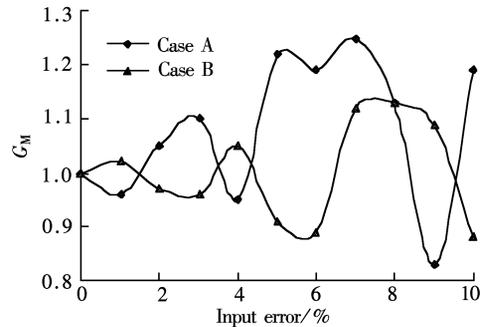


Fig. 6 Variations of single grand mean with input error

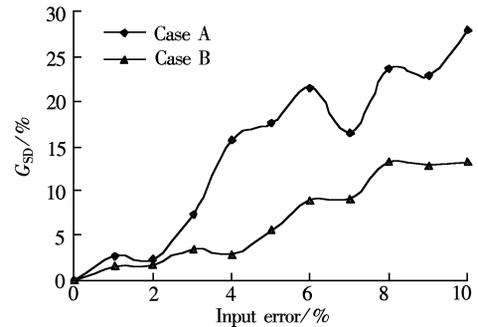


Fig. 7 Variations of single grand standard deviation with input error

Based on the results as shown in Fig. 7, it is reasonably assumed that $\sigma = 15\%$ and 4% in the baseline distribution $N_b(1, \sigma^2)$ in the face of a 4% measurement error for case A and case B, respectively. Then if we set $\alpha = 5\%$, we obtain $K_\alpha = 1.65$ and $C = 0.76$ and 0.93.

By using one sample from the estimated result, the status of the current structure is presented in Fig. 8. By hypothesis tests, three damaged members are identified as damaged and three undamaged members are also taken as damaged. Since the damage severity of the undamaged members is small compared with that of the damaged members, it is concluded that there is little possibility of damage in the undamaged members. Those results have 95% confidence.

4 Conclusion

Local damage not detected and not rectified may cause more damage and eventually lead to structural failure. This paper proposes the virtual work error estimator and develops the statistical evaluation scheme of

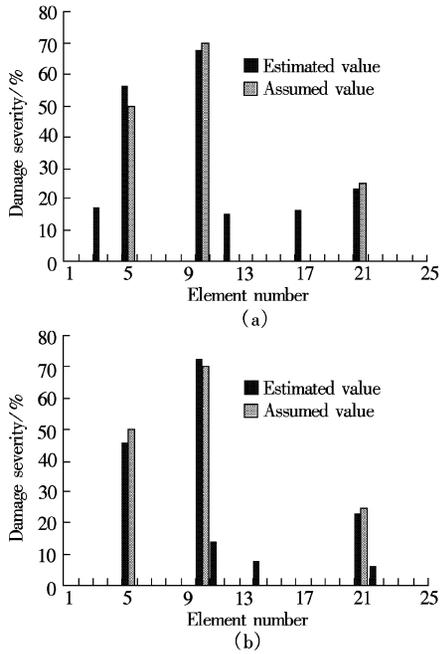


Fig. 8 Statistical status evaluation. (a) Case A; (b) Case B

an existing structure. The procedure is illustrated and tested using the Monte Carlo simulation method in numerical simulations.

In this paper, the expected result is theoretically verified that more measurement locations are better than fewer even though there are measurement errors as Eq. (18) shows and is proved in numerical simulations. It can be considered that the algorithm based on the proposed virtual work error estimator is an effective tool in evaluating the structural status^[9-10]. Future research is required to apply this method to experimental work.

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基于虚功误差估计算子服役结构状态评估的统计分析

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摘要: 针对服役结构状态评估问题提出了基于虚功误差估计算子的统计分析方法。首先定义虚功误差来表达实际结构与参数化分析模型之间的差别, 然后采用改进的牛顿算法推导分析模型结构识别算法。为了探讨在有测量误差的情况下算法的性能, 引用 Monte Carlo 方法模拟测量数据, 对测量数据的误差与识别结果的影响进行了详细的分析比较。根据识别结果确定它的概率分布, 通过假设检验对服役结构状态评估进行统计分析。最后应用双跨五层刚架结构进行了大量数值模拟, 计算结果显示了所提方法的有效性。

关键词: 虚功; 基本参数; 参数估计; 假设检验

中图分类号: TU317