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Fractal features of size distribution of Chinese intercity bus hubs

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Abstract: Size distribution characteristics of intercity bus hubs in China from 1997 to 2004 were analyzed regarding highway passenger volume as a size index of intercity bus hubs. Yearly fractal dimensions of intercity bus hub sizes were exactly calculated by a novel model. Fractal dimensions of the 200 biggest intercity bus hubs from 2000 to 2004 were 1. 486 2 to 1. 511 8, and that is consistent with fractal dimensions of Chinese urban system sizes. It showed that the size distribution of intercity bus hubs had fractal structure. Fractal dimensions from 1997 to 2004 indicated that intercity bus hub size distribution grew from bi-fractal to single fractal. It is concluded that the intercity bus hub system is in evolutionary progress, and the Central Government should support large intercity bus hubs more to optimize system structure.

Key words: intercity bus hub; fractal; size distribution; fractal dimension

Mandelbrot introduced the fractal concept in the "geometry of nature", the fractal dimension has been widely applied to describe spatial phenomena of urban geography, urban morphology, and transportation networks. Several studies about transportation networks, such as railway and public bus systems have been reported[1]. In large cities such as Moscow and Berlin, the fractal dimension is near 1. 7 and it seems that it corresponds to some optimal value. The case of Paris is an exceptional city[2], in which the transportation system is divided into two different networks: one providing transportation from suburbs to the center and the other giving transportation only within the center itself. The first network has a fractal dimension near 1. 5 and the second near 1. 8. It is clear that in other cities the transportation system plays two roles together, but the distinction is not always as visible as in the case of Paris. Batty and Longley reported results that the fractal dimension of bus transportation network of Lyon was between 1. 0 and 1. 45[3] in the past thirty years. However, the stations of any given transportation system have received little attention, except for two cases. One is Paris where the fractal dimension is determined in connection with the existence of two different networks, and it is found that the fractal dimension is $D = 2$ in the center of the city, but outside the city, the dimension decreases to 0. 5. The other is about Seoul, Kim et al. brought forward new data on the subway and surface rail transportation system of Seoul[4]. They measured the fractal density of subway and surface rail stations and ensemble lines. The findings show that the fractal dimensions of the stations ($D_S = 1.50$) are greater than those of the transit lines ($D_L = 1.35$). Based on the fractal dimension, the conclusion is that the mean distance between two stations decreases slowly when moving away from the center of the city.

It is important that data about intercity bus systems is brought forward to analyze the fractal structure of intercity bus hub size distribution. It is essential to verify whether Chinese intercity bus hubs' size distribution is fractal. It would be very important to find links between the state of development of an urban system and its intercity bus transportation fractal dimension. In this spirit, we undertook a case study of Chinese intercity bus hubs and measured size distribution fractal features of intercity bus hubs.

1 Fractal Dimension of Intercity Bus Hub's Size Distribution

Before analyzing the size distribution of Chinese intercity bus hubs, it is of value to provide some insight concepts of fractals and fractal structure. Then we explain how fractal dimension is determined. Fractal is a chaos and amorphous self-similarity system. In other words, one can say that a fractal object has the same pattern at different scales. Mandelbrot gave several examples of self-similar structures about given fractal objects. Thus, from the conclusion that a system is fractal, one can learn two things. First, the system is self-similar. In such a case, if one operates a zoom to see more details, the same pattern of the original object is

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observed. Second, one can determine a fractal structure when its dimension is lower than 2.

Self-similarity of a fractal object is induced to unsigned measuring of its spatial structure characters[5]. It means that different results $N(r)$ are obtained when a changeable scale r is used. The fractal dimension is an important parameter of complexity measurements. Unlike spatial distribution, fractal characteristics of size distribution are represented by Zipf dimensions of order-size rules or Pareto indices in practice. The intercity bus hub size r and intercity bus hub number $N(r)$ formulate the Pareto distribution:

$$N(r) = Ar^{\alpha} \tag{1}$$

where A is the constant, and parameter α is the Pareto index which is fractal[6]. Eq. (1) can be formulated as the Zipf expression of the size-distribution rule[7]

$$P_i = P_1 r^{-q} \tag{2}$$

where P_i is the size of the intercity bus hub if the order is i, P_1 is the theoretical intercity bus passenger volume of the largest intercity bus hub; r is the order number in the hub system, and q is parameter called the Zipf dimension.

The size-distribution rule of the urban system has been testified as a fractal structure, and the fractal dimension index $D \approx 1/q$[8]. Because the theoretical value of P_1 is larger than the practical value, q can be determined by

$$\ln P_i = \ln P_1 - q \ln r \tag{3}$$

Fitting intercity bus passenger volume statistical data of the intercity bus hub with the least-square method, Zipf dimension q and decisive coefficient R^2 can be acquired[9]. Fractal dimension D of intercity bus hub size distribution can be precisely determined by

$$Dq = R^2 \tag{4}$$

2 Fractal Dimensions of Chinese Intercity Bus Hubs

2.1 Fractal dimensions of Chinese urban system

We choose the 200 largest cities according to population statistical data in *the China Statistical Year Book* (1998 to 2005). Fig. 1 displays the fitting linear curve in log(number of cells)-log(size of a cell) plot ($\ln r$, $\ln P_i$) of the size distribution of the Chinese urban system(here $\ln r$ as horizontal x and $\ln P_i$ as vertical y), and Tab. 1 includes the main parameters q_{1-200}, R^2_{1-200} and D_{1-200} related to Fig. 1 coming from Eq. (4).

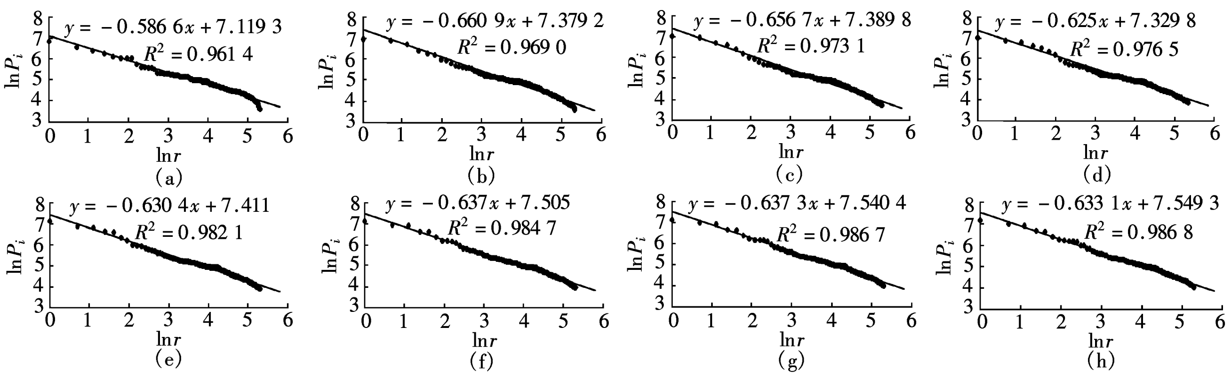

Fig. 1 Size distribution bi-logarithmic fitting curve of Chinese urban system. (a) 1997; (b) 1998; (c) 1999; (d) 2000; (e) 2001; (f) 2002; (g) 2003; (h) 2004

Tab. 1 Fractal dimensions of size of Chinese urban system (1997 to 2004)

Year	q_{1-200}	R^2_{1-200}	D_{1-200}
1997	0.586 6	0.961 4	1.638 9
1998	0.660 9	0.969 0	1.466 2
1999	0.656 7	0.973 1	1.481 8
2000	0.625 0	0.976 5	1.562 4
2001	0.630 4	0.982 1	1.557 9
2002	0.637 0	0.984 7	1.545 8
2003	0.637 3	0.986 7	1.548 3
2004	0.633 1	0.986 8	1.558 7

2.2 Fractal dimensions of Chinese intercity bus hubs

We choose the same 200 cities in *China Statistical Year Book* (1998 to 2005), and regard intercity bus passenger volume statistical data as size indices of intercity bus hubs. Under the proposed method, we can acquire Fig. 2 to display a fitting linear curve in log-log plot($\ln r$, $\ln P_i$) of a size distribution of the biggest 200 Chinese intercity bus hubs, and Tab. 2 includes three main parameters q'_{1-200}, R'^2_{1-200} and D'_{1-200} related to Fig. 1 coming from Eq. (4).

Tab. 2 Parameters of Chinese intercity bus hubs' size distribution (1997 to 2004)

Year	q'_{1-200}	R'^2_{1-200}	D'_{1-200}
1997	0.789 0	0.912 0	1.155 9
1998	0.741 2	0.923 7	1.246 2
1999	0.715 0	0.933 6	1.305 7
2000	0.644 7	0.969 5	1.503 8
2001	0.646 4	0.967 1	1.496 1
2002	0.649 9	0.965 9	1.486 2
2003	0.652 1	0.979 3	1.501 8
2004	0.643 2	0.972 4	1.511 8

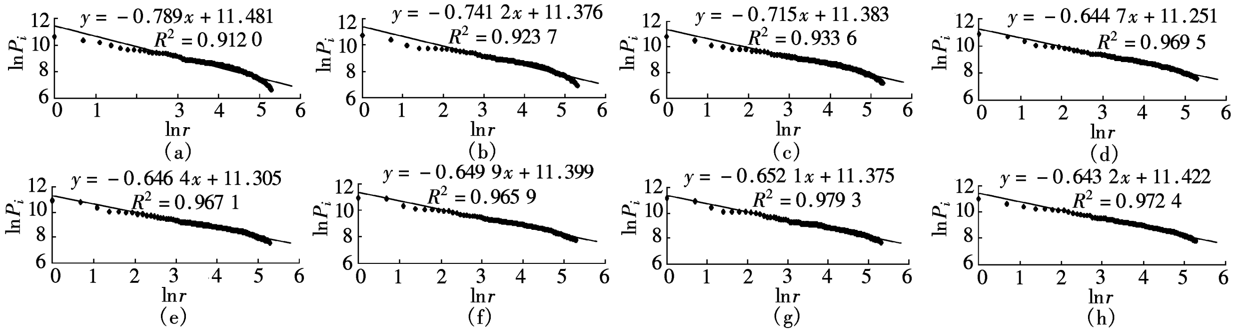

Fig. 2 Bi-logarithmic fitting curve of Chinese intercity bus hub size distribution. (a) 1997; (b) 1998; (c) 1999; (d) 2000; (e) 2001; (f) 2002; (g) 2003; (h) 2004

Because the decisive coefficient R'^2_{1-200} of fitting functions concerning size distribution points from 1997 to 1999 in Fig. 2 is less than 0. 95, we fit size distribution points in orders of 1 to 100 as one linear curve and 101 to 200 as another linear curve, respectively, in log-log plot($\ln r$, $\ln P_i$), as indicated in Fig. 3, and related parameters q'_{1-100}, $q'_{101-200}$ R'^2_{1-100}, $R'^2_{101-200}$ and D'_{1-100}, $D'_{101-200}$ are shown in Tab. 3.

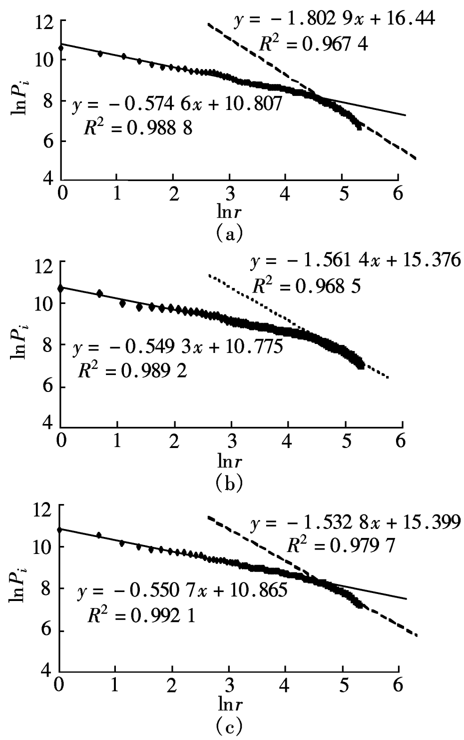

Fig. 3 Bi-fractal structure of Chinese intercity bus hub system. (a) 1997; (b) 1998; (c) 1999

Tab. 3 Bi-fractal dimensions of Chinese intercity bus hubs

Year	q'_{1-100}	$q'_{101-200}$	R'^2_{1-100}	$R'^2_{101-200}$	D'_{1-100}	$D'_{101-200}$
1997	0. 574 6	1. 802 9	0. 988 8	0. 967 4	1. 720 8	0. 536 58
1998	0. 549 3	1. 561 4	0. 989 2	0. 968 5	1. 800 8	0. 620 28
1999	0. 550 7	1. 532 8	0. 992 1	0. 979 7	1. 801 5	0. 639 16

3 Discussion

3. 1 Fractal structure of Chinese intercity bus hubs

One can determine a fractal object if its fractal dimension is lower than 2[10]. As fractal dimensions of Chinese intercity bus hub sizes distribution in Tab. 2 are all less than 2 from 1997 to 2004, and fractal dimensions in 2000 to 2004 are 1. 486 2 to 1. 511 8, we can conclude that the Chinese intercity bus hub size distribution has fractal features. Fitting size distribution points to two linear curves of each year in log-log($\ln r$, $\ln P_i$) plot in Fig. 3 show that intercity bus hub size distribution from 1997 to 1999 is bi-fractal. Fractal dimensions of Chinese intercity bus hub size distribution from 1997 to 2004 validate theoretical evolution progress from bi-fractal to single-fractal, and explain the reasonable size distribution structure of the Chinese intercity bus hub system. It is the result that central and local governments promote development of intercity bus hubs and subsidize more investments than before in recent years.

3. 2 Consistency between intercity bus hub system and urban system

Experimental results reveal that the size distribution of transportation systems are related to several factors such as urban population, economic and service industry levels. Urban system size distribution of developed countries concludes that fractal dimensions are always larger than 1 during earlier urbanization, and then decrease with development of the city. Fractal dimensions of intercity bus hub size distribution during 2000 to 2004 are 1. 486 2 to 1. 511 8 (as shown in Tab. 2), and those of the urban system are 1. 466 2 to 1. 638 9 (as shown in Tab. 1). Fractal dimension gaps between the two systems are 3. 00% to 3. 97%. Then we can conclude that the size distribution of intercity bus hubs is consistent with the urban system in China.

3. 3 Constructing larger intercity bus hubs

Object has optimum spatial configuration if its size distribution fractal dimension $D = 1$[11]. Based on results in section 2, we can conclude that Chinese intercity bus hubs are in a better stage than before, and are consistent with the development of urban systems. But those fractal dimensions of the size distribution of intercity bus hubs after 2000 are all larger than 1. 48 showing the growing tendency of the Chinese intercity bus hub system. The size distribution points in Fig. 2 indicate that there are fewer large intercity bus hubs and more small intercity bus hubs, and the agglomerative effect of large intercity bus hubs are not distinct. Large intercity bus hubs need to be promoted by the Central Government to act as national and regional transfer centers in intercity bus networks in China.

4 Conclusion

Taking intercity bus passenger volume as a size index of intercity bus hubs, fractal dimensions of the size distribution of the biggest 200 intercity bus hubs from 1997 to 2004 are obtained. The findings show that the size distribution of the Chinese intercity bus hub system has fractal structure and the development of the fractal structure is in accord with theoretical rule where the object system must evolve from single-fractal to bi-fractal. Case studies of Chinese intercity bus hubs explore that large hubs are absent and are unattractive to intercity highway passengers, while small hubs are in overabundance. Results indicate that the fractal structure of the Chinese intercity bus hub system is in a development progress and central and central and local government should pay more attention to enhancing the role of large intercity bus hubs in intercity passenger networks.

The size distribution feature is the main spatial characteristic of large intercity bus hubs. Location distribution feature analyses can be used to express the spatial characteristics as another assistant method of a topological analysis of intercity bus hubs. Multi-fractal dimensions of size distribution of intercity bus hubs should be considered in future research.

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中国公路客运枢纽规模分布的分形特征

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摘要:以公路客运量为规模参数分析了1997年至2004年中国前200位主要公路客运枢纽规模分布的分形特征,采用精确计算方法得到历年公路客运枢纽规模分布的分维数.发现中国公路客运枢纽体系的规模分布符合从双分形到单分形发展的结构特征,2000年至2004年规模分布维数介于1.486 2 ~ 1.511 8之间.这与城镇体系规模分布维数一致,说明现阶段中国公路客运枢纽体系与城镇体系规模分布结构相适应.结果表明,公路客运枢纽体系处于较好发展阶段,政府需加强大型公路客运枢纽建设推动其结构优化.

关键词:公路客运枢纽;分形;规模分布;分维数

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