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Fund budget model for multipurpose transit smart card systems

Zhang Ning Qian Zhendong Chen Kai Yang Liqiang

(Intelligent Transportation System Research Center, Southeast University, Nanjing 210096, China)

Abstract: The fund budget of multipurpose transit smart card systems is studied by stochastic programming to assign limited funds to different applications reasonably. Under the constraints of a gross fund, models of chance-constrained and dependent-chance for the fund budget of multipurpose transit smart card systems are established with application scale and social demand as random variables, respectively aiming to maximize earnings and satisfy the service requirements the furthest; and the genetic algorithm based on stochastic simulation is adopted for model solution. The calculation results show that the fund budget differs greatly with different system objectives which can cause the systems to have distinct expansibilities, and the application scales of some applications may not satisfy user demands with limited funds. The analysis results indicate that the forecast of application scales and application future demands should be done first, and then the system objective is determined according to the system mission, which can help reduce the risks of fund budgets.

Key words: multipurpose transit smart card systems; fund budgeting; stochastic programming; genetic algorithm

Multipurpose transit smart card systems are based on transit applications and can be expanded to other applications for service payments by smart cards[1−2]. Applications belonging to the public transportation sector include bus, ferry, taxi, light rail, metro, parking, gas stations, toll bridges or roads around the city, and even the management of drivers or vehicles. Applications of the non public transportation sector can be divided into several categories such as retail, finance, university, medical treatment, etc. In these applications, bus, metro, light rail and ferry services are transit applications. The multipurpose transit smart card systems greatly extend the ITS(intelligent transportation systems) application coverage, which can remarkably save user time, improve the efficiency of the application and more importantly, promote the use of public transportation. However, the funds required for the system construction is limited, and how to optimize it according to the characteristics of the city comes down to investment budget programming, that is, to reasonably allocate the limited funds into multiple applications.

In simple and certain conditions, cost-efficiency analysis[3] and integer programming with 0-1[4] can be applied to fund budgeting. During the decision making of complex systems with random parameters, methods can be adopted including the model of expected value, the chance-constrained programming model[5−7] and the dependent-chance programming model[8−9].

Multipurpose transit smart card systems are very complicated in implementation, and the fund budget is very important to systems development. As the most important factors such as application scale and user size contain great uncertainties, stochastic programming is applied to establish the fund budget model of multipurpose transit smart card systems.

1 Fund Budget Model for Multipurpose Transit Smart Card Systems

Suppose that the fund amount of the multipurpose transit smart card systems for allocation is a, the total number of applications being planned to be implemented at the same time is n, e. g., metro, bus, campus, retail and so on, and the life cycle of all the applications is T. In consideration of the costs of systems maintenance, operation and escalation, there is a certain investment for each application every year and the investment for the j-th year is P_j. Then

$$\sum_{j=0}^{T} P_j = a \tag{1}$$

Suppose that the i-th application can be divided into x_i unit systems and the average cost for every unit in the j-th year is a_{ij}, then there is a group of constraints:

$$\left.\begin{aligned} &a_{10}x_1 + a_{20}x_2 + \ldots + a_{n0}x_n \leqslant P_0 \\ &a_{11}x_1 + a_{21}x_2 + \ldots + a_{n1}x_n \leqslant P_1 \\ &\vdots \\ &a_{1T}x_1 + a_{2T}x_2 + \ldots + a_{nT}x_n \leqslant P_T \end{aligned}\right\} \tag{2}$$

Suppose that the process of the system fund budgeting is not limited by the investment of each year, the above constraints can be transformed into

$$\sum_{i=1}^{n} \sum_{j=0}^{T} a_{ij}x_i \leqslant \sum_{j=0}^{T} P_j \tag{3}$$

i. e.,

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n \leqslant a \tag{4}$$

where a_i is the average cost of unit system of the i-th application in the life cycle.

In addition, the number of the unit systems is limited to some level for each application. Then

$$x_i \leqslant b_i \qquad i = 1, 2, \ldots, n \tag{5}$$

where b_i is the total unit system number of the i-th applica-

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Biography: Zhang Ning (1972—), male, doctor, associate professor, ningzhang1972@yahoo.com.cn.

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tion.

Besides, we should also consider that the scale of each application should meet social requirements. The application scales and social requirements here respectively refer to the maximum numbers of customers that an application can cover and the customer numbers who require the service of the application (user size). Suppose that the scale of the unit system of application i is η_i and the future requirement of this application is ξ_i, then the constraint is

$$\eta_i x_i \geqslant \xi_i \qquad i = 1, 2, \dots, n \tag{6}$$

Suppose that the average earnings of the unit system of application i is c_{it} in the t-th year, then the benefits B of the multipurpose transit smart card systems in their lifecycle is

$$B = \sum_{i=1}^{n} \sum_{t=0}^{T} c_{it}(1 + i)^{-t} x_i \tag{7}$$

where i is the base earnings yield, which is used to discount the fund to the initial year of investment.

After picking up the factor x_i, Eq. (7) is predigested to

$$B = c_1 x_1 + c_2 x_2 + \dots + c_n x_n \tag{8}$$

where c_i is the accumulated value of the yearly unit system earnings of application i after being discounted to the initial year of investment.

The objectives of systems implementation may have two different definitions. On the one hand, with the constraint of the fund amount, the fund budget for the multipurpose transit smart card systems mainly solves the problem of how to reasonably allocate the funds to maximize the overall benefits during the development of each application, and in consideration of the uncertainty of future requirements and the requirements of multiple conflict objectives, the chance-constrained programming can be applied to solve this problem. The aim of the chance-constrained programming model is to maximize the benefits with random variables in constraints like application scale and user size, and the decision should be made before the random variables are observed. For the decision, it is allowed to dissatisfy the constraints to some certain extent, but the probability of the constraints to be true should be no less than a certain confidence level α_i. Then the chance-constrained programming model for the multipurpose transit smart card systems is set as follows:

$$\begin{aligned} &\max\ c_1 x_1 + c_2 x_2 + \dots + c_n x_n \\ &\text{s. t.} \\ &\quad a_1 x_1 + a_2 x_2 + \dots + a_n x_n \leqslant a \\ &\quad x_i \leqslant b_i \qquad i = 1, 2, \dots, n \\ &\quad \Pr\{\eta_i x_i \geqslant \xi_i\} \geqslant \alpha_i \qquad i = 1, 2, \dots, n \\ &\quad x_i,\ i = 1, 2, \dots, n, \quad \text{positive integer} \end{aligned} \tag{9}$$

On the other hand, what the multipurpose transit smart card systems provide is a kind of service, which aims to find the safest solution so as to maximize the probability that the service meets the future requirements. In the implementation of the multipurpose transit smart card systems, as limited by the total fund amount, those applications are dependent on and in conflict with each other. It is impossible to have a great investment for every application to ensure that it meets the requirements; it is only possible to make every applied service meet the requirements as much as possible, which means to maximize the probability of user size satisfying every application scale. Therefore, dependent-chance programming can be applied to model fund budgeting of multipurpose transit smart card systems. The model is as follows:

$$\begin{aligned} &\max\ \Pr\{\eta_i x_i \geqslant \xi_i\} \qquad i = 1, 2, \dots, n \\ &\text{s. t.} \\ &\quad a_1 x_1 + a_2 x_2 + \dots + a_n x_n \leqslant a \\ &\quad x_i \leqslant b_i \qquad i = 1, 2, \dots, n \\ &\quad x_i,\ i = 1, 2, \dots, n, \quad \text{positive integer} \end{aligned} \tag{10}$$

Due to the complexity of the target functions in formulae (9) and (10), the solving algorithm is required to be of high robustness so as to avoid plunging into local optimum solutions; the genetic algorithm has the merit of being adept at global searches, and it does not require conducting in-depth mathematical analysis on the nature of the optimization issue[10–12]. Hence, the genetic algorithm based on stochastic simulation is adopted as the solution method for the abovementioned model[13]. The specific algorithm is shown as follows:

① Assign the system parameters of the model and set the generation of evolution as 0;

② Generate the initial chromosome;

③ Check whether the individual meets the constraints. If no, go to step ②;

④ Calculate the target value and fitness function, and record the optimal individual;

⑤ Select the parents according to the fitness function;

⑥ Crossover process;

⑦ Check whether the individual meets the constraints. If no, go to step ⑥;

⑧ Mutation process;

⑨ Check whether the individual meets the constraints. If no, go to step ⑧;

⑩ Output the result of the current generation as required and add 1 to the generation of evolution;

⑪ Check if the generation of evolution is less than the maximum generation set in system parameters. If yes, go to step ④;

⑫ Output the best individual of the evolution.

2 Application Examples

One city, constructing multipurpose transit smart card systems, invests 120×10^6 yuan in six planned applications which are retail, ferry, taxi, parks and gardens, buses, and metro, respectively, that is $a = 12\,000$, $n = 6$. Suppose that the satisfactory confidence levels for formula (9) are all 90%; i. e., $\alpha_i = 0.90$; $i = 1, 2, \dots, 6$. The application scale η_i obeys a log normal distribution, whose parameters are (μ_i, σ_i); and the user size ξ_i obeys an exponential distribution, whose parameter is β_i. The elements of the arrays and other parameters a_i, b_i, c_i of formulae (9) and (10) are shown in Tab. 1.

Tab. 1 Computation parameters

Application	1	2	3	4	5	6
μ_i	3. 2	4. 5	4. 8	4. 0	3. 0	6. 0
σ_i	1. 2	1. 8	1. 8	1. 5	1. 0	2. 0
β_i	12	16	18	18	15	24
$a_i/(10^6$ yuan)	8	5	4	3	6	9
b_i	5	8	6	10	9	7
$c_i/(10^6$ yuan)	10	6	4. 8	4. 2	9	12

The genetic algorithm adopted by the program sets some system parameters including:

① Population size is 30.

② Maximum evolving generations are 300.

③ Crossover probability: $P_c = 0.4$.

④ Mutation probability: $P_m = 0.1$.

⑤ Cycle times for stochastic simulation are 3000.

⑥ If filial generation does not meet the constraints, repeat the mutation process. After some cycle times, if filial generation still does not meet the constraints, then preserve parent generation; the cycle times are limited to 50.

⑦ The parameters required for the computation of individual fitness: $P_a = 0.05$.

⑧ Integer variable generation: record the evolving generations, the initial value of the generation is 0.

By the genetic algorithm based on stochastic simulation, for the chance-constrained programming model after the evolution of 300 generations, the maximum earnings calculated by the program at last is 159×10^6 yuan, and the specific investment plan is $(x_1, x_2, x_3, x_4, x_5, x_6) = (3, 4, 4, 4, 5, 2)$; i. e., investment allocations for each application are 24×10^6, 20×10^6, 16×10^6, 12×10^6, 30×10^6, and 18×10^6 yuan, respectively.

For the dependent-chance programming model after the evolution of 300 generations, the optimal solution is obtained; i. e., the probability for the services provided by every application to meet the requirement is 82. 87% at most. The specific investment plan is $(x_1, x_2, x_3, x_4, x_5, x_6) = (3, 3, 3, 6, 4, 3)$; i. e., the investment allocations for each application are 24×10^6, 15×10^6, 12×10^6, 18×10^6, 24×10^6, and 27×10^6 yuan, respectively.

In the practice of the multipurpose transit smart card systems in that city, the real fund allocation is based on the objective of maximizing the benefits. The results indicate that different target settings cause differences in system development directions, so the fund budgeting processes should be closely connected to the system missions. On the other hand, results generating from the dependent-chance programming model show that under the condition of limited funds, some of the application scales may fail to satisfy customer requirements, which would indirectly reduce the service frequency of some applications and further extend the fund payback period. Hence, during fund budgeting, we should carefully take into consideration the scales of every application included in the multipurpose transit smart card systems and forecast each application's future demands in reason.

3 Conclusions

1) The models are applicable for fund budgeting of the systems and the solving algorithm is feasible.

2) The purpose of fund budgeting of the multipurpose transit smart card systems can be divided into two categories according to the requirements of the stakeholder: one is to maximize the total earnings with the constraint of the fund amount; the other is to maximize the service levels of the multipurpose transit smart card systems so as to maximize the probability of the services meeting future requirements.

3) The fund budget process should be closely connected to the system objective. Different target settings can cause big differences in system development directions.

4) Application scales with demands should be analyzed in detail before fund budgeting.

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城市公共交通一卡多用系统资金预算模型

张 宁 钱振东 陈 恺 杨利强

(东南大学 ITS 研究中心,南京 210096)

摘要:为将有限的资金合理分配到多个应用中,用随机规划理论研究了城市公共交通一卡多用系统资金预算问题. 在资金总量约束条件下,以应用规模和相应的社会需求作为随机变量,根据收益最大化和最大程度满足用户需求目标,分别建立了资金预算机会约束模型和相关机会模型,并采用基于随机模拟的遗传算法来求解模型. 实例分析表明:目标设定不一致,资金预算结果有较大差异,导致系统发展方向也不一致;资金有限的情况下,很可能造成一些应用规模不能满足用户的需求. 资金预算应首先对各应用的规模及业务量进行预测,根据系统使命确定系统目标,有助于降低资金预算的风险.

关键词:一卡多用系统;资金预算;随机规划;遗传算法

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