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Approach to obtaining weights of uncertain ordered weighted geometric averaging operator

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Abstract: The ordered weighted geometric averaging (OWGA) operator is extended to accommodate uncertain conditions where all input arguments take the forms of interval numbers. First, a possibility degree formula for the comparison between interval numbers is introduced. It is proved that the introduced formula is equivalent to the existing formulae, and also some desired properties of the possibility degree is presented. Secondly, the uncertain OWGA operator is investigated in which the associated weighting parameters cannot be specified, but value ranges can be obtained and the associated aggregated values of an uncertain OWGA operator are known. A linear objective-programming model is established; by solving this model, the associated weights vector of an uncertain OWGA operator can be determined, and also the estimated aggregated values of the alternatives can be obtained. Then the alternatives can be ranked by the comparison of the estimated aggregated values using the possibility degree formula. Finally, a numerical example is given to show the feasibility and effectiveness of the developed method.

Key words: interval numbers; uncertain ordered weighted geometric averaging operator; possibility degree

The ordered weighted averaging (OWA) operator introduced by Yager[1] provides for aggregation lying between the max and the min operators, and has received increasing attention[1-5] since its appearance. The OWA operator has also been extended to many other forms, such as the induced ordered weighted averaging (IOWA) operator[6-7], the generalized ordered weighted averaging (GOWA) operator[8], the weighted ordered weighted averaging (WOWA) operator[9], the uncertain ordered weighted averaging (UOWA) operator[10], and the linguistic ordered weighted averaging (LOWA) operator[11-12]. For a decision making problem, in the aggregation phase, another operator called the ordered weighted geometric averaging (OWGA) operator[13-14] is also an effective tool for aggregation information. In many situations, the OWGA operator reflects the fuzzy majority calculating its weighting vector by means of a fuzzy linguistic quantifier according to Yager's ideas[1], but in a real situation, the associated weighting parameters cannot be specified. Xu[15] proposed an approach to determine the weights by the OWGA operator, but it only deals with the input arguments in the form of exact number values. But in a real situation, people only provide their preference information denoted by interval values. Therefore, it is necessary to pay attention to this issue. The aim of this paper is to investigate the uncertain OWGA operator in which the associated weight information is incompletely known, and each input argument is given in the form of an interval of numerical values.

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1 Formulae for Comparing Interval Numbers

Definition 1[16] Let $\tilde{a} = [a^{L}, a^{U}] = \{x \mid a^{L} \leqslant x \leqslant a^{U}\}$, then $\tilde{a}$ is called an interval number. If $0 \leqslant a^{L} \leqslant a^{U}$, then $\tilde{a}$ is called a positive interval number. Especially, $\tilde{a}$ is a real number if $a^{L} = a^{U}$.

Let $N = \{1, 2, \dots, n\}$, and let Ω be the set of all interval numbers.

Definition 2 Let $\tilde{a} = [a^{L}, a^{U}]$ and $\tilde{b} = [b^{L}, b^{U}]$ be two positive interval numbers, and $\lambda \geqslant 0, k \geqslant 0$, then

① $\tilde{a} = \tilde{b}$, if and only if $a^{L} = b^{L}$ and $a^{U} = b^{U}$.
② $\tilde{a} + \tilde{b} = [a^{L} + b^{L}, a^{U} + b^{U}]$.
③ $\lambda \tilde{a} = [\lambda a^{L}, \lambda a^{U}]$. Especially, if $\lambda = 0$, then $\lambda \tilde{a} = 0$.
④ $\tilde{a}^{k} = [(a^{L})^{k}, (b^{L})^{k}]$.
⑤ $\tilde{a} \cdot \tilde{b} = [a^{L} b^{L}, a^{U} b^{U}]$.
⑥ $\dfrac{\tilde{a}}{\tilde{b}} = [a^{L}, a^{U}] \left[\dfrac{1}{b^{U}}, \dfrac{1}{b^{L}}\right] = \left[\dfrac{a^{L}}{b^{U}}, \dfrac{a^{U}}{b^{L}}\right]$.

Definition 3[17] Let $\tilde{a} = [a^{L}, a^{U}]$ and $\tilde{b} = [b^{L}, b^{U}]$, and let $\mathrm{len}(\tilde{a}) = a^{U} - a^{L}$ and $\mathrm{len}(\tilde{b}) = b^{U} - b^{L}$, then the degree of possibility of $\tilde{a} \geqslant \tilde{b}$ is defined as

$$p(\tilde{a} \geqslant \tilde{b}) = \min\left\{\max\left[\frac{a^{U} - b^{L}}{\mathrm{len}(\tilde{a}) + \mathrm{len}(\tilde{b})}, 0\right], 1\right\} \tag{1}$$

Definition 4 Let $\tilde{a} = [a^{L}, a^{U}]$ and $\tilde{b} = [b^{L}, b^{U}]$, and let $\mathrm{len}(\tilde{a}) = a^{U} - a^{L}$ and $\mathrm{len}(\tilde{b}) = b^{U} - b^{L}$; then $\mathrm{win}(\tilde{a}) = a^{L} + a^{U}$, $\mathrm{win}(\tilde{b}) = b^{L} + b^{U}$. Then the degree of possibility of $\tilde{a} \geqslant \tilde{b}$ is defined as

$$p(\tilde{a} \geqslant \tilde{b}) = \min\left\{\max\left[\frac{1}{2}\left(\frac{\mathrm{win}(\tilde{a}) - \mathrm{win}(\tilde{b})}{\mathrm{len}(\tilde{a}) + \mathrm{len}(\tilde{b})} + 1\right), 0\right], 1\right\} \tag{2}$$

Similarly, the degree of possibility of $\tilde{b} \geqslant \tilde{a}$ is defined as

$$p(\tilde{b} \geqslant \tilde{a}) = \min\left\{\max\left[\frac{1}{2}\left(\frac{\mathrm{win}(\tilde{b}) - \mathrm{win}(\tilde{a})}{\mathrm{len}(\tilde{a}) + \mathrm{len}(\tilde{b})} + 1\right), 0\right], 1\right\} \tag{3}$$

Theorem 1 Definition 3 is equivalent to definition 4; that is, (1)$\Leftrightarrow$(2).

Proof $p(\tilde{a} \geqslant \tilde{b}) = \min\left\{\max\left[\frac{1}{2}\left(\frac{\mathrm{win}(\tilde{a}) - \mathrm{win}(\tilde{b})}{\mathrm{len}(\tilde{a}) + \mathrm{len}(\tilde{b})} + 1\right), 0\right], 1\right\} = \min\left\{\max\left[\frac{1}{2}\left(\frac{a^{U} - 2b^{L}}{\mathrm{len}(\tilde{a}) + \mathrm{len}(\tilde{b})}\right), 0\right], 1\right\} = \min\left\{\max\left(\frac{a^{U} - b^{L}}{\mathrm{len}(\tilde{a}) + \mathrm{len}(\tilde{b})}, 0\right), 1\right\}$. From definition 3 or definition 4, we can obtain the following results easily.

Theorem 2 Let $\tilde{a}=[a^{\mathrm{L}}, a^{\mathrm{U}}]$, $\tilde{b}=[b^{\mathrm{L}}, b^{\mathrm{U}}]$, $\tilde{c}=[c^{\mathrm{L}}, c^{\mathrm{U}}]$; then

① $0 \leqslant p(\tilde{a} \geqslant \tilde{b}) \leqslant 1$.

② $p(\tilde{a} \geqslant \tilde{b})=1$, if and only if $b^{\mathrm{U}} \leqslant a^{\mathrm{L}}$.

③ $p(\tilde{a} \geqslant \tilde{b})=0$, if and only if $a^{\mathrm{U}} \leqslant b^{\mathrm{L}}$.

④ $p(\tilde{a} \geqslant \tilde{b})+p(\tilde{b} \geqslant \tilde{a})=1$. Especially, $p(\tilde{a} \geqslant \tilde{a})=\frac{1}{2}$.

⑤ $p(\tilde{a} \geqslant \tilde{b}) \geqslant \frac{1}{2}$ if and only if $a^{\mathrm{U}}+a^{\mathrm{L}} \geqslant b^{\mathrm{U}}+b^{\mathrm{L}}$. Especially, $p(\tilde{a} \geqslant \tilde{b})=\frac{1}{2}$, if and only if $a^{\mathrm{U}}+a^{\mathrm{L}}=b^{\mathrm{U}}+b^{\mathrm{L}}$.

⑥ $p(\tilde{a} \geqslant \tilde{b}) \geqslant \frac{1}{2}$ and $p(\tilde{b} \geqslant \tilde{c}) \geqslant \frac{1}{2}$, then $p(\tilde{a} \geqslant \tilde{c}) \geqslant \frac{1}{2}$.

Suppose that there are n input arguments $\tilde{a}_i (i \in N)$ taking the forms of interval numbers, and $\tilde{a}_i=[a_i^{\mathrm{L}}, a_i^{\mathrm{U}}]\ (i \in N)$. To rank these arguments, we first compare each argument $\tilde{a}_i$ with all arguments $\tilde{a}_j (j \in N)$ by Eqs. (3) and (4), and

$$p(\tilde{a}_i \geqslant \tilde{a}_j)= \min\left\{\max\left[\frac{1}{2}\left(\frac{\operatorname{win}(\tilde{a}_i)-\operatorname{win}(\tilde{a}_j)}{\operatorname{len}(\tilde{a}_i)+\operatorname{len}(\tilde{a}_j)}+1\right), 0\right], 1\right\} \quad j \in N \tag{4}$$

For simplicity, we let $p_{ij}=p(\tilde{a}_i \geqslant \tilde{a}_j)$. Then we can construct a complementary matrix as

$$\boldsymbol{P}=\begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix}$$

where $p_{ij} \geqslant 0, p_{ij}+p_{ji}=1, p_{ii}=1/2$ and $i, j \in N$.

Summing all the elements in each line of matrix $\boldsymbol{P}$, we have

$$p_i=\sum_{j=1}^{n} p_{ij} \qquad i \in N \tag{5}$$

Then we can rank the arguments $\tilde{a}_i (i \in N)$ in descending order in accordance with the value of $p_i, i \in N$.

2 Model to Determine the Weights

Definition 4[16] An uncertain OWGA operator of dimension is a mapping $g: \Omega^n \rightarrow \Omega$ that has an associated vector $\boldsymbol{w}=\{w_1, w_2, \ldots, w_n\}^{\mathrm{T}}$, such that $w_i \in[0,1]$ and $\sum_{i=1}^{n} w_i=1$, Furthermore, $g(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)=\prod_{j=1}^{n}(\tilde{b}_j)^{w_j}$, where $\tilde{b}_j$ is the j-th largest of $\tilde{a}_i$, and all of the $\tilde{a}_i (i \in N)$ are interval numbers.

In this section, we shall give a programming model that can be used to obtain the weights associated with the uncertain OWGA operator by utilizing the given partial weight information and the arguments.

Given a collection of m samples (observations), each consists of an n tuple of arguments $(\tilde{a}_{k1}, \tilde{a}_{k2}, \ldots, \tilde{a}_{kn})$ and an associated aggregated value $\tilde{s}_k$, where $\tilde{a}_{ki}=[a_{ki}^{\mathrm{L}}, a_{ki}^{\mathrm{U}}]$, $\tilde{s}_k=[s_k^{\mathrm{L}}, s_k^{\mathrm{U}}]$, $k=1,2, \ldots, m$. And partial weight information is

$$\boldsymbol{w}=\left\{\{w_1, w_2, \ldots, w_n\}^{\mathrm{T}} \mid 0 \leqslant \alpha_j \leqslant w_j \leqslant \beta_i \leqslant 1, \sum_{j=1}^{n} \alpha_j \leqslant 1, \sum_{j=1}^{n} \beta_j \geqslant 1, \sum_{j=1}^{n} w_j=1\right\}$$

We need an uncertain OWGA operator, a weighing vector $\boldsymbol{w}$, such that for the entire collection of data we satisfy the following conditions as faithfully as possible. $g(\tilde{a}_{k1}, \tilde{a}_{k2}, \ldots, \tilde{a}_{kn})=\tilde{s}_k, k=1,2, \ldots, m$. We can utilize Eq. (3) or Eq. (4) to compare the k-th sample arguments $\tilde{a}_{ki}, i \in N$, and utilize Eq. (5) to obtain $p_i^{(k)}, i \in N$. Then we can rank the arguments of the k-th sample by $\tilde{b}_{k1}, \tilde{b}_{k2}, \ldots, \tilde{b}_{kn}$ in descending order in accordance with the value of $p_i^{(k)}, i \in N$. Using these re-ordered arguments, we need to find a vector of the uncertain OWGA weights $\boldsymbol{w}=\{w_1, w_2, \ldots, w_n\}^{\mathrm{T}}$ such that

$$\prod_{j=1}^{n}(\tilde{b}_{kj})^{w_j}=\tilde{s}_k \qquad k=1,2, \ldots, m$$

that is

$$\prod_{j=1}^{n}(b_{kj}^{\mathrm{L}})^{w_j}=s_k^{\mathrm{L}}, \quad \prod_{j=1}^{n}(b_{kj}^{\mathrm{U}})^{w_j}=s_k^{\mathrm{U}} \tag{6}$$

Since Eq. (6) is a non-linear form, it is difficult to obtain the weighting vector $\boldsymbol{w}$. We take the logarithm on both sides of Eq. (5), then we can obtain

$$\sum_{j=1}^{n} w_j \ln(b_{kj}^{\mathrm{L}})=\ln s_k^{\mathrm{L}}, \quad \sum_{j=1}^{n} w_j \ln(b_{kj}^{\mathrm{U}})=\ln s_k^{\mathrm{U}} \tag{7}$$

In the real life, however, there always exit some differences; that is, Eq. (7) in general does not hold. Here, we introduce the deviation elements $e_k^{\mathrm{L}}(\boldsymbol{w})$ and $e_k^{\mathrm{U}}(\boldsymbol{w})$; i. e., let

$$e_k^{\mathrm{L}}=\left|\sum_{j=1}^{n} w_j \ln(b_{kj}^{\mathrm{L}})-\ln s_k^{\mathrm{L}}\right|, e_k^{\mathrm{U}}=\left|\sum_{j=1}^{n} w_j \ln(b_{kj}^{\mathrm{U}})-\ln s_k^{\mathrm{U}}\right| \tag{8}$$

To determine the vector of weights $\boldsymbol{w}=\{w_1, w_2, \ldots, w_n\}^{\mathrm{T}}$, we can construct the following multi-objective programming (MOP) model:

$$\begin{aligned} &\min e_k^{\mathrm{L}}=\left|\sum_{j=1}^{n} w_j \ln(b_{kj}^{\mathrm{L}})-\ln s_k^{\mathrm{L}}\right| \qquad k=1,2, \ldots, m \\ &\min e_k^{\mathrm{U}}=\left|\sum_{j=1}^{n} w_j \ln(b_{kj}^{\mathrm{U}})-\ln s_k^{\mathrm{U}}\right| \qquad k=1,2, \ldots, m \\ &\text{s. t.} \quad 0 \leqslant \alpha_j \leqslant w_j \leqslant \beta_i \leqslant 1 \\ &\qquad \sum_{j=1}^{n} \alpha_j \leqslant 1, \sum_{j=1}^{n} \beta_j \geqslant 1, \sum_{j=1}^{n} w_j=1 \end{aligned}$$

Generally, all the objectives are fairly competitive and there is no preference relationship among them; therefore, the above model can be transformed into the following linear goal programming problem:

$$\min J=\sum_{k=1}^{m}\left[(e_k^{\mathrm{L}})^{+}+(e_k^{\mathrm{L}})^{-}+(e_k^{\mathrm{U}})^{+}+(e_k^{\mathrm{U}})^{-}\right]$$

s. t.

$$\sum_{j=1}^{n} w_j \ln(b_{kj}^{\mathrm{L}})-\ln s_k^{\mathrm{L}}-(e_k^{\mathrm{L}})^{+}+(e_k^{\mathrm{L}})^{-}=0 \quad k=1,2, \ldots, m$$

$$\sum_{j=1}^{n} w_j \ln(b_{kj}^{\mathrm{U}})-\ln s_k^{\mathrm{U}}-(e_k^{\mathrm{U}})^{+}+(e_k^{\mathrm{U}})^{-}=0 \quad k=1,2, \ldots, m$$

$$0 \leqslant \alpha_j \leqslant w_j \leqslant \beta_i \leqslant 1, \sum_{j=1}^{n} \alpha_j \leqslant 1, \sum_{j=1}^{n} \beta_j \geqslant 1, \sum_{j=1}^{n} w_j=1$$

$$(e_k^{\mathrm{L}})^{+} \geqslant 0,(e_k^{\mathrm{L}})^{-} \geqslant 0,(e_k^{\mathrm{L}})^{+}(e_k^{\mathrm{L}})^{-}=0 \quad k=1,2, \ldots, m$$

$(e_k^{\mathrm{U}})^+ \geqslant 0, (e_k^{\mathrm{U}})^- \geqslant 0, (e_k^{\mathrm{U}})^+ (e_k^{\mathrm{U}})^- = 0 \quad k = 1, 2, \ldots, m$

where $(e_k^{\mathrm{L}})^+$ and $(e_k^{\mathrm{L}})^-$ are the upper and lower deviation variables of s_k^{L}, respectively; $(e_k^{\mathrm{U}})^+$ and $(e_k^{\mathrm{U}})^-$ are the upper and lower deviation variables of s_k^{U}, respectively.

By solving the model, we can obtain the vector of the uncertain OWGA weights $\boldsymbol{w} = \{w_1, w_2, \ldots, w_n\}^{\mathrm{T}}$.

3 Illustrative Example

The samples of data are listed in Tab. 1. Each sample consists of three arguments and the relevant aggregated value, all having the forms of interval numbers. The known partial weight information is $0.2 \leqslant w_1 \leqslant 0.6, 0.3 \leqslant w_2 \leqslant 0.5, 0.1 \leqslant w_3 \leqslant 0.4$.

Tab. 1 Samples of data

Sample	Argument values	Aggregate value
1	[0.4, 0.7], [0.2, 0.5], [0.7, 0.8]	[0.3, 0.7]
2	[0.3, 0.4], [0.6, 0.8], [0.3, 0.5]	[0.4, 0.5]
3	[0.2, 0.6], [0.3, 0.4], [0.5, 0.8]	[0.3, 0.6]
4	[0.5, 0.8], [0.3, 0.5], [0.3, 0.4]	[0.4, 0.6]

Utilize Eq. (4) to compare the k-th sample of data, and construct the complementary matrix P_k,

$$\boldsymbol{P}_1 = \begin{bmatrix} 0.5 & 0.833 & 0 \\ 0.167 & 0.5 & 0 \\ 1 & 1 & 0.5 \end{bmatrix}, \boldsymbol{P}_2 = \begin{bmatrix} 0.5 & 0 & 0.333 \\ 1 & 0.5 & 1 \\ 0.667 & 0 & 0.5 \end{bmatrix}$$

$$\boldsymbol{P}_3 = \begin{bmatrix} 0.5 & 0.6 & 0.143 \\ 0.4 & 0.5 & 0 \\ 0.857 & 1 & 0.5 \end{bmatrix}, \boldsymbol{P}_4 = \begin{bmatrix} 0.5 & 1 & 1 \\ 0 & 0.5 & 0.667 \\ 0 & 0.333 & 0.5 \end{bmatrix}$$

By Eq. (5), we can obtain

$$p_1^{(1)} = 1.333, \quad p_2^{(1)} = 0.667, \quad p_3^{(1)} = 2.5$$
$$p_1^{(2)} = 0.833, \quad p_2^{(2)} = 2.5, \quad p_3^{(2)} = 1.167$$
$$p_1^{(3)} = 1.243, \quad p_2^{(3)} = 0.9, \quad p_3^{(3)} = 2.357$$
$$p_1^{(4)} = 2.5, \quad p_2^{(4)} = 1.167, \quad p_3^{(4)} = 0.833$$

Then we rank the k-th sample arguments $\tilde{a}_{k1}, \tilde{a}_{k2}, \tilde{a}_{k3}$ in descending order in accordance with the values of $p_1^{(k)}, p_2^{(k)}, p_3^{(k)}$, and we can obtain $\tilde{b}_{k1}, \tilde{b}_{k2}, \tilde{b}_{k3}$.

$$\tilde{b}_{11} = [0.7, 0.8], \quad \tilde{b}_{12} = [0.4, 0.7], \quad \tilde{b}_{13} = [0.2, 0.5]$$
$$\tilde{b}_{21} = [0.6, 0.8], \quad \tilde{b}_{22} = [0.3, 0.5], \quad \tilde{b}_{23} = [0.3, 0.4]$$
$$\tilde{b}_{31} = [0.5, 0.8], \quad \tilde{b}_{32} = [0.2, 0.6], \quad \tilde{b}_{33} = [0.3, 0.4]$$
$$\tilde{b}_{41} = [0.5, 0.8], \quad \tilde{b}_{42} = [0.3, 0.5], \quad \tilde{b}_{43} = [0.3, 0.4]$$

Utilizing the LOP model, we have

$$\min J = \sum_{k=1}^{4} [(e_k^{\mathrm{L}})^+ + (e_k^{\mathrm{L}})^- + (e_k^{\mathrm{U}})^+ + (e_k^{\mathrm{U}})^-]$$

s. t.

$$\ln(0.7) w_1 + \ln(0.4) w_2 + \ln(0.2) w_3 - \ln(0.3) - (e_1^{L})^+ + (e_1^{L})^- = 0$$
$$\ln(0.8) w_1 + \ln(0.7) w_2 + \ln(0.5) w_3 - \ln(0.7) - (e_1^{\mathrm{U}})^+ + (e_1^{\mathrm{U}})^- = 0$$
$$\ln(0.6) w_1 + \ln(0.3) w_2 + \ln(0.3) w_3 - \ln(0.4) - (e_2^{\mathrm{L}})^+ + (e_2^{\mathrm{L}})^- = 0$$
$$\ln(0.8) w_1 + \ln(0.5) w_2 + \ln(0.4) w_3 - \ln(0.5) - (e_2^{\mathrm{U}})^+ + (e_2^{\mathrm{U}})^- = 0$$
$$\ln(0.5) w_1 + \ln(0.2) w_2 + \ln(0.3) w_3 - \ln(0.3) - (e_3^{\mathrm{L}})^+ + (e_3^{\mathrm{L}})^- = 0$$
$$\ln(0.8) w_1 + \ln(0.6) w_2 + \ln(0.4) w_3 - \ln(0.6) - (e_3^{\mathrm{U}})^+ + (e_3^{\mathrm{U}})^- = 0$$
$$\ln(0.5) w_1 + \ln(0.3) w_2 + \ln(0.3) w_3 - \ln(0.4) - (e_4^{\mathrm{L}})^+ + (e_4^{\mathrm{L}})^- = 0$$
$$\ln(0.8) w_1 + \ln(0.5) w_2 + \ln(0.4) w_3 - \ln(0.6) - (e_4^{\mathrm{U}})^+ + (e_4^{\mathrm{U}})^- = 0$$
$$0.2 \leqslant w_1 \leqslant 0.6, \quad 0.3 \leqslant w_2 \leqslant 0.5, \quad 0.1 \leqslant w_3 \leqslant 0.4$$
$$(e_k^{\mathrm{L}})^+ \geqslant 0, \quad (e_k^{\mathrm{L}})^- \geqslant 0, \quad (e_k^{\mathrm{L}})^+ (e_k^{\mathrm{L}})^- = 0 \qquad k = 1, 2, 3, 4$$
$$(e_k^{\mathrm{U}})^+ \geqslant 0, \quad (e_k^{\mathrm{U}})^- \geqslant 0, \quad (e_k^{\mathrm{U}})^+ (e_k^{\mathrm{U}})^- = 0 \qquad k = 1, 2, 3, 4$$

By solving the above model, we obtain the vector of the uncertain OWGA weights:

$$\boldsymbol{w} = \{0.266, 0.334, 0.4\}^{\mathrm{T}}$$

Therefore, we obtain the estimated aggregated values $\hat{s}_k$ of $\tilde{s}_k$ as follows:

$$\hat{s}_1 = g(\tilde{a}_{11}, \tilde{a}_{12}, \tilde{a}_{13}) = (\tilde{b}_{11})^{0.266} \times (\tilde{b}_{12})^{0.334} (\tilde{b}_{13})^{0.4} = [0.352, 0.634]$$
$$\hat{s}_2 = g(\tilde{a}_{21}, \tilde{a}_{22}, \tilde{a}_{23}) = (\tilde{b}_{21})^{0.266} \times (\tilde{b}_{22})^{0.334} \times (\tilde{b}_{23})^{0.4} = [0.361, 0.518]$$
$$\hat{s}_3 = g(\tilde{a}_{31}, \tilde{a}_{32}, \tilde{a}_{33}) = (\tilde{b}_{31})^{0.266} \times (\tilde{b}_{32})^{0.334} \times (\tilde{b}_{33})^{0.4} = [0.3, 0.551]$$
$$\hat{s}_4 = g(\tilde{a}_{41}, \tilde{a}_{42}, \tilde{a}_{43}) = (\tilde{b}_{41})^{0.266} \times (\tilde{b}_{42})^{0.334} \times (\tilde{b}_{43})^{0.4} = [0.344, 0.518]$$

By Eqs. (4) and (5), we can rank the estimated aggregated values, $\hat{s}_k$ in descending order:

$$\boldsymbol{P} = \begin{bmatrix} 0.5 & 0.6219 & 0.6266 & 0.636 \\ 0.3781 & 0.5 & 0.5343 & 0.5257 \\ 0.3734 & 0.4657 & 0.5 & 0.4871 \\ 0.3640 & 0.4743 & 0.5129 & 0.5 \end{bmatrix}$$

$$p_1 = 2.3845, \quad p_2 = 1.9318, \quad p_3 = 1.8262, \quad p_4 = 1.8512$$
$$\hat{s}_1 \succ \hat{s}_2 \succ \hat{s}_4 \succ \hat{s}_3$$

4 Conclusion

We have investigated the uncertain ordered weighted geometric averaging (UOWGA) operator in which the associated weighting parameters cannot be specified, but value ranges can be obtained. Each input argument is given in the form of an interval of numerical values, which develops the theory of the OWA operator introduced by Yager. We introduce an equivalent possibility degree formula for the comparison between interval numbers and study its properties. To determine the specified weights of the OWGA operator, we establish a linear objective-programming model. By solving this model, we can not only obtain the weights but also can rank the alternatives.

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不确定有序加权几何平均算子的赋权方法

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摘要:把有序加权几何平均(OWGA)算子推广到所给定的数据信息均为区间数形式的不确定环境之中.首先给出了区间数两两比较的可能度的一个公式,证明了该公式与现有的公式是等价的,并给出了该公式的一些优良性质.其次,研究了不确定有序加权几何平均算子,这里算子的权重参数不能够确定,但是值的范围是给定的,并且不确定 OWGA 算子的集结值是已知的.建立了一个线性目标规划模型,求解该模型,不仅可以得到不确定 OWGA 算子的权重向量而且可得到方案的估计值,然后用可能度公式通过对估计集结值的比较来对方案进行排序.最后通过实例说明了该方法的有效性和可行性.

关键词:区间数;不确定有序加权几何平均算子;可能度

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