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Modal identification to non-stationary random excitation based on wavelet transform and co-integration theory

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Abstract: A kind of method of modal identification subject to ambient excitation is presented. A new synthesis stationary signal based on structural response wavelet transform and wavelet coefficient processes co-integration is obtained. The new signal instead of structural response is used in identifying the modal parameters of a non-stationary system, combined with the method of modal identification under stationary random excitation—the NExT method and the adjusted continuous least square method. The numerical results show that the method can eliminate the non-stationarity of structural response subject to non-stationary random excitation to a great extent, and is highly precise and robust.

Key words: modal identification; wavelet transform; non-stationary random excitation; co-integration; NExT method

Modal identification of engineering structures has wide application prospects. The identified modal parameters can be used to estimate and locate structural damage, and forecast structural future response; therefore, it can be used in structural health monitoring of bridges and large buildings. The traditional models and methods on modal identification, which rely only on structural response, are based on stationary exterior random excitation (especially white noise). In fact, random loads, such as wind load and vehicle load which act on bridges and large buildings, are almost non-stationary. Because of lack of cognition and understanding of non-stationary random excitation, the relevant research results in the literature are rather few now. The primary results are classified into the following categories: ① Identifying modal parameters by using the ARIMA model and the VARMA process which sample output response as a time series[1-2]; ② Identifying modes based on the autocorrelation and cross-correlation cohen time-frequency transform of structural response[3-4]; analogously, Zhang et al.[5] expanded structural response through Gabor transform, then identified modal parameters just as in Ref. [3]; ③ Restructuring signal by wavelet multi-scale denoising[6]; ④ In Refs. [7 – 8], authors first decomposed random excitation into white noise and non-white noise, and computed the cross-correlation function of response, then identified modal parameters based on modal functions and residual of the EMD method.

Based on the characteristics of wavelet transform, this paper puts forward a new modal identification method for a system under non-stationary random excitation. The key idea is to construct a new stationary signal based on the wavelet coefficients of non-stationary response and co-integration theory, then to identify modal parameters through the stationary signal, combined with the method of modal identification under stationary random excitation—the NExT method and the adjusted continuous least square method. The new identification method can be applied in all non-stationary linear systems.

1 NExT[9] Method under Stationary Excitation and Adjusted Continuous LS Method

The n-dimensional structural dynamic equation is given by

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{f}(t) \tag{1}$$

where $\boldsymbol{x}(t)=\{x_1(t),\ \dots,\ x_n(t)\}^{\mathrm{T}}$ is an n-dimensional displacement vector; $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are mass, damping and stiffness matrices, respectively; $\dot{\boldsymbol{x}}(t)$ and $\ddot{\boldsymbol{x}}(t)$ are velocity and acceleration vectors; $\boldsymbol{f}(t)$ denotes stationary exterior random excitation.

The NExT method is described as follows.

Post-multiplying Eq. (1) by a reference scalar response process $x_i(s)$ $(s<t)$, and taking expectations of each side, let $\tau=t-s$, then we obtain

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$$\boldsymbol{M\ddot{R}}(\tau) + \boldsymbol{C\dot{R}}(\tau) + \boldsymbol{KR}(\tau) = \boldsymbol{0} \tag{2}$$

where $\boldsymbol{R}(\tau) = E[\boldsymbol{x}(t)\ x_i(t-\tau)] = \{R_1(\tau), \ldots, R_n(\tau)\}^{\mathrm{T}}$.

The adjusted continuous least square method of modal identification, which was based on the continuous least square method in Refs. [9 – 10], has high precision both in theory and in practice. In order to illustrate the method, an undamped architecture under non-stationary random excitation is discussed. Assuming that the mass matrix of the structure is known, the stiffness matrix $\boldsymbol{K}$ is as follows[11]:

$$\boldsymbol{K} = \begin{bmatrix} K_{x_1}+K_{x_2} & -K_{x_2} & & & \\ -K_{x_2} & K_{x_2}+K_{x_3} & -K_{x_3} & & \\ & & \ddots & & \\ & & -K_{x_{n-1}} & K_{x_{n-1}}+K_{x_n} & -K_{x_n} \\ & & & -K_{x_n} & K_{x_n} \end{bmatrix}$$

In order to estimate the stiffness coefficients $\boldsymbol{K}_X = \{K_{x_1}, \ldots, K_{x_n}\}^{\mathrm{T}}$, we rewrite Eq. (2) in the form

$$\boldsymbol{A}(\tau)\boldsymbol{K}_X = -\boldsymbol{M\ddot{R}}(\tau) \tag{3}$$

where

$$\boldsymbol{A}(\tau) = \begin{bmatrix} R_1(\tau) & R_1(\tau)-R_2(\tau) & & \\ & R_2(\tau)-R_1(\tau) & R_2(\tau)-R_3(\tau) & \\ & & \ddots & \\ & & & R_{n-1}(\tau)-R_n(\tau) \\ & & & R_n(\tau)-R_{n-1}(\tau) \end{bmatrix}$$

Sampling from $\boldsymbol{R}(\tau)$ and substituting the sample data of size N into Eq. (3) yields

$$\boldsymbol{AK}_X = \boldsymbol{R}_{\mathrm{d}} \tag{4}$$

where $\boldsymbol{A} = \{(\boldsymbol{A}(h))^{\mathrm{T}}, \ldots, (\boldsymbol{A}(Nh))^{\mathrm{T}}\}^{\mathrm{T}}$; $\boldsymbol{R}_{\mathrm{d}} = -\{(\boldsymbol{M\ddot{R}}(h))^{\mathrm{T}}, \ldots, (\boldsymbol{M\ddot{R}}(Nh))^{\mathrm{T}}\}^{\mathrm{T}}$; h is the sampling period. Then the LSE of $\boldsymbol{K}_X$ is given by

$$\boldsymbol{K}_X = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{R}_{\mathrm{d}} \tag{5}$$

2 Modal Identification Method under Non-stationary Random Excitation

2.1 Wavelet transform

Assume that the following system is an LTI system under non-stationary random excitation, which is due to various ambient excitations out of control. Therefore, the system's structural response is also non-stationary because of the linearity of the system.

The n-dimensional structural dynamic equation is given by

$$\boldsymbol{M\ddot{x}}(t) + \boldsymbol{C\dot{x}}(t) + \boldsymbol{Kx}(t) = \boldsymbol{u}(t) \tag{6}$$

where $\boldsymbol{x}(t)$, $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the same as in section 1, and $\boldsymbol{u}(t)$ is non-stationary exterior excitation. Let $W_{\mathrm{f}i}(b, a)$ be the continuous wavelet coefficient of $x_i(t)$, $W_{\mathrm{f}i}(b,a) = \int_{\mathbf{R}} x_i(t)\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)\mathrm{d}t$, where $\psi(t)$ is a given mother wavelet and satisfies the admissibility condition; $a > 0$ is the dilation scale; $b \in \mathbf{R}$ is the translation scale. Let $\boldsymbol{W}_{\mathrm{f}}(b,a) = \{\boldsymbol{W}_{\mathrm{f}1}(b,a), \ldots, \boldsymbol{W}_{\mathrm{f}n}(b,a)\}^{\mathrm{T}}$, then one can obtain the following theorems.

Theorem 1[12] If function $\psi(t)$ is at least two-order differentiable, then $\boldsymbol{W}_{\mathrm{f}}(b,a)$ satisfies

$$\boldsymbol{M}\frac{\mathrm{d}^2\boldsymbol{W}_{\mathrm{f}}(b,a)}{\mathrm{d}b^2} + \boldsymbol{C}\frac{\mathrm{d}\boldsymbol{W}_{\mathrm{f}}(b,a)}{\mathrm{d}b} + \boldsymbol{KW}_{\mathrm{f}}(b,a) = \boldsymbol{s}(b,a) \tag{7}$$

where $\boldsymbol{s}(b,a) = \int_{\mathbf{R}} \boldsymbol{u}(t)\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)\mathrm{d}t$ is the wavelet transform of $\boldsymbol{u}(t)$.

Generally, discrete wavelet transform is adopted for computing concision. Applying dyadic wavelet transform $\boldsymbol{W}_{\mathrm{f}}(b,m) = \int_{\mathbf{R}} \boldsymbol{x}(t)2^{-\frac{m}{2}}\psi\left(\frac{t-b}{2^m}\right)\mathrm{d}t$ ($m \in \mathbf{Z}$) to Eq. (7) yields

$$\boldsymbol{M}\frac{\mathrm{d}^2\boldsymbol{W}_{\mathrm{f}}(b,m)}{\mathrm{d}b^2} + \boldsymbol{C}\frac{\mathrm{d}\boldsymbol{W}_{\mathrm{f}}(b,m)}{\mathrm{d}b} + \boldsymbol{KW}_{\mathrm{f}}(b,m) = \boldsymbol{s}(b,m) \tag{8}$$

Theorem 2[12] If $\psi(t)$ is just as in theorem 1 and $\{\boldsymbol{W}_{\mathrm{f}}(b,m): b \in \mathbf{R}\}$ is the m-th scale wavelet coefficient process of $\boldsymbol{x}(t)$, then linear combination $\sum_m \alpha_m \boldsymbol{W}_{\mathrm{f}}(b,m)$ satisfies Eq. (8).

2.2 Co-integration theory

Wavelet transform decomposes a signal through time-frequency analysis. When a signal is non-stationary, different scale wavelet coefficient processes are also generally non-stationary. However, the present better methods of ambient vibration are mainly based on stationary exterior random excitation. If we can transform non-stationary response into a stationary signal, the modal identification methods under stationary excitation can then be used. Here we introduce co-integration theory first.

Co-integration describes the long-term balance relation of the system, particularly with the economic system. In detail, it describes the balance relation of not fewer than two non-stationary processes. Although every process can be non-stationary, a particular linear combination of these processes can be stationary.

2.3 Modal identification

On the basis of the above results, in order to eliminate the influence of non-stationary excitation on response, we decompose $\boldsymbol{x}(t)$ by wavelet transform and construct linear combination $\sum_m \alpha_m \boldsymbol{W}_{\mathrm{f}}(b,m)$. If the linear combination is tested as stationary, we can replace structural response $\boldsymbol{x}(t)$ with it to identify modal parameters. However, the combined new signal should include original responses as much as possible. In order to realize it, we choose wavelet coefficient processes from the point of view of energy, ensuring that the difference between the linear combination of wavelet coefficient processes and $\boldsymbol{x}(t)$ is as small as possible, and the linear combination is stationary.

The whole procedure is as follows:

① Decomposing response $\boldsymbol{x}(t)$ by wavelet transform, let $\{\boldsymbol{W}_{\mathrm{f}}(b,m): b\in\mathbf{R}\}$ be the m-scale wavelet coefficient process of $\boldsymbol{x}(t)$.

② Choose wavelet coefficient processes to construct a new signal, and ensure that $\left|\int\sum_m \|\boldsymbol{W}_{\mathrm{f}}(b,m)\|^2\mathrm{d}b - \int\|\boldsymbol{x}(t)\|^2\mathrm{d}t\right|$ is controlled within a given range. Assume that the chosen wavelet coefficients are $\{\boldsymbol{W}_{\mathrm{f}}(b,k_1): b\in\mathbf{R}\}$, $\{\boldsymbol{W}_{\mathrm{f}}(b,k_2): b\in\mathbf{R}\}$, …, $\{\boldsymbol{W}_{\mathrm{f}}(b,k_l): b\in\mathbf{R}\}$.

③ Construct linear combination $\boldsymbol{y}(b) = \sum_{i=1}^{l}\alpha_i\boldsymbol{W}_{\mathrm{f}}(b,k_i)$, where linear coefficients $\alpha_m(m=1,2,\dots,l)$ can be obtained by the LSE method. Let $\boldsymbol{v}(b) = \sum_{i=1}^{l}\alpha_i\boldsymbol{s}(b,k_i)$. By theorem 2 we know that $\boldsymbol{y}(b)$ satisfies

$$\boldsymbol{M}\ddot{\boldsymbol{y}}(b)+\boldsymbol{C}\dot{\boldsymbol{y}}(b)+\boldsymbol{K}\boldsymbol{y}(b)=\boldsymbol{v}(b) \tag{9}$$

④ If the stationarity of the linear combination can be tested, we replace original response $\boldsymbol{x}(t)$ with it to identify modal parameters just as in section 1.

2.4 Choosing mother wavelet

Whether the mother wavelet is suitable for analysis will directly influence the precision and robustness of modal identification. Therefore, before we decompose $\boldsymbol{x}(t)$ by wavelet transform, we should first know about some properties of $\boldsymbol{x}(t)$, such as the extent of non-stationarity and symmetry, then choose mother wavelet combining with the properties of wavelet, such as orthogonality, vanishing moments, regularity and symmetry.

3 Numerical Simulations

Undamped architecture under non-stationary random excitation is discussed in this paper. Our goal is to identify the stiffness matrix by structural response under the condition that the mass matrix of structure is known. We validate the feasibility and rationality of the above method. The following discussed is an ideal architecture of a three-floor frame structure. The ideal architecture is formed under the assumption that each of the floors is perfectly rigid and braces are massless and restricted to translation in a plane parallel to the floors, and it has the following characteristics:

1) The mass center of each floor does not locate in the principal axis of the structure;

2) The stiffnesses of x and y directions are K_{x_i} and K_{y_i} respectively, and their values may be different for each floor.

For the i-th ($i=1,2,3$) floor, the displacements in x and y directions are x_i and y_i respectively. The corresponding motion equation is

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{u}(t) \tag{10}$$

where $\boldsymbol{x}(t)=\{x_1(t),y_1(t),x_2(t),y_2(t),x_3(t),y_3(t)\}^{\mathrm{T}}$ is a 6-dimensional displacement vector; $\boldsymbol{M}=\mathrm{diag}(m_1,m_1,m_2,m_2,m_3,m_3)$; $\boldsymbol{u}(t)$ is non-stationary random excitation; $\boldsymbol{K}$ is as follows[11]:

$$\boldsymbol{K}=\begin{bmatrix} K_{x_1}+K_{x_2} & 0 & -K_{x_2} & 0 & 0 & 0\\ 0 & K_{y_1}+K_{y_2} & 0 & -K_{y_2} & 0 & 0\\ -K_{x_2} & 0 & K_{x_2}+K_{x_3} & 0 & -K_{x_3} & 0\\ 0 & -K_{y_2} & 0 & K_{y_2}+K_{y_3} & 0 & -K_{y_3}\\ 0 & 0 & -K_{x_3} & 0 & K_{x_3} & 0\\ 0 & 0 & 0 & -K_{y_3} & 0 & K_{y_3}\end{bmatrix}$$

Assume that white noise plus Ramp signal and periodic impulse excitation is used as non-stationary excitation, where the mean of white noise is 0, and the variance is 100 N^2; the slope of Ramp signal and the amplitude of periodic impulse excitation are 1 000 N, and the sampling frequency is 1 000 Hz; simulation time is 30 s. Down sampling to 100 Hz, Fig. 1 shows the structural response $x_1(t)$. Obviously, it is non-stationary.

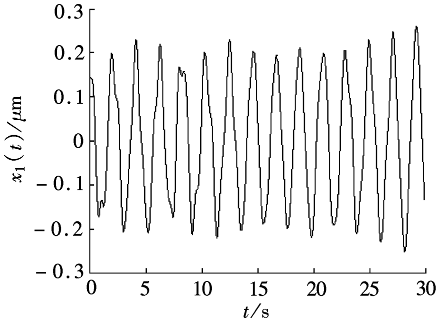

Fig. 1 Structural response under non-stationary excitation

After analyzing the response, we use "bior" and "db" wavelets, respectively. We adopts the 3rd to the 16th scale wavelet coefficient processes after comparing the energy to construct a new stationary signal. Because big scale means low frequency and structural natural frequencies are generally small, we adopt the 16th scale wavelet coefficient process as a dependent variable, and other scale wavelet coefficient processes as independent variables in regression analysis to construct $\boldsymbol{y}(b)$. After testing the stationarity of $\boldsymbol{y}(b)$, post-multiplying $\boldsymbol{y}(b)$ by the 3rd floor response and taking expectation we obtain a cross-correlation function (to ensure that all of the modes are observed). Finally, the method in section 1 is used to identify the stiffness matrix. Tab. 1 shows the theoretical and computed values when "db8" wavelet is used. The analogous result is obtained with "bior" wavelet.

Tab. 1 Theoretical and computed values of physical parameters of 3-floor frame structure

Floor	$M/(10^7\,\text{kg})$	Theoretical value		Computed value	
		$K_{x_i}/(10^8\,\text{GN}\cdot\text{m}^{-1})$	$K_{y_i}/(10^8\,\text{GN}\cdot\text{m}^{-1})$	$K_{x_i}/(10^8\,\text{GN}\cdot\text{m}^{-1})$	$K_{y_i}/(10^8\,\text{GN}\cdot\text{m}^{-1})$
1	2.0	0.90	0.85	0.864 40	0.828 09
2	1.9	0.85	0.75	0.819 34	0.730 71
3	1.9	0.85	0.75	0.835 10	0.726 16

During simulation, we also adjust the slope of Ramp and the amplitude of periodic impulse excitation time after time. The results indicate that when amplitude changes in a given range, the estimated value of modal parameters hardly change. This suggests that the new method is robust in identifying modal parameters.

In Ref. [12], we once presented a kind of modal identification method based on wavelet transform. Compared with this paper, the method in Ref. [12] has two flaws. One is that we must test all scale wavelet coefficient processes in order to choose stationary ones, which increases estimation errors to some extent; the other is that many wavelet coefficient processes can be relatively stationary, so different choices may result in the volatility of the estimator. This paper just overcomes these problems by use of co-integration theory.

4 Conclusion

We can conclude from the above analysis that the method of wavelet transform and co-integration theory may identify the modal parameters of non-stationary LTI system only through structural response, and if mother wavelet is chosen as suitable, a highly precise (error is less than 5%) and robust estimator can be acquired. We can also conclude that the method can apply to the situation of stationary excitation. It occurs to us that we can identify mode directly according to structural response whether or not the excitation is stationary or non-stationary.

References

[1] Yang Shuzi, Wu Ya. *Engineering application of time series analysis*[M]. Wuhan: Huazhong University of Science and Technology Press, 1991. (in Chinese)

[2] Larbi N, Lardies J. Experimental modal analysis of a structure excited by a random force[J]. *Mechanical Systems and Signal Processing*, 2000, **14**(2): 181 − 192.

[3] Ueng Jinmin, Lin Chichang, Lin Paolung. System identification of torsionally coupled buildings[J]. *Computers and Structures*, 2000, **74**(6): 667 − 686.

[4] Bonato P, Ceravold R, de Stefano A, et al. Use of cross-time-frequency estimators for structural identification in non-stationary conditions and under unknown excitation[J]. *Journal of Sound and Vibration*, 2000, **237**(5): 775 − 791.

[5] Zhang Z Y, Hua H X, Xu X Z, et al. Modal parameter identification through Gabor expansion of response signals[J]. *Journal of Sound and Vibration*, 2003, **266**(5): 943 − 955.

[6] Donoho D L. De-noising by soft-thresholding[J]. *IEEE Transactions on Information Theory*, 1995, **41**(3): 613 − 627.

[7] Li Zhongfu, Hua Hongxing, Song Hanwen, et al. Modal parameters identification of linear structures undergoing non-stationary ambient excitation [J]. *Journal of Vibration Engineering*, 2002, **15**(2): 139 − 143. (in Chinese)

[8] Xu Xiuzhong, Hua Hongxing, Li Zhongfu, et al. Identification of modal parameters with linear structure under non-stationary ambient excitation[J]. *Journal of Donghua University: English Edition*, 2004, **21**(1): 146 − 151.

[9] Cailedo Juan Martin, Dyke Shirley J, Johnson Erik A. Natural excitation technique and eigensystem realization algorithm for phase 1 of the IASC-ASCE benchmark problem: simulated data[J]. *Journal of Engineering Mechanics*, 2004, **130**(1): 49 − 61.

[10] Yao Zhiyuan. Method of identification of structural physical parameters based on continuous time model[J]. *Journal of Southeast University: Natural Science Edition*, 2003, **33**(5): 617 − 620. (in Chinese)

[11] Ventura C E, Liam Finn W D, Lord J F. Dynamic characteristics of a base isolated building from ambient vibration measurements and low level earthquake shaking [J]. *Soil Dynamics and Earthquake Engineering*, 2003, **23**(4): 313 − 322.

[12] Du Xiuli, Wang Fengquan. Method of identification based on wavelet transform in LTI system under non-stationary random excitation [J]. *Journal of Southeast University: Natural Science Edition*, 2006, **36**(2): 319 − 321. (in Chinese)

非平稳随机激励下结构模态识别的小波协整方法

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摘要:提出一种环境激励下的模态识别方法. 具体方法是通过对非平稳随机激励下的线性时不变系统的结构响应进行小波变换,对各级小波系数利用协整理论进行线性合成得到新的信号. 若此信号是平稳的,则以它代替原始的结构响应,然后结合平稳随机激励下的模态识别方法——NExT 方法和修正的连续最小二乘法,只通过输出信号就可以实现对非平稳随机激励下系统模态参数的识别. 仿真结果表明,该方法可以极大程度上消除非平稳随机激励所引起的结构响应的非平稳性,而且具有很高的精度和稳健性.

关键词:模态识别;小波变换;非平稳随机激励;协整理论;NExT 方法
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