

# Statistical segmentation model on lattices

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**Abstract:** To reduce the difficulty of implementation and shorten the runtime of the traditional Kim-Fisher model, an entirely discrete Kim-Fisher-like model on lattices is proposed. The discrete model is directly built on the lattices, and the greedy algorithm is used in the implementation to continually decrease the energy function. First, regarding the gray values in images as discrete-valued random variables makes it possible to make a much simpler estimation of conditional entropy. Secondly, a uniform method within the level set framework for two-phase and multiphase segmentations without extension is presented. Finally, a more accurate approximation to the curve length on lattices with multi-labels is proposed. The experimental results show that, compared with the continuous Kim-Fisher model, the proposed model can obtain comparative results, while the implementation is much simpler and the runtime is dramatically reduced.

**Key words:** image segmentation; curve evolution; conditional entropy; lattice; labelling problem

Being the basis of many other applications, image segmentation has received much attention. Among image segmentation techniques, statistical approaches together with information theory play an important role. A nonparametric statistical model for image segmentation from an information-theoretical perspective was proposed in Ref. [1]. We call it the Kim-Fisher model in this paper. The model can obtain satisfactory segmentation results, whereas, due to the complexity of the level set approach and the Parzen estimation, the implementation is difficult and it takes much time to complete the segmentation.

In this paper, we establish a discrete Kim-Fisher-like model on lattices. The discrete model is different from the Kim-Fisher model in several major ways. First, regarding gray values as discrete-valued random variables makes it possible to estimate the probabilities with a much simpler method. Secondly, when the Kim-Fisher model and all the other segment models within the level set framework are applied to multiphase segmentation, some extensions must be built. In the Kim-Fisher model and Ref. [2], the extension was via multiple level set functions. In Ref. [3], the extension was built by introducing an additional region indication function. In this paper, we develop a uniform method without introducing any additional functions. Thirdly, we add an area term in the energy function, which can keep the evolution moving in the proper direction. Unlike the energy function of the Kim-Fisher model, all the terms in the objective function of the proposed discrete model are defined on lattices and formulated as functions of the labels, namely, a

discrete level set. And we do not use traditional level set techniques to evolve the curve; instead, we accomplish the segmentation by updating the discrete level set. The idea of a discrete level set function was first built in Refs. [2, 4].

The difference between our work and Ref. [2] is that we aim at a fully discrete Kim-Fisher-like model instead of only a fast algorithm. Besides, the formulation to approximate the curve length on lattices in this paper is more accurate than that in Ref. [2]. Furthermore, as mentioned above, in Ref. [2], some extensions must be built to do multiphase segmentation. While we develop a uniform method within the level set framework for two-phase and multiphase segmentations without extension. The last difference is that we evolve only boundary points while Ref. [2] sweeps every pixel on the image.

## 1 Kim-Fisher Model<sup>[1]</sup>

In this section, we will cite some illustrations in Ref. [1] to explain the Kim-Fisher model. The regions of an image are distinct in the sense that they have different probability density functions for the pixel intensities. The Kim-Fisher model is based on the assumption that the pixel intensities in each region are independent and identically distributed (i. i. d.). The assumption can be formulated by

$$\{G(x) | x \in R_1\} \sim p_1, \quad \{G(x) | x \in R_2\} \sim p_2 \quad (1)$$

where  $G(x)$  is the image intensity at pixel  $x$ ;  $R_1$  and  $R_2$  denote the two regions whose associated probability density functions are  $P_1$  and  $P_2$ , respectively. To segment the image, we hope that the evolution of the segment curve  $\tilde{C}$  can match the boundary between  $R_1$  and  $R_2$ . Suppose that  $\tilde{C}$  divides the image into two regions: the region inside the curve  $R_+$  and the region outside the curve  $R_-$ . And a binary label  $L_C: \Omega \rightarrow \{L_+, L_-\}$  is defined as

$$L_C(x) = \begin{cases} L_+ & \text{if } x \in R_+ \\ L_- & \text{if } x \in R_- \end{cases} \quad (2)$$

Supposing that  $X$  is a uniformly distributed random location in the image domain, then the image intensity  $G(x)$  and the binary label  $L_C(x)$  are two interrelated random variables. Ref. [1] explained that the mutual information  $I(G(X); L_C(X))$  can be used as a segmentation criterion and proved that  $I(G(X); L_C(X))$  is maximized if and only if  $\tilde{C}$  is the correct segmentation, i. e., if  $R_+ = R_1$ ,  $R_- = R_2$  (or, equivalently,  $R_+ = R_2$ ,  $R_- = R_1$ ).

The energy function in Ref. [1] is defined as

$$\tilde{E}(C) = -|\Omega| \hat{I}(G(X); L_C(X)) + \gamma \oint_C ds \quad (3)$$

where  $|\Omega|$  is the area of the image domain and  $\hat{I}(G(X);$

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$L_{\hat{c}}(X)$ ) is an approximation to  $I(G(X); L_{\hat{c}}(X))$ . In Ref. [1], nonparametric Parzen density estimates are used to obtain the approximation.

## 2 Discrete Kim-Fisher-Like Model

### 2.1 Objective function

The idea of using  $I(G(X); L_{\hat{c}}(X))$  as a segmentation criterion is creative. Whereas, in the Kim-Fisher model, regarding  $G(x)$ , the image density at pixel  $x$ , as a continuous-valued random variable makes it difficult to approximate  $I(G(X); L_{\hat{c}}(X))$ .

The discrete Kim-Fisher-like model is also based on the assumption that the pixel intensities in each region are independent and identically distributed. The difference is that  $G(x)$  in the discrete model is a discrete-valued random variable. Suppose that there are  $T$  regions in the image. Then for an image defined on the lattice of size  $M \times N$  ( $1 \leq m \leq M, 1 \leq n \leq N$ ),

$$\{G_{mn} \mid (m, n) \in R_t\} \sim P_t \quad 0 \leq t \leq T-1 \quad (4)$$

where  $R_0, R_1, \dots, R_{T-1}$  is the virtual partition of the image;  $P_t$  is the associated probability mass function of  $R_t$ ;  $G_{mn}$  is the gray value of pixel  $(m, n)$ , and  $G_{mn} \in \{0, 1, 2, \dots, K\}$ , where  $K$  is the maximum gray level in the image. The discreteness of  $G_{mn}$  makes it possible to estimate the probability by the frequency of occurrence, and it is a simple and effective method. The label function  $L$  is a discrete-valued function and  $L_{mn} \in \{0, 1, 2, \dots, T-1\}$ .

In this paper, we use  $R_t^L$  to denote the region of  $L = t$ . For the uniformly distributed random location  $X$ , the maximization of  $I(G(X); L(X))$  is equivalent to the minimization of conditional entropy  $H(G(X); L(X))$ . We use the latter as the segmentation criterion. And  $H(G(X); L(X))$  is minimized if and only if  $R_{\{i\}}^L = R_{\{j\}}$  and  $\{i\}$  and  $\{j\}$  are arbitrary permutations of  $\{0, 1, 2, \dots, T-1\}$ . Although this proposition can be proved by extending the proof in Ref. [1], for the sake of integrality, we present our proof, which is slightly different from that of Ref. [1].

**Proof** For any  $0 \leq t \leq T-1$ , we have

$$H(G(X) \mid L(X) = t) = - \sum_{k=0}^K P(G(X) = k \mid L(X) = t) \log P(G(X) = k \mid L(X) = t) \quad (5)$$

where

$$\begin{aligned} P(G(X) = k \mid L(X) = t) &= \sum_{s=0}^{T-1} P(G(X) = k \mid X \in R_s, \\ &L(X) = t) P(X \in R_s \mid L(X) = t) = \sum_{s=0}^{T-1} P(G(X) = k \mid X \in R_s) P(X \in R_s \mid L(X) = t) = \\ &\sum_{s=0}^{T-1} P_s(k) \frac{|R_s \cap R_t^L|}{|R_t^L|} = \sum_{s=0}^{T-1} P_s(k) \alpha_{st} \end{aligned} \quad (6)$$

where  $P_s$  is the mass function in formula (4) and  $\alpha_{st} = \frac{|R_s \cap R_t^L|}{|R_t^L|}$ . The second equality is based on the assumption described by formula (4).

$$\begin{aligned} H(G(X) \mid L(X)) &= \sum_{t=0}^{T-1} H(G(X) \mid L(X) = t) \cdot \\ P(L(X) = t) &= \sum_{t=0}^{T-1} H(G(X) \mid L(X) = t) \frac{|R_t^L|}{|\Omega|} \\ &= - \sum_{t=0}^{T-1} \left\{ \sum_{k=0}^K \left[ \sum_{s=0}^{T-1} P_s(k) \alpha_{st} \right] \log \left[ \sum_{s=0}^{T-1} P_s(k) \alpha_{st} \right] \right\} \cdot \\ &\frac{|R_t^L|}{|\Omega|} \geq - \sum_{t=0}^{T-1} \sum_{k=0}^K \sum_{s=0}^{T-1} \alpha_{st} P_s(k) \log P_s(k) \frac{|R_t^L|}{|\Omega|} \quad (7) \end{aligned}$$

The inequality holds due to Jensen's inequality. If  $R_{\{i\}}^L = R_{\{j\}}$ , we have  $P(G(X) \mid L(X) = t) = P_u(k)$ ,  $u$  is the label satisfying  $R_t^L = R_u$ . If we use  $H_1(G(X) \mid L(X))$  to denote  $H(G(X) \mid L(X))$  under the condition of  $R_{\{i\}}^L = R_{\{j\}}$ , then

$$H_1(G(X) \mid L(X)) = - \sum_{u=0}^{T-1} \frac{|R_u|}{|\Omega|} \sum_{k=0}^K P_u(k) \log P_u(k) \quad (8)$$

Now, we come back to Eq. (7),

$$\begin{aligned} H(G(X) \mid L(X)) &\geq - \sum_{t=0}^{T-1} \sum_{k=0}^K \sum_{s=0}^{T-1} \alpha_{st} P_s(k) \cdot \\ \log P_s(k) \frac{|R_t^L|}{|\Omega|} &= - \sum_{s=0}^{T-1} \sum_{t=0}^{T-1} \frac{|R_t^L|}{|\Omega|} \cdot \\ \frac{|R_s \cap R_t^L|}{|R_t^L|} \sum_{k=0}^K P_s(k) \log P_s(k) &= - \sum_{s=0}^{T-1} \frac{|R_s^L|}{|\Omega|} \cdot \\ \sum_{k=0}^K P_s(k) \log P_s(k) &= H_1(G(X) \mid L(X)) \end{aligned} \quad (9)$$

In many segment models, including the Kim-Fisher model, there is a length term in the objective function. The main purpose of this term is to avoid redundant curves. Besides the length term, we add an area term to keep the evolution moving in the proper direction. Then, our objective function can be formulated as

$$E_1(L) = MN \hat{H}(G(X) \mid L(X)) + \alpha_1 \text{len}(L) + \beta_1 \text{area}(L) \quad (10)$$

where  $\alpha_1$  and  $\beta_1$  are parameters and  $\alpha_1 > 0$ .  $\hat{H}(G(X) \mid L(X))$  is the estimation of the conditional entropy  $H(G(X); L(X))$ .  $\text{len}(L)$  is the length of the segment curve, that is, the boundaries between every two adjacent regions. And  $\text{area}(L)$  is the sum area of the regions with either odd region labels or even labels. This area term can be viewed as the generalization of the constant term<sup>[5]</sup> in the traditional level set approach.

### 2.2 Estimation of conditional entropy

Owing to the discreteness of gray values, we can use the frequency of occurrence to approximate the conditional probability in  $H(G(X); L(X))$ . Before presenting the formulation of estimation, we first introduce some notations. Suppose that the image is defined on a lattice of  $M \times N$  ( $1 \leq m \leq M, 1 \leq n \leq N$ ).

$K$  is the maximum gray level in the image;  $T$  is the total number of regions in the image;  $a_{ij}$  is the cardinality of the set of  $\{(m, n) \mid L_{mn} = i, G_{mn} = j\}$ , ( $0 \leq i \leq T-1, 0 \leq j \leq K$ );  $Q_i$  is the total number of pixels with label  $i$ , ( $0 \leq i \leq T-1$ ).

Obviously, we have

$$\sum_{j=0}^K a_{ij} = Q_i, \quad \sum_{i=0}^{T-1} Q_i = MN$$

For the sake of simplicity, we drop the index  $X$  in the following formulae. Then we can estimate the conditional entropy by

$$\begin{aligned} \hat{H}(G(X) | L(X)) &= \sum_{i=0}^{T-1} P(L = i) \hat{H}(G | L = i) = \\ &= - \sum_{i=0}^{T-1} \frac{Q_i}{MN} \sum_{j=0}^K P(G = j | L = i) \log P(G = j | L = i) = \\ &= - \sum_{i=0}^{T-1} \frac{Q_i}{MN} \sum_{j=0}^K \frac{a_{ij}}{Q_i} \log \frac{a_{ij}}{Q_i} = - \frac{1}{MN} \sum_{\substack{0 \leq i \leq T-1 \\ 0 \leq j \leq K}} a_{ij} \log \frac{a_{ij}}{Q_i} \quad (11) \end{aligned}$$

In fact, in the discrete model, we need not calculate  $\hat{H}(G(X) | L(X))$ . Only the increment of  $\hat{H}(G(X) | L(X))$  after one boundary point has changed its label makes sense. And as we will see later, this increment is easy to calculate.

### 2.3 Estimation of curve length

After the estimation of the entropy, we will come to the length term. The segment curve consists of the boundaries between every two adjacent regions. It is difficult to calculate the curve length on lattices with multi-labels, so we use a function of  $L$  to approximate the sum length of the boundaries. Before presenting the formulation, we present a definition frequently used in this paper.

**Definition 1** Given a discrete-valued function  $L$  on an  $M \times N$  lattice, we say that pixel  $(m, n)$  is a boundary point if there exists  $(p, q) \in S(m, n)$  such that  $L_{mn} \neq L_{pq}$ , where  $S(m, n)$  is the eight neighbors set of pixels  $(m, n)$ .

Then, our approximation to the sum length of the boundaries is

$$\begin{aligned} \text{len}(L) &= \frac{1}{12} \sum_{m,n} \sum_{(p,q) \in S(m,n)} (1 - L_{mn} \circ L_{pq}) \\ L_{mn} \circ L_{pq} &= \begin{cases} -1 & \text{if } L_{mn} = L_{pq} \\ 1 & \text{if } L_{mn} \neq L_{pq} \end{cases} \quad (12) \end{aligned}$$

Among the neighbors of  $(m, n)$ , only those whose  $L$  values are different from  $L_{mn}$  can attribute to the sum. If pixel  $(m, n)$  is not a boundary point, the term  $\sum_{(p,q) \in S(m,n)} (1 - L_{mn} \circ L_{pq})$  is equal to 0; otherwise, the term is two times the number of neighbors who have different  $L$  values from  $L_{mn}$ . So, the term  $\sum_{m,n} \sum_{(p,q) \in S(m,n)} (1 - L_{mn} \circ L_{pq})$  is a weighted sum of all the boundary points on lattices. However, the weighted sum enlarges the actual length. For a horizontal or vertical line segment, the weighted sum magnifies the length 12 times. So a factor of  $1/12$  is needed in Eq. (12).

A special case of Eq. (12), e. g., on the lattice with two labels, is proposed by us in Ref. [6], and the advantage over other methods has been explained. In this paper, we propose a generalized formulation for the curve length on the lattice with multi-labels. The method in Ref. [7] cannot calculate the curve length on the lattice with multi-labels. The calculation of the curve length in Ref. [2] is the direct discretization of  $\sum_i \iint_{\Omega} |\nabla H(\phi_i)| d\chi$ , ( $\phi_i$  is the level set function)

using upwind difference. The advantage of Eq. (12) is illustrated in Fig. 1. The square brackets in Fig. 1 denote the values of two level sets, namely,  $[\phi_1, \phi_2]$ . The curve consists of  $OA$ ,  $OB$  (quarter circle with radius 100) and  $OC$  (line segment of length 95). The length of the curve should be  $100\pi + 95 \approx 409.16$ . The result of Eq. (12) is 403.67, with the relative error of 1.36%, while the result of the method in Ref. [2] is 598.22, with the relative error of 31.6%. The big discrepancy of Ref. [2] is partially due to the inaccuracy of discretization and partially because of the fact that the quarter circle  $OB$  is counted twice.

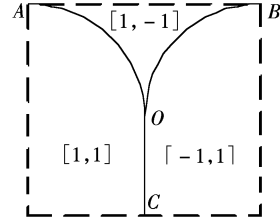


Fig. 1 Curve on the lattice with two level sets

### 2.4 Formulation of the area

In order to accelerate the segmentation and keep the evolution moving in the desired direction, we add an area term to the energy function. The area term is formulated as

$$\text{area}(L) = \frac{1}{2} \sum_{m,n} (1 + \text{mod}(L_{mn}, 2) \circ 1) \quad (13)$$

If  $L_{mn}$  is an even number,  $1 + \text{mod}(L_{mn}, 2) \circ 1 = 0$ . Otherwise,  $1 + \text{mod}(L_{mn}, 2) \circ 1 = 2$ . So the sum is the area of regions with odd labels. This area term can be viewed as the generalization of the constant term in the traditional level set approach, which can be either the area inside or outside the curve. Minimizing this area term alone will result in the shrinking of odd-labelled regions. Of course, minimizing the area term is not the purpose, but we can keep the evolution moving in the desired direction by adjusting the parameter of this area term.

## 3 Optimization of the model

### 3.1 Optimization of the objective

For simplicity, we present a slightly simpler formulation of the energy by dropping the constants in  $\text{len}(L)$  and  $\text{area}(L)$ .

$$\begin{aligned} E(L) &= - \sum_{i,j} a_{ij} \log \frac{a_{ij}}{Q_i} - \alpha \sum_{m,n} \sum_{(p,q) \in S(m,n)} L_{mn} \circ L_{pq} + \\ &\quad \beta \sum_{m,n} \text{mod}(L_{mn}, 2) \circ 1 \quad (14) \end{aligned}$$

As mentioned above, we implement the segmentation by refreshing the label  $L$ . To ensure the stability, we only update the labels of boundary points. To optimize Eq. (14), the greedy algorithm is used. That is, refresh every boundary point  $(m, n)$  by setting

$$L_{mn} = \arg \min_{0 \leq t \leq T-1} (E(L) | L_{mn} = t) \quad (15)$$

The calculation of the energy  $E(L)$  defined by Eq. (14) is time consuming. Fortunately, we only need to know the relative value of  $E(L) | L_{mn} = t, 0 \leq t \leq T-1$ , which is equiv-

alent to knowing the difference of  $E(L)$  for different  $L$ .

We first discuss the conditional entropy term. Suppose that  $G_{mn} = p$  and  $L_{mn} = x$ . If  $L_{mn}$  changes to  $y$ ,  $\hat{H}(G(X) | L(X))$  in Eq. (11) changes to  $\hat{H}_2(G(X) | L(X))$ , then the change of the entropy is

$$\begin{aligned} \Delta H = & \hat{H}_2(G(X) | L(X)) - \hat{H}(G(X) | L(X)) = \\ & -\frac{1}{MN} \left[ Q_x \log \frac{Q_x - 1}{Q_x} + Q_y \log \frac{Q_y + 1}{Q_y} + \right. \\ & a_{xp} \log a_{xp} - (a_{xp} - 1) \log (a_{xp} - 1) - \log (Q_x - 1) + \\ & \left. a_{yp} \log a_{yp} - (a_{yp} + 1) \log (a_{yp} + 1) + \log (Q_y + 1) \right] \quad (16) \end{aligned}$$

Compared with the change of the entropy, the changes of the length term and the area term are much simpler. When  $L_{mn}$  changes from  $x$  to  $y$ , only items including  $L_{mn}$  in the length term and the area term of Eq. (14) change while others remain unchanged. After we have calculated the change of  $E(L)$  for different  $L$ , it is easy to compute  $\arg \min_{0 \leq t \leq T-1} (E(L) | L_{mn} = t)$ .

### 3.2 Algorithm

The steps of the algorithm are as follows:

**Step 1** Find all the boundary points according to definition 1.

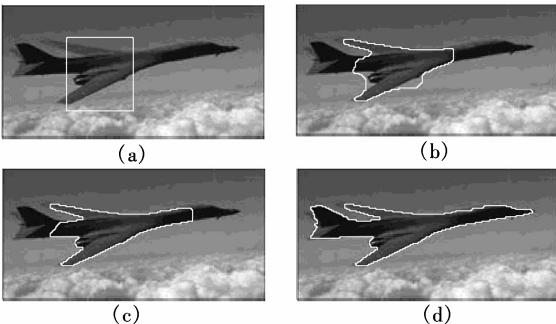
**Step 2** For every boundary point  $(m, n)$ , set  $L_{mn} = \arg \min_{0 \leq t \leq T-1} (E(L) | L_{mn} = t)$ .

**Step 3** If labels of some boundary points have been changed in step 2, go to step 1; else, finish.

## 4 Experimental Results

All experiments run on a 2.4 GHz Intel Pentium 4 CPU with 256 MB memory and the programs are written in C language together with Matlab. Experimental results are shown in Figs. 2 to 7.

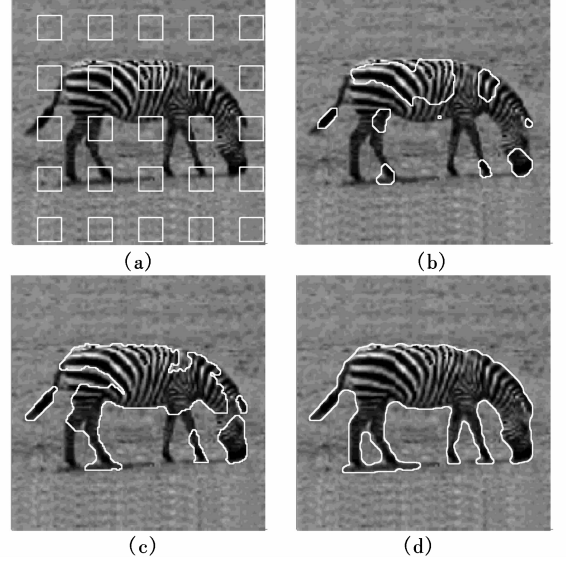
In Figs. 2 to 4,  $T=2$ . That is, the segment curve partitions the image into two areas:  $L=0$  and  $L=1$ . Figs. 3 and 4 are two challenging segmentation problems. Compared with Ref. [1], we obtain comparable results with the proposed method, which improves speed by more than 100 times, and one can refer to Ref. [1] for the comparison of results. The notable progress in runtime is partly due to the evolution policy and partly due to our much easier estimation of the entropy.



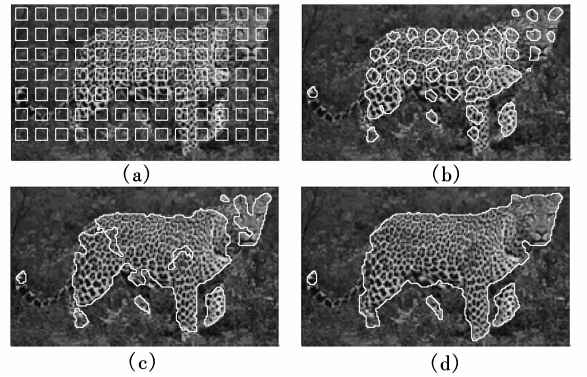
**Fig. 2** Segmentation of plane image (Image size is  $200 \times 100$ ,  $\alpha = 0.35$ ,  $\beta = -0.08$ ,  $L \in \{0, 1\}$ , runtime is 0.484 s.) (a) Initial curve; (b) 20 iterations; (c) 50 iterations; (d) 100 iterations

In Fig. 5, there are three regions, the sky, the cloud and the tree. In Fig. 6, there are the background, the white matter and the gray matter. So we choose  $T=3$  for the two images.

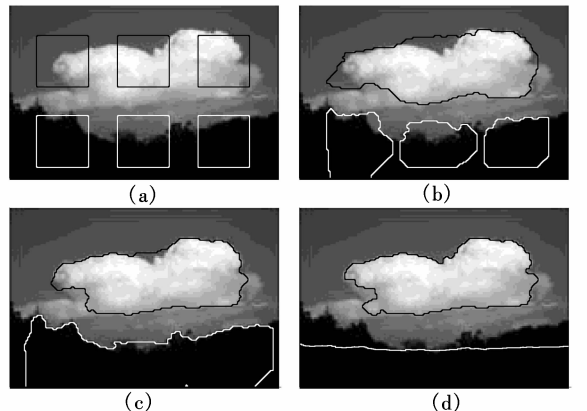
In Fig. 7, the background and the three shapes have four different gray values, so we choose  $T=4$ . Different from the above results, we use the image of label function to display the evolution process. As shown in Fig. 7(d), different regions in the original image have different label values.



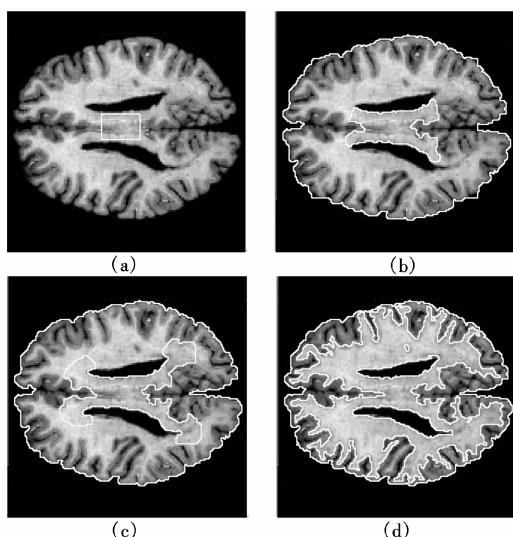
**Fig. 3** Segmentation of zebra image (Image size is  $200 \times 200$ ,  $\alpha = 0.636$ ,  $\beta = -0.41$ ,  $L \in \{0, 1\}$ , runtime is 1.454 s.) (a) Initial curve; (b) 50 iterations; (c) 80 iterations; (d) 140 iterations



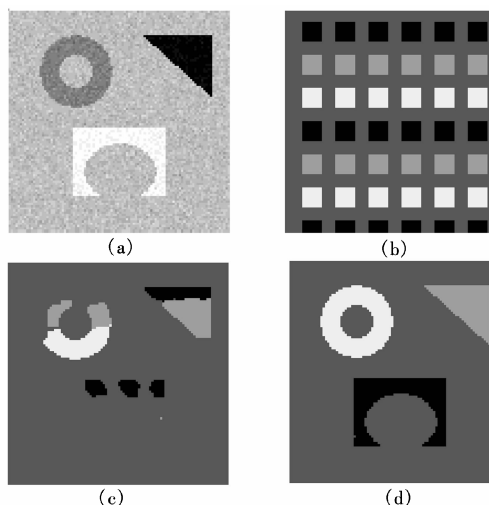
**Fig. 4** Segmentation of leopard image (Image size is  $200 \times 118$ ,  $\alpha = 0.45$ ,  $\beta = -0.28$ ,  $L \in \{0, 1\}$ , runtime is 0.625 s.) (a) Initial curve; (b) 14 iterations; (c) 28 iterations; (d) 50 iterations



**Fig. 5** Segmentation of cloud image (Image size is  $150 \times 100$ ,  $\alpha = 0.4$ ,  $\beta = 0$ ,  $L \in \{0, 1, 2\}$ , runtime is 0.344 s.) (a) Initial curve; (b) 10 iterations; (c) 20 iterations; (d) 35 iterations



**Fig. 6** Segmentation of brain image (Image size is  $160 \times 160$ ,  $\alpha = 0.25$ ,  $\beta = 0.5$ ,  $L \in \{0, 1, 2\}$ , runtime is 1.843 s.) (a) Initial curve; (b) 20 iterations; (c) 40 iterations; (d) 80 iterations



**Fig. 7** Segmentation of artificial image (Image size is  $100 \times 100$ ,  $\alpha = 0.3$ ,  $\beta = -0.1$ ,  $L \in \{0, 1, 2, 3\}$ , runtime is 0.328 s.) (a) Initial image; (b) initial labels; (c) 6 iterations; (d) 34 iterations

## 5 Conclusion

In this paper, we propose an entirely discrete Kim-Fisher-like model on lattices. First, regarding the gray values as discrete-valued random variables makes it possible to make a much simpler estimation of conditional entropy. Secondly, we present a uniform method within the level set framework for two-phase and multiphase segmentations without any extension. Finally, we present a more accurate formulation to calculate the curve length on the lattices with multi-labels. Compared with the continuous Kim-Fisher model, the model proposed in this paper can obtain comparable results, while the runtime is dramatically reduced.

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## 格点上的一种统计分割模型

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**摘要:** 为了克服 Kim-Fisher 模型实现难度大、运行速度慢的问题, 提出了离散的近似 Kim-Fisher 模型. 该离散模型的目标函数直接定义在格点上, 采用贪心法进行优化. 首先, 把图像的灰度值视为离散的随机变量, 从而可以采用更为简单的方法估计条件熵. 其次, 针对基于水平集技术的二区域和多区域图像分割, 提出一种无须扩展的统一的方法. 最后, 还提出一种多标号格点上曲线长度的近似方法, 该方法比现有的方法更加准确. 实验结果表明, 同传统的连续 Kim-Fisher 模型相比, 所提出的模型在取得相当的分割效果的同时, 简化了实现过程, 并大大降低了运行时间.

**关键词:** 图像分割; 曲线演化; 条件熵; 格点; 标号问题

**中图分类号:** TP391