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Calculation connectivity reliability of road networks based on recursive decomposition arithmetic

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Abstract: In order to decrease the calculation complexity of connectivity reliability of road networks, an improved recursive decomposition arithmetic is proposed. First, the basic theory of recursive decomposition arithmetic is reviewed. Then the characteristics of road networks, which are different from general networks, are analyzed. Under this condition, an improved recursive decomposition arithmetic is put forward which fits road networks better. Furthermore, detailed calculation steps are presented which are convenient for the computer, and the advantage of the approximate arithmetic is analyzed based on this improved arithmetic. This improved recursive decomposition arithmetic directly produces disjoint minipaths and avoids the non-polynomial increasing problems. And because the characteristics of road networks are considered, this arithmetic is greatly simplified. Finally, an example is given to prove its validity.

Key words: recursive decomposition arithmetic; road network; connectivity reliability; disjoint minipath; topological structure

The study of the reliability of road networks is more recent than that of other networks. It has been no more than twenty years since road network reliability was paid attention to[1]. The contents of road network reliability have changed along with changes in society, economy, traffic technology etc. Their contents mainly include three aspects: connectivity reliability, travel time reliability and capacity reliability[2]. Although road connectivity reliability has only two extreme states, the method of combining topological structure with path searching is the foundation of road reliability. So road connectivity reliability is the foundation of road network reliability. It has an important meaning that a better connectivity reliability calculation method is presented which takes into account actual road network characteristics.

The topological structure reliability of common networks has many mature calculation methods. They can be used in the calculation of road connectivity reliability given some transformations. These calculation methods can be classified into a random simulation method and a probability resolution method. The theory of the random simulation method is simpler and its realization is easier. But its precision is difficultly forecasted. The probability resolution method gives a precise or half precise model using the technology of network analysis. And it can be classified into the minipath sets method, the probability analysis method, the cut graph method and the real value method, etc. The minipath set method is often used. It first searches the minipath set of road networks, and then disjoints; then it finally calculates reliability by using the probability sum formula. The disadvantage of this method is that it needs more operation time in disjointing. Because it needs to search all the minipaths, and then all-around disjointing must be applied to them. An improved recursive decomposition arithmetic is put forward to calculate the road connectivity reliability. It directly searches the disjoint minipaths and the disjoint minicuts. Based on the characteristics of actual road networks, the proposed method is improved and optimized. The detailed steps and flowchart are also presented which is applicable to computer programming.

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1 Basic Theory of Recursive Decomposition Arithmetic

This paper is based on the following hypotheses:

1) The links in a road network merely include two states: reliable state and disabled state;

2) Only the links in a road network are likely to be disabled, and the nodes are fine.

The recursive decomposition arithmetic is a probability resolution method which is based on the minipath method. The basic theory is introduced[3-5]. A road network is regarded as a kind of topological structure. The structure function (Structure function is the logical merge and denotes system reliable state) of any OD pair can be expressed as

$$\Psi(S) = \bigcup_{i=1}^{m} A_i \tag{1}$$

where $\Psi(\cdot)$ is the system structure function; A_i is the i-th minipath, and m is the total number.

By the use of the laws of absorbing and mutually excluding, Eq. (1) is transformed into

$$\Psi(S) = A_1 \cup (\bigcup_{i=2}^{m} A_i) = A_1 + \overline{A_1}\Psi(S) \tag{2}$$

where $\overline{A_i}$ is the logical reverse of A_i. If $A_i = a_1 a_2 \ldots a_k$, a_i is the i-th reliable link of minipath A_i, then

$$\overline{A_1} = \overline{a_1} + a_1\overline{a_2} + \ldots + a_1 a_2 \ldots \overline{a_k} \tag{3}$$

where $\overline{a_i}$ denotes the disabled state of link a_i.

Combining Eq. (3) with Eq. (2), and by the use of Bull's law, Eq. (2) becomes

$$\Psi(S) = A_1 + \overline{a_1}\Psi(S_{-a_1}) + a_1\overline{a_2}\Psi(S_{-a_2}) + a_1 a_2 \ldots \overline{a_k}\Psi(S_{-a_k}) \tag{4}$$

where $S_{-a_i}(i=1,2,\dots,m)$ is the sub-network in which a_i is deleted in the original network. The items including structure function $\Psi(S_{-a_i})$ of non-connected sub-network S_{-a_i} are deleted in Eq. (4). But if sub-network S_{-a_i} is connected, the structure function $\Psi(S_{-a_i})$ is decomposed according to Eqs. (2) and (3). Then Bull's operation is done in Eq. (4). So the decomposition format of original reliability function is obtained. Do this repeatedly, until the decomposed formula of $\Psi(S)$ cannot be decomposed further. The topological structure function which is expressed by disjoint minipaths is obtained as

$$\Psi(S) = A_1 + \sum_{i=2}^{M} L_i = \sum_{i=1}^{M} L_i \tag{5}$$

where L_i is the i-th minipath, $L_1 = A_1$; and M is the total number of disjoint minipaths.

In the course of decomposition of Eq. (1), the Bull's clusters before structure function $\Psi(S_{-a_i})$ with non-connected sub-networks is a disjoint minicut of the original network. Sum up all these disjoint minicuts and have

$$1 - \Psi(S) = \sum_{j=1}^{N} C_j \tag{6}$$

where C_j is the j-th disjoint minicut, and N is the total number.

According to Eqs. (5) and (6), the reliability $p_r(S)$ and disabled probability $p_f(S)$ of a network system can be expressed as

$$p_r(S) = \sum_{i=1}^{M} p_r(L_i), \quad p_f(S) = \sum_{j=1}^{N} p_r(C_j)$$

where $p_r(\cdot)$ is the probability of random events. Fig. 1 is a simple network with link weights. And the graphic process of analyzing this example reliability by way of the recursive decomposition method is given in Fig. 2[6-7]. According to Fig. 2, the reliability and disabled rate of the topological structure is

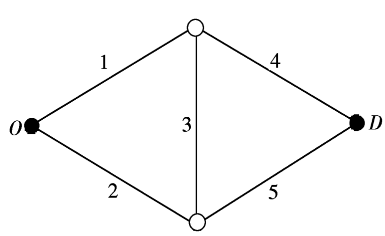

Fig. 1 Example of link weight network

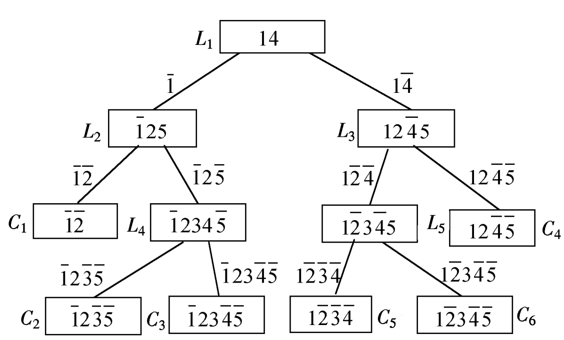

Fig. 2 Recursive decomposition process of the example

$$p_r\{S\} = \sum_{i=1}^{5} p_r\{L_i\} = p_r\{14\} + p_r\{\bar{1}25\} + p_r\{12\bar{4}5\} + p_r\{\bar{1}2345\} + p_r\{\bar{1}234\bar{5}\} \tag{7}$$

$$p_f\{S\} = \sum_{j=1}^{6} p_r\{C_j\} = p_r\{\bar{1}\bar{2}\} + p_r\{12\bar{4}\bar{5}\} + p_r\{\bar{1}2\bar{3}\bar{5}\} + p_r\{\bar{1}23\bar{4}\bar{5}\} + p_r\{1\bar{2}\bar{3}\bar{4}\} + p_r\{1\bar{2}3\bar{4}5\} \tag{8}$$

2 Improved Recursive Decomposition Arithmetic and Application in Road Networks

2.1 Characteristic analysis of road networks

Road networks do not only share common characteristics of other networks, but have some special ones. These characteristics mainly include:

1) A travel route choice takes the initiative in the road networks. The medium in the road networks is greatly different from the ones in other networks. The medium in the road networks is humans who take the initiative when a route is chosen. Although a road network is also a network-carrying medium, it is very different from other engineering networks, such as an electricity network, a water network, etc. A human can choose his own routes initially. The routes with the fewest impedances are always selected (They may be time, fare and comfort.). Then the next path with more impedance may be taken into account.

2) Road networks can be simplified according to the actual states. A road network is a kind of large-scale network, and it may have thousands of links. The premise of reliability calculation is simplification. Compared with other networks, the links disabled in road networks are not absolutely random, but they are correlative. Such as, it is not all links with tolerably large disabled rate. That is to say that disabled links only occur to some local units, and others have greater reliability. Suppose that the links which have greater disabled rates can be confirmed. For the sake of simplifying calculations, we consider that the disabled road networks are only due to those links which have greater disable rates, and other links are regarded as fully reliable. That is to say, these reliable links are only considered when searching the shortest path, and are not considered when calculating reliability. In this way, complexity is greatly reduced, and efficiency is improved. The precision of the results is reasonable.

3) These disabled links are correlative in road networks. The correlation of these disabled links includes two categories: links correlation and region correlation. The former indicates that these links are correlative in nature, and the latter indicates that these links are influenced by the same region. These correlative links are blocked and discharged together[2]. So these links have correlative reliability and disable rates. When carrying through reliability analysis, it is necessary to confirm the correlative segments first.

Under these conditions, road network reliability about routes and OD can be subsequently calculated.

2.2 Improved recursive decomposition arithmetic and application in road networks

2.2.1 Precise arithmetic

According to the characteristics of road networks and considering the actual states of links, an improved recursive decomposition arithmetic is presented, which fits connectivity reliability calculation of road networks better. The detailed calculation steps are as follows:

1) Data preparation and procedure initialization

Step 1 Input known data, including links impedance, reliability index, and other correlative information. Limit the searching area of the shortest path. Initialize parameter $C = 1$.

2) Search the shortest paths between the given *OD* pair and judge *C*.

Step 2 If $C = 1$, search the shortest path (It can be the shortest distance, time or total fare.) in the remainder network (The remainder network is so-called because this network is taken off these decomposed links).

Step 3 Judge whether the spare shortest path (namely the shortest path in the remainder network) is in the search area or not. If it is, skip to step 4; otherwise, skip to step 7.

Step 4 Judge whether these units in the spare shortest path are correlative with the former decomposed units or not. If they are, skip to step 6, otherwise skip to step 5.

Step 5 Decompose the new added units in the shortest path, and decomposition only done among the known disabled units. In the process of decomposition, the units with the least reliability are first decomposed, then the second one, etc.

Step 6 Judge whether the remainder network in the search area is connected or not. If it is, set $C = 1$ and go to step 2, otherwise set $C = 0$ and skip to step 7.

3) The connectivity reliability calculation between the given *OD* pair.

Step 7 Classify and judge the decomposition results, calculate the connectivity reliability and disabled rate.

Step 8 Output the results of the connectivity reliability between the given *OD* pair.

The flowchart of the improved recursive decomposition arithmetic is shown in Fig. 3.

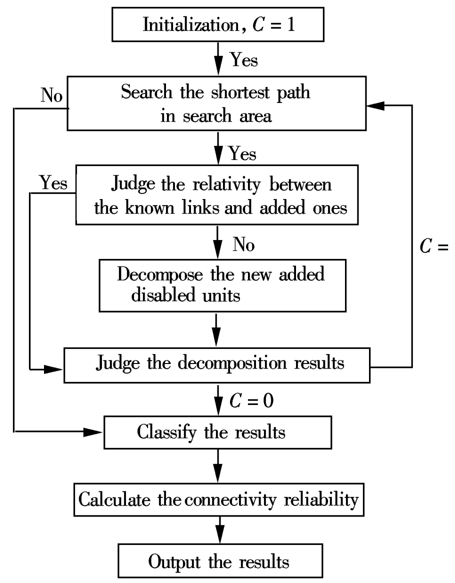

Fig. 3 The flowchart of improved recursive decomposition arithmetic

The improved recursive decomposition arithmetic fits actual road networks better than that of the original one. The advantages are given below:

① In the improved arithmetic, the minipath is replaced by the shortest path. In this way, blindness in searching is avoided, and this method fits actual road networks better. At the same time the searching area is easily limited when the minipath is replaced by the shortest path;

② In the decomposition process of the shortest path units, the actual states are considered. Those units having greater disabled rates are just decomposed. The number being decomposed is greatly decreased, and so is operation time. The speed of calculation is improved;

③ The correlation among links is taken into account when the shortest path in the remainder network is searched. It is not necessary that all the rational sets be found at one time and replaced step by step[2].

So the improved recursive decomposition arithmetic is more compatible with actual road networks, and it is in favor able in calculating connectivity reliability of road network within rational bounds.

2.2.2 Approximate arithmetic

A road network is a kind of large-scale topological structure, and it is always made up of thousands of links and nodes. It is difficult to obtain precise results by usual arithmetic. So it is necessary to find an effective approximate method to reduce the complexity and save operation time. In the improved recursive decomposition arithmetic disjoint minipaths and minicuts are obtained in turn. So it is more convenient to make use of the Bonforrieni inequality, especially regarding complicated road networks[8-9]. The bounds of reliability are as

$$p_r\{\sum_{i=1}^{M_A} L_i\} \leqslant R_{sys} \leqslant 1 - p_r\{\sum_{j=1}^{N_B} C_j\} \quad (9)$$

where M_A, N_B are the numbers of disjoint minipaths and disjoint minicuts, respectively; L_i is a disjoint minipath, and C_j is a disjoint minicut. The up and down bounds of road network reliability are given respectively. If the differences of two sides is less than a known error, the value of connectivity reliability can be obtained.

The approximate method in the improved recursive decomposition arithmetic has obvious advantages:

Because the recursive decomposition arithmetic can obtain the minipaths the minicuts at the same time, they need not be disjointed. By Eq. (9) we can calculate the approximate results at any moment in the process of decomposition.

3 Numerical Example

In this section we calculate the connectivity reliability of Sioux Falls road networks (see Fig. 4) using the improved recursive decomposition arithmetic described above. And we compare this method with the minipath sets method and the original recursive decomposition method. For simplification, the correlation is not considered in this example.

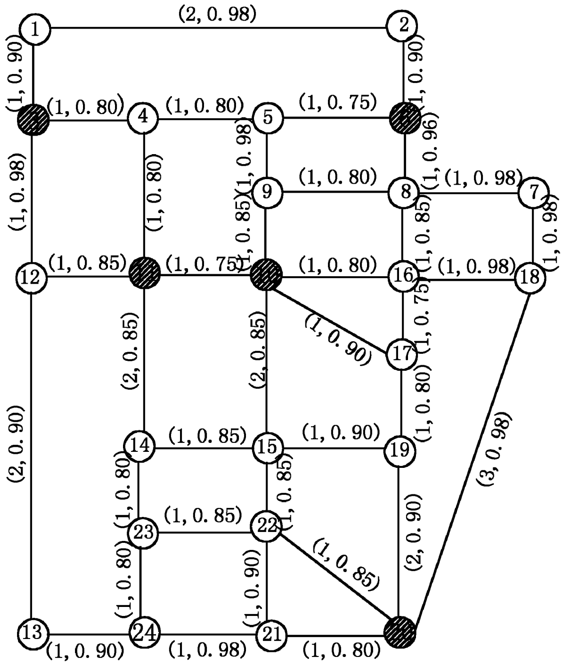

Fig. 4 Road network of Sioux Falls

Suppose the following are known: ① There are 24 nodes and 38 links, and the links are bidirectional. Nodes 3, 6, 10, 11 and 20 are original and destination points, and the other nodes are central points; ② In Fig. 4, impedance and reliability of links are marked in brackets. The former is impedance and the latter is reliability. The Matlab language is used to calculate the connectivity reliability of the road network. Suppose that the search bound is $L \leqslant 1.5L_0$, where L_0 is the shortest path impedance in the original network and L is the shortest path impedance in the remainder network. And we suppose that the paths searched are not less than two.

In Tab. 1, there is operation time using different arithmetics, including minipath sets arithmetic, original recursive decomposition arithmetic and improved recursive decomposition arithmetic introduced in this paper. Compile the Matlab program and run them in Matlab. It is obvious that the improved recursive decomposition arithmetic has the highest efficiency. Tabs. 2 and 3 give the reliability results using different calculation methods, the original recursive decomposition arithmetic and the improved one. In the improved arithmetic, those links reliabilities which are greater than 0.95 are ignored. By comparison, the improved recursive decomposition arithmetic can fully satisfy the precision.

Tab. 1 Operation time of different arithmetics

Methods	Disjoint minipath	Original recursive decomposition	Improved recursive decomposition
Operation time/s	4.358	1.956	1.129

Tab. 2 Results of original recursive decomposition arithmetic

O	D				
	3	6	10	11	20
3	—	0.893	0.852	0.940	0.951
6	—	—	0.935	0.920	0.949
10	—	—	—	0.883	0.936
11	—	—	—	—	0.949
20	—	—	—	—	—

Tab. 3 Results of improved recursive decomposition arithmetic

O	D				
	3	6	10	11	20
3	—	0.901	0.858	0.946	0.965
6	—	—	0.944	0.932	0.956
10	—	—	—	0.886	0.945
11	—	—	—	—	0.950
20	—	—	—	—	—

4 Conclusion

In this paper, the improved recursive decomposition arithmetic is presented to calculate network connectivity considering actual road network characteristics. The improved arithmetic fits the road networks better. Detailed steps and flowchart are given. Improving the operation time of calculation is always the most important problem involving road network connectivity. However, the improved recursive decomposition arithmetic is based on minipath sets, and the operation time is greatly limited. Further studies may focus on new calculation methods which are not based on minipath sets.

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基于递推分解法的路网连通可靠度计算

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摘要:为了降低道路网连通可靠度计算的复杂度,提出了基于递推分解法的可靠度计算方法.首先阐述了递推分解算法的基础理论,然后对道路网不同于常规网络的特性进行了分析,最后提出了适合于道路网络连通可靠度计算的改进的递推分解算法,同时给出了方便计算机编程实现的具体求解步骤,并对相应的上下限近似算法的优越性进行了分析.改进的递推分解算法打破了传统的先搜索最小路然后进行不交化的连通可靠度求解步骤,直接生成计算中涉及到的不交最小路,并充分考虑了道路网的实际特性,大大简化了计算,避免了可靠度计算中的 NP 难题.最后通过一简例,说明该算法的实用性.

关键词:递推分解算法;路网;连通可靠度;不交最小路;拓扑结构

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