

Theoretical framework for distributed reduction in concept lattice

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Abstract: In order to reduce knowledge reasoning space and improve knowledge processing efficiency, a framework of distributed attribute reduction in concept lattices is presented. By employing the idea similar to that of the rough set, the characterization of core attributes, dispensable attributes and unnecessary attributes are described from the point of view of local formal contexts and virtual global contexts. A determinant theorem of attribute reduction is derived. Based on these results, an approach for distributed attribute reduction is presented. It first performs reduction independently on each local context using the existing approaches, and then local reducts are merged to compute reducts of global contexts. An algorithm implementation is provided and its effectiveness is validated. The distributed reduction algorithm facilitates not only improving computation efficiency but also avoiding the problems caused by the existing approaches, such as data privacy and communication overhead.

Key words: distributed reduction; knowledge processing; formal context

Formal concept analysis (FCA) has been proved to be an effective tool for data analysis and knowledge processing^[1-2]. It allows us to identify meaningful groupings of objects that have common attributes. These groupings form a complete partial order called a concept lattice. Each node in such a lattice is a formal concept and the lattice can be interpreted as the possibility of generalizing or specializing a concept. FCA has been developed very rapidly in recent years. It has been applied to a wide variety of fields^[3-9], such as machine learning, information retrieval, software engineering and knowledge discovery.

One of the important problems in knowledge processing is attribute reduction. Attribute reduction in a concept lattice involves finding minimal attribute sets which determine a concept lattice that is isomorphic to the one determined by all the attributes while the objects set remains unchanged^[2, 10]. A few works have focused on the reduction in concept lattices^[11-15]. However, to the best of our knowledge, no proposal for distributed attribute reduction in concept lattices exists. In a distributed environment, the virtual global dataset is geographically distributed across multiple sites. For example, the sale datasets of hundreds of chain stores are stored at different locations. To compute the reduction in these datasets, we can convey all distributed data-

sets to a central site, and these materialize the whole dataset and perform the reduction tasks with the existing approaches. However, due to expensive communication overhead and privacy concerns, the datasets cannot be moved to a centralized site. Moreover, because of huge storage requirements, a central site may not be able to mine the datasets as a whole, but it would have to repartition it in some way. Thus it is important to develop a distributed algorithm to solve this problem.

This paper presents an approach to computing attribute reduction in a distributed environment. The basic idea of the approach is to utilize local results, which are extracted independently on each web site, in order to obtain global results. The core attributes, dispensable attributes and unnecessary attributes are characterized from the point of view of local contexts and global contexts. Based on these results, an algorithm is developed and its effectiveness is validated.

1 Basic Notions

Concept analysis starts with a formal context $K = (O, A, I)$, where O and A are the set of objects and the set of attributes, respectively, and $I \subseteq O \times A$ is a binary relationship between O and A .

For any set of objects $X \subseteq O$, the set of common attributes contained in X is defined as

$$X' = \{a \in A \mid \forall o \in X, (o, a) \in I\} \quad (1)$$

Similarly, the set of common objects for any set of attributes $Y \subseteq A$ is defined as

$$Y^* = \{o \in O \mid \forall a \in Y, (o, a) \in I\} \quad (2)$$

A pair of the set of objects and the set of attributes (X, Y) is called a concept iff $Y^* = X$ and $X' = Y$. The set of concepts is partially ordered via $(X_1, Y_1) \leq (X_2, Y_2) \Leftrightarrow X_1 \subseteq X_2 \Leftrightarrow Y_2 \supseteq Y_1$. The set of all concepts and the partial order \leq form a concept lattice, denoted by $L(K)$.

Given a formal context $K = (O, A, I)$ and $D \subseteq A$, we define $I_D = I \cap (O \times D)$. Then $K_D = (O, D, I_D)$ is also a formal context. It is easy to prove that there certainly exists $(X, Y) \in L(K)$ for any $(X, Z) \in L(K_D)$. If for any $(X, Y) \in L(K)$ there exists a concept $(X, Z) \in L(K_D)$, then D is called a consistent set of K . And further, if for any $d \in D$, $D - \{d\}$ is not a consistent set, then D is called a reduct of K ^[2, 11].

The reducts for any finite formal contexts certainly exist but are unnecessarily unique. This paper mainly focuses on finite formal contexts.

Let the set of all reducts of K be denoted by $\text{Red} = \{D_i \mid D_i \text{ is a reduct, } i \in J\}$ (J is an index set). Assume that $C = \bigcap_{i \in J} D_i$, $R = \bigcup_{i \in J} D_i - \bigcap_{i \in J} D_i$, $U = A - \bigcup_{i \in J} D_i$. The attributes of K

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can be classified into three types^[11]:

- ① a is a core attribute if $a \in C$;
- ② a is a dispensable attribute if $a \in R$;
- ③ a is an unnecessary attribute if $a \in U$.

The attribute a is called a reducible attribute if it is not the core attribute. That is to say, it is either a dispensable attribute or an unnecessary attribute.

2 Attribute Reduction in Distributed Concept Lattice

Ref. [1] suggested that the reduction of a concept lattice can be transformed into the reduction of a formal context. In this section, we will discuss the reduction from the point of view of a formal context. In the following, we write a^* instead of $\{a\}^*$ for the sake of simplicity.

Given a finite formal context $K = (O, A, I)$ and $D \subseteq A$, it is obvious that D is a consistent set of K iff for any $a \in A - D$, $(a^* \cap D)^* = a^*$. D is a reduct iff $(d^* \cap (D - \{d\}))^* \neq d^*$ for any $d \in D$ ^[15-16]. For any attribute $a \in A$, a is a reducible attribute iff $(a^* \cap (A - \{a\}))^* = a^*$. a is a core attribute iff $(a^* \cap (A - \{a\}))^* \neq a^*$.

Definition 1 Let $K = (O, A, I)$ be a formal context. For any $a, b \in A$, a is equivalent to b iff $a^* = b^*$. The equivalence class of a is denoted by

$$[a] = \{b \in A \mid b^* = a^*\} \quad (3)$$

Obviously, the attributes within an equivalence class are of the same attribute type. That is, if a is a reducible attribute, then all the attributes in $[a]$ are also reducible attributes.

Theorem 1^[15] Let $K = (O, A, I)$ be a formal context. $\forall a \in A$, the following two propositions hold:

- ① a is an unnecessary attribute iff $(a^* \cap (A - [a]))^* = a^*$;
- ② a is a dispensable attribute iff $(a^* \cap (A - [a]))^* \neq a^*$ and $[a] \supset \{a\}$.

Definition 2 Let $K = (O, A, I)$ be a formal context. For any $a \in A$, the super set of a in K is defined as

$$S(a) = \{b \in A \mid b^* \supset a^*\} \quad (4)$$

Definition 3 Let $K = (O, A, I)$ be a formal context and $E \subseteq A$. E is called a determinant set of a , if for $a \in A$, $E \cap [a] = \emptyset$ and $E^* = a^*$. And further, E is called a minimal determinant set if for any $b \in E$, $(E - \{b\})^* \neq a^*$.

The determinant sets of core attributes and dispensable attributes do not exist, whereas the minimal determinant sets of unnecessary attributes certainly exist but are not unique.

In a distributed environment, the virtual whole formal context is partitioned into N parts. Each part is stored in different websites. For the sake of simplicity, we limit the following discussion to a context that is vertically partitioned into only two parts. It can easily generalize the results to N partitions, for any value of N .

Definition 4 Given a formal context $K = (O, A, I)$, the contexts $K_1 = (O_1, A_1, I_1)$ and $K_2 = (O_2, A_2, I_2)$ are called two vertical partitions of K , if $A_1 \cup A_2 = A$, $I_1 \cup I_2 = I$, $O_1 \cup O_2 = O$ and $O_1 \cap O_2 = \emptyset$.

Assume that K_1 and K_2 are two vertical partitions of K .

Let the mapping $*$ on the attributes set and the mapping $'$ on objects set in K_1, K_2, K be denoted by $*_1, *_2, *$ and $'_1, '_2, '$, respectively. The equivalence class of a in K_1, K_2, K are respectively denoted by $[a]_1, [a]_2, [a]$. Thus, $[a] = [a]_1 \cap [a]_2$, $a^* = a^{*1} \cup a^{*2}$, $a^{*'} = a^{*1'} \cap a^{*2'}$. Since $O_1 \cap O_2 = \emptyset$, it follows that $o' = o'_1$ if $o \in O_1$. Otherwise, $o' = o'_2$ if $o \in O_2$. In the following, the mappings $'_1, '_2, '$ in K_1, K_2, K are denoted by $'$ for brevity.

Theorem 2 Let K_1 and K_2 be two vertical partitions of context $K = (O, A, I)$, then the core attributes of K_1, K_2 are also the core attributes of K .

Proof Let the sets of the core attributes of K_1, K_2, K be denoted respectively by C_1, C_2, C , it needs to be proved that for any $a \in C_1 \cup C_2$, $(a^{*'} \cap (A - \{a\}))^* \neq a^*$.

If $a \in C_1$, then $(a^{*'} \cap (A - \{a\}))^{*1} = (a^{*1'} \cap a^{*2'} \cap (A - \{a\}))^{*1} \supseteq (a^{*1'} \cap (A - \{a\}))^{*1} \supseteq a^{*1}$. Thus, $(a^{*'} \cap (A - \{a\}))^* = (a^{*'} \cap (A - \{a\}))^{*1} \cup (a^{*'} \cap (A - \{a\}))^{*2} \supseteq (a^{*1} \cup (a^{*'} \cap (A - \{a\}))^{*2}) \supseteq a^*$, i. e., $(a^{*'} \cap (A - \{a\}))^* \supset a^*$. Similarly, we can prove that $(a^{*'} \cap (A - \{a\}))^* \neq a^*$ if $a \in C_2$.

Theorem 3 Let K_1 and K_2 be two vertical partitions of $K = (O, A, I)$. For any $a \in A$, the super sets and the set of all the minimal determinant sets of a in K_1, K_2 are denoted by $S_1(a), S_2(a)$ and $EQ_1(a), EQ_2(a)$, respectively. Then the following two propositions hold:

- ① a is an unnecessary attribute of K if one of the following conditions holds;
- ② a meets at least one of the following conditions if it is an unnecessary attribute of K .
 - (i) $EQ_1(a) \cap EQ_2(a) \neq \emptyset$;
 - (ii) $[a]_1 \cap S_2(a) \neq \emptyset$ and $[a]_2 \cap S_1(a) \neq \emptyset$;
 - (iii) $[a]_1 \cap S_2(a) \neq \emptyset$, $[a]_2 \cap S_1(a) = \emptyset$ and $\exists E \in EQ_2(a), E \subseteq ([a]_1 \cup S_1(a)) \cap S_2(a)$;
 - (iv) $[a]_1 \cap S_2(a) = \emptyset$, $[a]_2 \cap S_1(a) \neq \emptyset$ and $\exists E \in EQ_1(a), E \subseteq ([a]_2 \cup S_2(a)) \cap S_1(a)$.

Proof It needs to be proved that $a^* = (a^{*'} \cap (A - [a]))^*$. Since $a^{*'} = a^{*1'} \cap a^{*2'} = ([a]_1 \cup S_1(a)) \cap ([a]_2 \cup S_2(a))$, $[a] = [a]_1 \cap [a]_2$, then $a^{*'} \cap (A - [a]) = ([a]_1 \cap S_2(a)) \cup (S_1(a) \cap S_2(a)) \cup ([a]_2 \cap S_1(a))$.

① For any $B \in EQ_1(a) \cap EQ_2(a)$, $B \subseteq S_1(a) \cap S_2(a)$, $B^{*1} = a^{*1}$ and $B^{*2} = a^{*2}$, therefore, $a^{*1} \subseteq (a^{*'} \cap (A - [a]))^{*1} \subseteq (S_1(a) \cap S_2(a))^{*1} \subseteq B^{*1} = a^{*1}$, $a^{*2} \subseteq (a^{*'} \cap (A - [a]))^{*2} \subseteq B^{*2} = a^{*2}$, i. e., $a^* = (a^{*'} \cap (A - [a]))^*$.

② It directly follows that $a^{*1} \subseteq (a^{*'} \cap (A - [a]))^{*1} \subseteq ([a]_1 \cap S_2(a))^{*1} = a^{*1}$, $a^{*2} \subseteq (a^{*'} \cap (A - [a]))^{*2} \subseteq ([a]_2 \cap S_1(a))^{*2} = a^{*2}$. That is, $a^* = (a^{*'} \cap (A - [a]))^*$.

③ As $[a]_1 \cap S_2(a) \neq \emptyset$, then $a^{*1} = (a^{*'} \cap (A - [a]))^{*1}$. According to the given condition, it follows that $a^{*2} \subseteq (a^{*'} \cap (A - [a]))^{*2} \subseteq (([a]_1 \cap S_2(a)) \cup (S_1(a) \cap S_2(a)))^{*2} \subseteq E^{*2} = a^{*2}$. Thus, $a^* = (a^{*'} \cap (A - [a]))^*$.

④ The proof is similar to ③.

If a is a reducible attribute, then $a^* = (a^{*'} \cap (A - [a]))^*$, i. e., $a^{*1} = (a^{*'} \cap (A - [a]))^{*1}$, $a^{*2} = (a^{*'} \cap (A - [a]))^{*2}$.

$[a]) \}^*$. As $a^* \cap (A - [a]) = ([a]_1 \cap S_2(a)) \cup (S_1(a) \cap S_2(a)) \cup ([a]_2 \cap S_1(a))$, Thus, we can easily derive the conclusion by decomposing and comparing the two sides of the equations.

By theorem 3, the super set for attribute $a \in A$ in K is $S(a) = ([a]_1 \cap S_2(a)) \cup (S_1(a) \cap S_2(a)) \cup ([a]_2 \cap S_1(a))$.

On the condition that a is not an unnecessary attribute, it follows that a is a dispensable attribute if $[a] \supset \{a\}$. Otherwise, if $[a] = \{a\}$, a is a core attribute.

Theorem 4 Let K_1 and K_2 be two vertical partitions of K . The family of the minimal determinant sets for any $a \in A$ in K can be denoted by

$$\begin{aligned} EQ(a) = & (EQ_1(a) \cap EQ_2(a)) \cup \\ & \{ \{b, c\} \mid b \in [a]_1 \cap S_2(a), c \in [a]_2 \cap S_1(a) \} \cup \\ & \{ E \in EQ_1(a) \mid E \subseteq S_1(a) \cap ([a]_2 \cup S_2(a)) \} \cup \\ & \{ E \in EQ_2(a) \mid E \subseteq S_2(a) \cap ([a]_1 \cup S_1(a)) \} \end{aligned}$$

Proof According to the proof of theorem 3 and definitions 2 and 3, it can easily be proved.

The set of all dispensable attributes R for a given context $K = (O, A, I)$ can be denoted by $R = \bigcup_{i=1}^n R_i$, where $R_i \cap R_j = \emptyset$ and $\exists a \in R, R_i = [a]$, for $1 \leq i, j \leq n$. Let C be the set of the core attributes of K and $F = \{ \{f_1, f_2, \dots, f_n\} \mid f_i \in R_i, 1 \leq i \leq n \}$. The set of all the attribute reducts can be denoted by $Red = \{ F_i \cup C \mid F_i \in F \}^{[15]}$. Thus, we can easily derive all reducts if we obtain the core attributes and the dispensable attributes.

The results can easily be generalized to N partitions. If there exist N subcontexts, we first independently compute local reducts on each web site, then merge any two local reducts to obtain the results of context union. Thus, by merging and computing iteratively, the global attribute reducts can be obtained.

3 Algorithm Implementation

Based on the above discussions, we develop an algorithm to compute distributed attribute reduction in concept lattices. The pseudo code is shown in algorithm 1.

Assume that context K is vertically partitioned into K_i and K_j . Algorithm 1 is used to compute all the reducts of K . In algorithm 1, the sets of the core attributes of K_i, K_j, K are denoted by C_i, C_j, C , respectively. The sets of dispensable attributes and unnecessary attributes of K are denoted by R and U . All the attributes are initialized not to be visited.

Algorithm 1 A framework for distributed attribute reduction in a concept lattice

Input: Reducts information about K_i, K_j ;

Output: All reducts of K .

for $a \in C_i \cup C_j$ and a not visited

$C = C \cup \{a\}$

visit a

end for

for $a \in A$ and a not visited

$[a] = [a]_i \cap [a]_j$

$S(a) = (S_i(a) \cup [a]_i) \cap (S_j(a) \cup [a]_j) - [a]$

calculate $EQ(a)$

if $EQ(a) \neq \emptyset$

$U = U \cup [a]$

else if $[a] \neq \{a\}$

$R = R \cup [a]$

else $C = C \cup \{a\}$

end if

end if

visit $[a]$

end for

by R and U , compute all the reducts of K

Given a context, we can employ an idea similar to an *a priori* algorithm^[17] to compute minimal determinant sets for attributes. The pseudo code is shown in algorithm 2.

Algorithm 2 Determinant set for attributes

Input: A context $K = (O, A, I)$;

Output: The super set and the minimal determinant set of all the attributes.

for $a \in A$ and a not visited

$S(a) = \{b \in A \mid b^* \supset a^*\}$

if a is not an unnecessary attribute

$EQ(a) = \emptyset$

else

$k = 2$, flag = false

$CDS_1 = \emptyset$ // candidate determinant sets

while ($k \leq \|S(a)\|$ and not flag)

$CDS_k = \{S \subseteq S(a) \mid \|S\| = k \text{ and not exist } E \in CDS_{k-1}$

such that $E \subset S\}$

if $CDS_k = \emptyset$

flag = true

else $EQ(a) = EQ(a) \cup \{S \in CDS_k \mid S^* = a^*\}$

end if

$k = k + 1$

end while

end if

visit $[a]$

end for

To obtain the global core attributes and the dispensable attributes, algorithm 1 needs to perform operations on set unions and set intersections. Assuming that attributes are partially ordered, the time complexity for computing an attribute's super set and minimal determinant set is $O((p + q) \times C_k^{\lfloor k/2 \rfloor})$, where p and q are the average cardinalities of its minimal determinant sets in K_i, K_j , respectively. Therefore, in the worst case, the time complexity of algorithm 1 is $O(\|A\| \times (p + q) \times C_k^{\lfloor k/2 \rfloor})$.

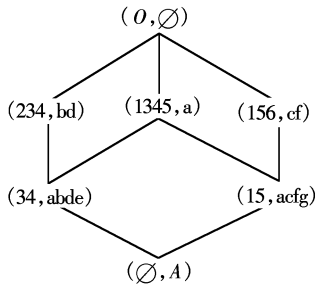
4 Example

Tab. 1 shows a formal context $K = (O, A, I)$, where $O = \{1, 2, 3, 4, 5, 6\}$ and $A = \{a, b, c, d, e, f, g\}$. The context contains seven formal concepts and the lattice $L(O, A, I)$ is shown in Fig. 1.

Tab. 1 Formal context (O, A, I)

Object	Attribute						
	a	b	c	d	e	f	g
1	x		x			x	x
2		x		x			
3	x	x		x	x		
4	x	x		x	x		
5	x		x			x	x
6			x			x	

Assume that context K is vertically partitioned into two contexts $K_1 = (O_1, A, I_1)$ and $K_2 = (O_2, A, I_2)$, where $O_1 = \{1,$

Fig. 1 $L(O, A, I)$

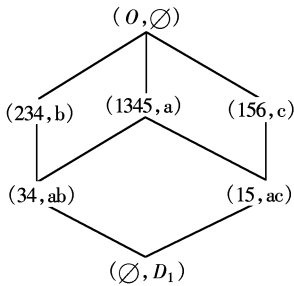
2, 3} and $O_2 = \{4, 5, 6\}$. By performing independent reduction on K_1 and K_2 , we obtain that $C_1 = C_2 = \{a\}$, $R_1 = \{c, f, g\} \cup \{b, d\}$, $R_2 = \{b, d, e\} \cup \{c, f\}$, $U_1 = \{e\}$ and $U_2 = \{g\}$. The set of

equivalence classes, the super set and the minimal determinant set for attributes in K_1 and K_2 are shown in Tab. 2.

Considering the attributes in the global context K , a is a core attribute because it is also a core attribute of K_1 and K_2 . As $[b] = [d] = \{b, d\}$, $[c] = [f] = \{c, f\}$ and b, d and c, f are not unnecessary attributes, thus they are dispensable attributes in the global context. As $[e]_1 \cap S_2(e) = \emptyset$, $[e]_2 \cap S_1(e) = \{a\}$, $\{a, b\} \in EQ_1(e)$ and $\{a, b\} \subseteq ([e]_2 \cup S_2(e)) \cap S_1(e) = \{a, b, d\}$, thus e is an unnecessary attribute. Similarly, g is also an unnecessary attribute. Thus, $C = \{a\}$, $R = \{b, d\} \cup \{c, f\}$ and $U = \{e, g\}$. The attribute reducts of K are $D_1 = \{a, b, c\}$, $D_2 = \{a, b, f\}$, $D_3 = \{a, d, c\}$ and $D_4 = \{a, d, f\}$. The concept lattice $L(O, D_1, I_{D_1})$ is shown in Fig. 2.

Tab. 2 Results from K_1 and K_2

Attribute	K_1			K_2		
	$[\cdot]_1$	$S_1(\cdot)$	$EQ_1(\cdot)$	$[\cdot]_2$	$S_2(\cdot)$	$EQ_2(\cdot)$
a	$\{a\}$	\emptyset	\emptyset	$\{a\}$	\emptyset	\emptyset
b	$\{b, d\}$	\emptyset	\emptyset	$\{b, d\}$	\emptyset	\emptyset
c	$\{c, f, g\}$	\emptyset	\emptyset	$\{c, f\}$	\emptyset	\emptyset
d	$\{b, d\}$	\emptyset	\emptyset	$\{b, d\}$	\emptyset	\emptyset
e	$\{e\}$	$\{a, b, d\}$	$\{\{a, b\}, \{a, d\}\}$	$\{b, d, e\}$	$\{a\}$	\emptyset
f	$\{c, f, g\}$	$\{a\}$	\emptyset	$\{c, f\}$	\emptyset	\emptyset
g	$\{c, f, g\}$	$\{a\}$	\emptyset	$\{g\}$	$\{a, c, f\}$	$\{\{a, c\}, \{a, f\}\}$

Fig. 2 $L(O, D_1, I_{D_1})$

5 Conclusion

This paper presents an approach to computing attribute reduction in a distributed environment. It can be decomposed into two steps. First, local reductions are independently performed on each website, and then the local results are merged to obtain global results. In this paper we assume that a virtual single formal context is partitioned into many parts and stored in multiple websites. We limit our discussion on vertical partition in that their object sets have no intersections and their unions of object sets cover all objects. However, in a distributed environment, a context can be horizontally partitioned into many parts. These parts may have the same object sets while their attribute unions of object sets cover all attributes. We plan to consider a reduction in these kinds of contexts and perform data analysis on a reduced concept lattice. Additionally, further work also includes applying and testing our approach with real-world datasets where possible.

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基于概念格的分布式约简理论框架

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摘要:为了缩减知识推理空间,提高分布式环境下知识处理的效率,提出分布式概念格属性约简的理论框架.基于粗糙集理论的思想,从子形式背景和全局形式背景的角度,刻画了核心属性、相对必要属性和绝对不必要属性的属性特征,给出属性约简的判定定理.在此基础上,给出概念格的分布式属性约简方法:首先,使用现有的约简方法分别计算各子形式背景的约简,然后,逐一利用各子背景的约简,通过合并计算得到全局形式背景的约简.给出了算法的实现并用实例验证了它的有效性.分布式约简有效避免了使用现有方法而引起的数据安全和网络通信等问题,提高了约简的计算效率.

关键词:分布式约简;知识处理;形式背景

中图分类号:TP391