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Application of uncertain type of AHP to condition assessment of cable-stayed bridges

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Abstract: In order to guarantee the safety service and life-span of long-span cable-stayed bridges, the uncertain type of analytic hierarchy process (AHP) method is adopted to access the bridge condition. The correlative theory and applied objects of uncertain type of AHP are introduced, and then the optimal transitive matrix method is chosen to calculate the interval number judgment matrix, which makes the weights of indices more reliable and accurate. Finally, with Harbin Songhua River Cable-Stayed Bridge as an example, an index system and an assessment model are proposed for the condition assessment of this bridge, and by using uncertain type of AHP, the weights of assessment indices are fixed and the final assessment results of the bridge are calculated, which proves the feasibility and practicability of this method. The application of this assessment method can provide the scientific basis for maintenance and management of long-span cable-stayed bridges.

Key words: cable-stayed bridge; condition assessment; uncertain type of analytic hierarchy process; interval number judgment matrix

Long-span bridges, an important part of the country's infrastructure, play a pivotal role in communications and maintain the lifelines of the national economy. With the increasingly growing number of long-span cable-stayed bridges, people are more concerned with the safety, durability and applicability of these bridges. The present *Code for Maintenance of Highway Bridges and Culvers* (JTG H11—2004)[1] is mainly for medium and mini-sized span beam bridges and arch bridges. However, there are no mature theories and methods concerning the maintenance, management and assessment system of long-span bridges which are still at probing and researching stages. In order to guarantee the safety service and life-span of long-span cable-stayed bridges and assess bridge conditions scientifically, a system is developed for condition assessment of long-span cable-stayed bridges. The assessment system applies system engineering theory and improves the traditional expert assessment methods. The assessment modes of the system are as follows:

1) Establish a hierarchical model of condition assessment and make a list of assessment indices;

2) Determine the final weight of each assessment index in the whole assessment system;

3) Determine the value of each bottom assessment index by the investigation and inspection of experts or technical personnel;

4) Use the variable weight synthesizing method to obtain the final assessment results.

In the whole assessment, the core is to obtain the weight of each assessment index by an uncertain type of analytic hierarchy process (abbreviated as AHP). This paper describes it in detail and takes Harbin Songhua River Bridge as an example to establish the condition assessment system.

1 Uncertain Type of AHP

1.1 A brief introduction to AHP

AHP was proposed by American operational researcher Saaty in the 1970s[2], and it is applicable to decision-making problems that involve complex hierarchies and multiple indices. AHP can deal with the qualitative and quantitative factors of the decisions, and it is practical, systematic and terse[3]. The condition assessment of long-span cable-stayed bridges is a complex decision-making problem. AHP can make the factors that affect bridges orderly and hierarchical, establish relationship models with multiple hierarchies, and obtain the upper assessment indices from the lower ones, by which the assessment results of the whole bridge can be finally obtained[4]. Generally speaking, four steps are involved while assessing long-span cable-stayed bridges by AHP:

① Establish the orderly hierarchy of the bridge

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assessment. Put similar or closely related assessment indices in the same lay and propose relation assessment models with multiple hierarchies. Besides, the numbers of assessment indices cannot exceed nine in the same lay[5].

② Construct a judgment matrix. Assessment indices in each lay are compared in pairs. If there are n indices in a certain lay, they will construct the judgment matrix $\boldsymbol{A}$: $\boldsymbol{A}=(a_{ij})_{n\times n}$. The properties of $\boldsymbol{A}$ are as follows:

$$a_{ij}>0;\ a_{ij}=\frac{1}{a_{ji}};\ a_{ii}=1$$

The value of a_{ij} is from 1 to 9. Their definitions are described in Tab. 1.

③ Obtain the relative weights of the assessment indices by calculating the judgment matrix; namely, get the weights of indices in their lay by calculating the judgment matrix constructed by the indices in each lay.

④ Obtain the final weights of the assessment indices of each lay; namely, get the relative weights of all the indices in each lay to the final index.

Tab. 1 Measurement of judgment matrix $\boldsymbol{A}$

Intensity of relative importance	Definition
1	Equal importance
3	Weak importance (of one over the other)
5	Strong importance
7	Demonstrated importance over the other
9	Absolute importance
Other numbers from 1 to 9	Intermediate values between

1.2 Uncertain type of AHP

In the assessment of long-span cable-stayed bridges, due to the complexity and uncertainty of objective factors, it is difficult for experts to render accurate judgments to the relative importance of assessment indices. Namely, it is difficult for experts to describe the importance between any two indices with accurate numbers. Consequently, interval numbers are adopted to describe the relative importance while comparing assessment indices in pairs, for instance, the interval numbers of importance of one assessment index to another are [3.5, 4.5]. The interval numbers reflect the uncertainty and fuzziness of things better and help experts express their opinions more easily[6]. Subsequently, obtain the weight of each assessment index by calculating the interval number judgment matrix. The method introduced here is called an uncertain type of AHP.

The properties of an interval number judgment matrix in an uncertain type of AHP are the same as those of a judgment matrix in AHP. The only difference lies in that, in an uncertain type of AHP, each matrix element is composed of interval numbers (see Tab. 2).

Tab. 2 Judgment matrix of uncertain type of AHP $\boldsymbol{A}=(a_{ij})_{n\times n}$

a_j	a_i			
	a_1	a_2	…	a_n
a_1	$[1,1]$	$[a_{12}^-, a_{12}^+]$	…	$[a_{1n}^-, a_{1n}^+]$
a_2	$\left[\frac{1}{a_{12}^+}, \frac{1}{a_{12}^-}\right]$	$[1,1]$	…	$[a_{2n}^-, a_{2n}^+]$
⋮	⋮	⋮		⋮
a_n	$\left[\frac{1}{a_{1n}^+}, \frac{1}{a_{1n}^-}\right]$	$\left[\frac{1}{a_{2n}^+}, \frac{1}{a_{2n}^-}\right]$	…	$[1,1]$

1.3 Analysis and selection of several algorithms of the uncertain type of AHP

In fact, it is a complex process to get the weight of each assessment index by calculating the interval number judgment matrix, and the calculation methods are not uniform or accurate. There are many methods to calculate the interval number judgment matrix, such as the interval number random sample method, the interval number eigenvalue method (IME), the consistency approximation and weight calculation method of the judgment matrix, the interval number logarithm least second multiplication (ILLSM) and so on[7-9]. The analysis and contrast of these methods are made and the conclusions are as follows:

The interval number random sample method was adopted in Ref. [4]. However, its workload is too large to include the whole information of judgment matrices, and the results of the decisions cannot fully reflect the bases of the decisions. Ref. [8] proposed the consistency approximation and weight calculation method of the judgment matrix which can make full use of all judgment information given by experts to construct the consistency judgment matrix, and applied the error transfer formula to calculate the weights. Nevertheless, the results will be incorrect, even negative when the judgment matrix given by experts lack consistency. The method in Ref. [7] gives too large an interval to use when the judgment matrix given by experts lacks consistency. After analysis, the optimal transitive matrix method[10] is adopted in this paper to calculate the judgment matrix and obtain the weight of each assessment index. This method is easy to use and it is more accurate; and it can include all the information of the judgment matrix. Besides, it depends little on the consistency of the judgment matrix given by experts.

1.4 Theoretical introduction of optimal transitive matrix of uncertain type of AHP

Based on the original optimal transitive matrix method[10], the improved calculation process is as follows:

① Experts give the interval number judgment matrix $\boldsymbol{A} = (a_{ij})_{n\times n}$ according to the importance of each assessment index (see Tab. 2).

② Divide the matrix $\boldsymbol{A}$ into two matrices: $(a_{ij}^-)_{n\times n}$ and $(a_{ij}^+)_{n\times n}$, and calculate them respectively (herein, both $(a_{ij}^-)_{n\times n}$ and $(a_{ij}^+)_{n\times n}$ are denoted by $(a_{ij})_{n\times n}$). Let $\boldsymbol{B} = \ln\boldsymbol{A} = (\ln a_{ij})_{n\times n}$, then $\boldsymbol{B}$ is the dissymmetry interval matrix.

③ Based on the theorem[10], $\boldsymbol{C} = (c_{ij})_{n\times n}$ is the optimal transitive matrix of $\boldsymbol{B}$, $c_{ij} = \dfrac{1}{n}\sum_{k=1}^{n}(b_{ik} - b_{jk})$.

④ Let $\boldsymbol{A}^* = \exp(\boldsymbol{C}) = (\mathrm{e}^{c_{ij}})_{n\times n}$, then $\boldsymbol{A}^*$ is the consistency interval matrix.

⑤ Normalize $\boldsymbol{A}^*$, and then calculate the weight of each assessment index by averaging the sum:

$$w_i = \frac{\sum_{j=1}^{n} a_{ij}^*}{\sum_{k=1}^{n}\sum_{j=1}^{n} a_{kj}^*}$$

⑥ Calculate the two matrices $(a_{ij}^-)_{n\times n}$ and $(a_{ij}^+)_{n\times n}$, respectively, and combine their results w_i^- and w_i^+ together. Thus, the weight interval of the uncertain type of AHP judgment matrix can be fixed, that is $(w_i^-, w_i^+)_n$. Finally, calculate the weights of the assessment indices by averaging the interval numbers.

2 Harbin Songhua River Cable-Stayed Bridge Condition Assessment

Harbin Songhua River Cable-Stayed Bridge is the first long-span cable-stayed bridge in Heilongjiang province, and it is the symbolic project on the mainline of the highway around Harbin city. The main bridge is a double-tower, double-plain cable-stayed bridge with steel-concrete girder; its spans are 44 + 136 + 336 + 136 + 44 m with a 33. 2 m width. For such a large bridge, how to maintain and accomplish condition assessments are the main tasks.

Next, Harbin Songhua River Cable-Stayed Bridge will be taken as an example to introduce the process of calculating the weight of each assessment index by the uncertain type of AHP and by conducting the condition assessment.

2. 1 Calculation of the condition assessment indices and weights

Affected and restricted by many factors, Harbin Songhua River Cable-Stayed Bridge has the characteristics of multiple hierarchies and uncertainty. Therefore, its condition assessment index system is a complex system with multiple hierarchies. The steps of calculating the weight of each assessment index by the uncertain type of AHP are as follows.

1) Establish an orderly hierarchy of condition assessment

A condition assessment model is proposed with the aim of safety and endurance of the Harbin Songhua River Cable-Stayed Bridge, and it is an index model with multiple targets and hierarchies. Tower, cable, girder, abutment and foundation and affiliated facilities are chosen as the first lay assessment indices; twenty-two elements such as profile and stress are chosen as the second lay indices; twenty independent elements including chloride ions and concrete compressive are chosen as the third lay indices. Assessment indices in each lay and their mutual relations model are shown in Fig. 1[11].

2) Establish the judgment matrices

When experts establish the judgment matrices, the interval number judgment matrix model is adopted in order to reflect the fuzzy and uncertain relationships among assessment indices. In the following, the first lay indices judgment matrix established by some experts is cited as an instance, which is shown in Tab. 3. $\boldsymbol{A} = (a_{ij})_{5\times 5}$ is an interval number judgment matrix, in which the element $a_{11} = [1, 1]$ means that the "abutment and foundation" index is as important as the "abutment and foundation" index (see Tab. 1). The element $a_{15} = [8, 9]$ means that compared with the "affiliated facilities" index, the importance of the "abutment and foundation" index lies between "demonstrated importance" and "absolute importance". The meanings of other elements in the matrix are like those above.

3) Calculate the relative weight of each assessment index

The relative weight of each index in its lay can be obtained by adopting the optimal transitive matrix method of the uncertain type of AHP to calculate the

Tab. 3 Indices judgment matrix of the first lay established by some experts

a_j	a_i				
	Abutment and foundation	Tower	Girder	Cable	Affiliated facilities
Abutment and foundation	[1, 1]	[1, 2. 5]	[3. 5, 4. 5]	[4, 5]	[8, 9]
Tower	[0. 4, 1]	[1, 1]	[1, 2]	[3, 4]	[6. 5, 8]
Girder	[0. 22, 0. 29]	[0. 5, 1]	[1, 1]	[1. 5, 2. 5]	[4, 5]
Cable	[0. 20, 0. 25]	[0. 25, 0. 33]	[0. 4, 0. 67]	[1, 1]	[3, 4]
Affiliated facilities	[0. 11, 0. 13]	[0. 13, 0. 15]	[0. 2, 0. 25]	[0. 25, 0. 33]	[1, 1]

judgment matrices established by experts. For example, the relative weights of the first lay assessment indices are obtained by calculating the judgment matrix (see Tab. 3):

$$w_{a\&f} = 0.438\,0,\ w_{tower} = 0.272\,0,\ w_{girder} = 0.159\,8$$

$$w_{cable} = 0.092\,7,\ w_{affiliated\ facilities} = 0.037\,5$$

All the relative weights of the assessment indices of Songhua River Cable-Stayed Bridge calculated by the method above are shown in Fig. 1. The underlined numbers are the relative weights.

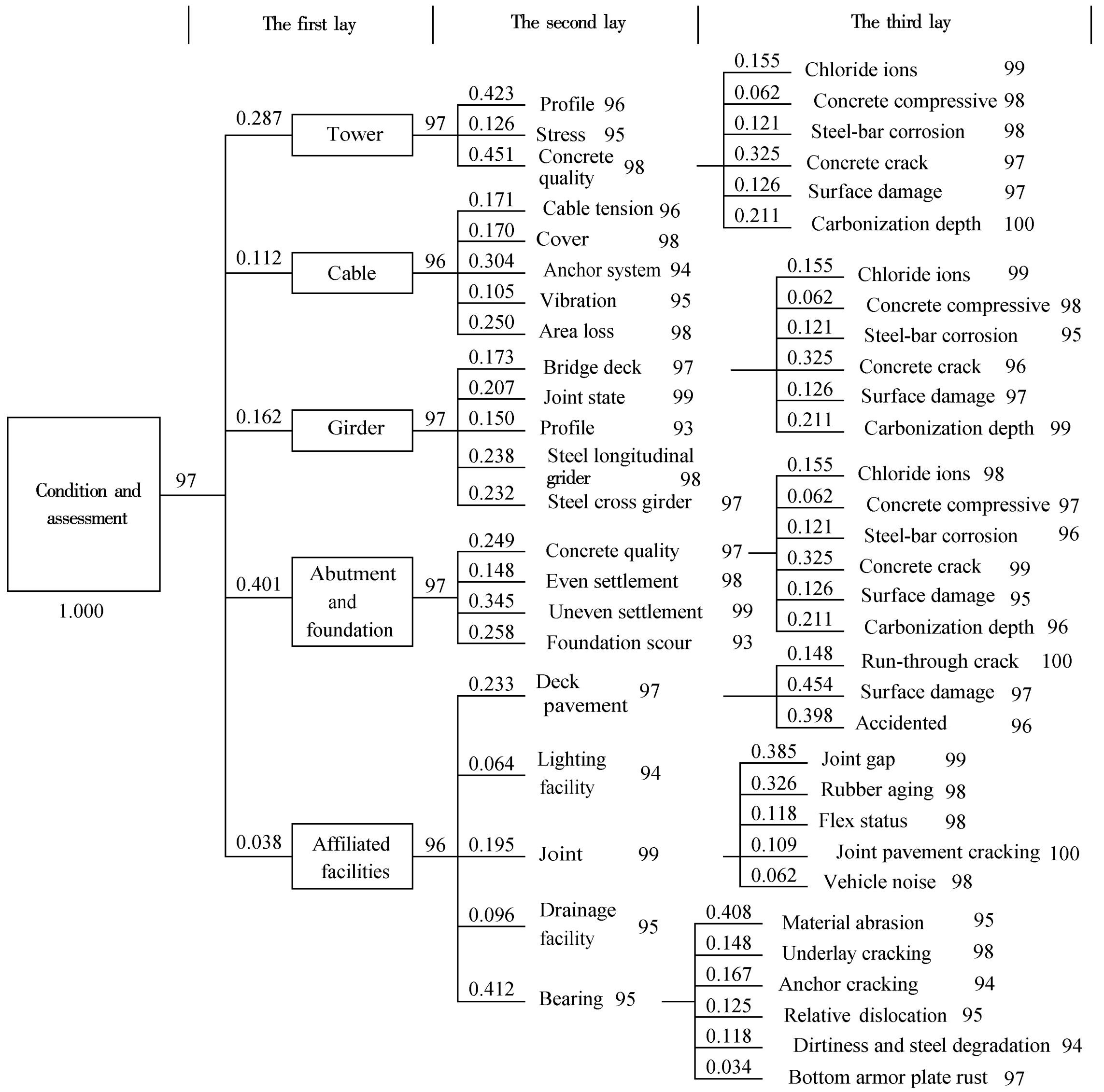

Fig. 1 Frame of condition assessment of Harbin Songhua River Cable-Stayed Bridge

4) Calculate the final weight of each assessment index

The final weights of all assessment indices can be calculated according to the above results. For example, the final weight of the third lay assessment index "surface damage" to the second lay indices "deck pavement" and even to the first lay index "affiliated facilities" is $0.454 \times 0.233 \times 0.038 = 0.004\,02$. The calculation of other indices' final weights is the same.

2.2 Condition assessment results of Harbin Songhua River Cable-Stayed Bridge

After obtaining the weight of each assessment index of Harbin Songhua River Cable-Stayed Bridge, each index is scored on the basis of expert advice, *Code for Maintenance of Highway Bridges and Culverts*[1], history inspection and manual examination. The score of each assessment index is shown in Fig. 1. The full score of each assessment index is 100. For example, the score of drainage equipment is 95, which means that the drainage equipment of the bridge is in an excellent state.

In the light of Ref. [1], there are five ranks as to the assessment results: excellent (88 to 100), good (60 to 87), medium (40 to 59), bad (20 to 39), worse (0 to 19). From Fig. 1, with condition assessment scoring 97, Harbin Songhua River Cable-Stayed Bridge is in excellent condition.

3 Conclusion

In view of the defects of the traditional AHP, the uncertain type of AHP was adopted in the condition assessment of long-span cable-stayed bridges so as to reflect the fuzzy and uncertain relationships among assessment indices better. The improved optimal transitive matrix method was first applied to the condition assessment of long-span cable-stayed bridges. This method is easy to use, and its calculating results of the weights are authentic and credible for various judgment matrices. Besides, the main factors which affect the safety and endurance of the bridges can be fully considered by applying the uncertain type of AHP to the condition assessment of long-span cable-stayed bridges. Furthermore, it can guarantee the safety and endurance of the long-span cable-stayed bridges and prolong their lives by establishing a complete and reasonable assessment system to maintain and manage them in a timely manner. In the end, by illustrating the condition assessment of Harbin Songhua River Cable-Stayed Bridge, the application of the uncertain type of AHP to long-span cable-stayed bridges is introduced. A hierarchy model for cable-stayed bridges with steel-concrete girder is proposed, and the feasibility and applicability of uncertain type of AHP are proved.

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不确定型层次分析在斜拉桥状态评估中的应用

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摘要:为了确保大跨度斜拉桥的安全运营,保证桥梁的使用寿命,采用不确定型层次分析法对桥梁的状态进行评估.介绍了不确定型层次分析法的相关理论和应用对象,在众多计算区间数判断矩阵的方法中,经过对比分析选用了最优传递矩阵理论,因为其计算结果精确可靠.以哈尔滨松花江斜拉桥为背景桥,利用不确定型层次分析法建立了该桥状态评估的指标体系和评估模型,确定了评估指标的权重并通过计算得到该桥最终的状态评估值,证明了该方法的可行性与实用性.这种评估方法的应用将会为大跨度斜拉桥的养护管理提供更加科学的依据.

关键词:斜拉桥;状态评估;不确定型层次分析;区间数判断矩阵
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