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# Real option pricing method for R&D investment under changing risk-free rate and discount rate

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**Abstract:** The polynomial spline model, which belongs to the static term structure model of interest rates, is studied. Every cash flow of the project is discounted relatively accurately by obtaining the discount rate from the static term structure model of interest rates. A simple basic model, which belongs to the dynamic term structure model, is studied, and the option pricing formula under changing risk-free rates is obtained by bringing it into the option pricing formula. Both dynamic and static term structure models are estimated by the use of the data of buy-back rates and the Shanghai Stock Exchange, and an example is given to compare the differences between the traditional method and the method under the changes in the interest rates and the discount rates.

**Key words:** risk-free interest rate; discount rate; polynomial spline; real option

In order to overcome the shortcomings of the net present value (NPV) method in project evaluation, Myers and Ross proposed the real option-pricing method to evaluate the operational flexibility of an investment project. Kester[1] discussed the problems conceptually on the strategy of growth opportunities and competition. Other universal conceptual frameworks of real options were proposed by Trigeorgis et al. [2] and Kulatilaka et al[3]. Many improvements were made based on those research results. The current method is to introduce the financial option pricing model into the real option pricing model, replacing current stock price $S$ with the present value of a project's future income, replacing exercise price $X$ with investment cost $V$, replacing effective duration $T$ with the duration of a project's investment opportunity $T$, replacing stock volatility $\sigma$ with project volatility $\theta$, and the meaning of the risk-free interest rate is the same. The project income and investment costs do not occur as soon as the project begins, but will occur for several times year by year later on, and the cash flows should be discounted at a certain discount rate. Because interest rate marketization is underway in China, researches on the term structure of interest rates and discount rates are still underway. A simplification is often made when using real option theory to estimate the investment value of a project, in which the risk-free interest rates and the discount rates are regarded as constants and not distinguished in many cases. But in many cases, especially some multi-stage projects on a large scale, the real option has a long term and there are many uncertain factors affecting it. So the reality is that the risk-free interest rate and the discount rate may change. Hu and Liu[4] researched the real option pricing model on the condition of the changing risk-free interest rate. They did not distinguish between the risk-free interest rate and the discount rate, and did not give out a specific parameter estimation but made a parameter assumption directly. He et al. [5] studied the R& D investment by combining game theory and the term structure of interest rates. But they have not applied the term structure model to option pricing formula. Many others researched option pricing based on stochastic interest rates, but the results are difficult to be applied in practice.

This paper studies the dynamic and static term structure models of interest rates. Through the analysis of heteroscedasticity in the static term structure model, a method to deal with heteroscedasticity is proposed, which calculates variance according to the trade volume in bonds. The risk-free interest rate and the discount rate are distinguished. The real option pricing method for R& D investment under changing risk-free interest rates and discount rates are studied. Assuming that the changes of risk-free interest rates obey a general dynamic model, we obtain the option pricing formula under the dynamic change in risk-free interest rates. Every cash flow of a project is discounted relatively accurately by obtaining the discount rate from the static term structure model of interest rates. Both the dynamic and static term structure models of risk-free interest rates are estimated by the use of the data from Chinese buy-back interest rates and the Shanghai Stock Exchange, and an example is given to compare the differences between the traditional method and the new method under the change in the interest rates and the discount rates.

## 1 Theory of Term Structure of Interest Rates

The study of the term structure of interest rates is about the relationship between gains of risk-free bonds, which have different maturities. According to the previous researches, there are two kinds of term structure models of interest rates, which are the dynamic and the static ones.

### 1.1 Dynamic term structure model of interest rates

The dynamic term structure model of interest rates occurred after the 1970s. Inspired by the research method of Black and Scholes, it studies the dynamic characteristics of changing interest rates by using continuous-time mathematical tools[6].

This paper adopts a simple and basic dynamic model of interest rates as follows:

$$\mathrm{d}r_t = (\alpha + \beta r_t)\mathrm{d}t + \sigma \mathrm{d}W_t \tag{1}$$

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where $dW_t$ is the increment in the Wiener process; $r_t$ is the interest rate at the time $t$; $\sigma$ is the rate of fluctuation of interest rates; $\alpha, \beta, \sigma$ are the parameters to be estimated. According to the economic meaning, $\beta$ is the adjustment speed of interest rates and $\alpha$ is the product of the adjustment speed and the average interest rate.

Parameters can be estimated by the discretization of Eq. (1)[7-9]. Let the time from $t$ to $t+1$ be $\delta$, so

$$r_{t+1} - r_t = (\alpha + \beta r_t)\delta + \sigma\delta^{0.5}\xi_t \tag{2}$$

where $\xi_t$ is the white noise; $E[\xi_t] = 0$ and $D[\xi_t] = 1$. The parameter can be estimated by the least squares method.

The term structure of interest rates obtained by this method can only be the theoretical discussion on the condition of no arbitrage in an efficient market, which can be applied in derivative pricing and risk management. But it is difficult to fit the actual data of bond prices and yield rates.

### 1.2 Static term structure model of interest rates

The static term structure model of interest rates is to estimate term structure on the basis of essence, no matter how the economic situation is. Namely, the term structure of interest rates is fitted by the actual data acquired from the market[10-11]. The polynomial spline method is a very important and widely used method among them. In the polynomial spline method, the discount factor $E(t_0, t)$ is represented as a piecewise spline function. Let

$$E(t_0, t) = \begin{cases} 1 + b_0\Delta t + c_0(\Delta t)^2 + d_0(\Delta t)^3 & \Delta t \in [0,5] \\ 1 + b_0\Delta t + c_0(\Delta t)^2 + d_0(\Delta t)^3 + d_1(\Delta t - 5)^3 & \\ \quad \Delta t \in [5,10] & \\ 1 + b_0\Delta t + c_0(\Delta t)^2 + d_0(\Delta t)^3 + & \\ \quad d_1(\Delta t - 5)^3 + d_2(\Delta t - 10)^3] & \Delta t \in [10,20] \end{cases} \tag{3}$$

From the above expression, we can see that $E(t_0, t)$ is determined by $\alpha(b_0, c_0, d_0, d_1, d_2)$, which is the solution of the following model:

$$\left.\begin{array}{ll} \boldsymbol{P}_{t_0} = \hat{\boldsymbol{P}}_{t_0} + \boldsymbol{\varepsilon} & \varepsilon \in \{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_i, \ldots, \varepsilon_n\}^{\mathrm{T}} \\ E(\varepsilon_i) = 0 & \\ \operatorname{var}(\varepsilon_i) = \sigma^2 q_i^2 & \sigma \in \mathbf{R} \\ \operatorname{cov}(\varepsilon_i, \varepsilon_j) = 0 & i \neq j \end{array}\right\} \tag{4}$$

where $\boldsymbol{P}_{t_0} = (p_{t_0}^j)$ and $\hat{\boldsymbol{P}}_{t_0} = (\hat{p}_{t_0}^j)$, $j = 1, 2, \ldots, n$ are represented relatively as price vectors. $p_{t_0}^j$ is the market price of $j$ bonds at the time $t_0$; $\hat{p}_{t_0}^j$ is the theoretical price of $j$ bonds at the time $t_0$, which can be calculated by the formula $\hat{p}_{t_0}^j = \sum_{t_i} F_{t_i}^{(j)} E(t_0, t_i)$; $F_{t_i}^j$ is the cash flow of $j$ bonds at the time $t_i$; $q_i^2$ is the proportional coefficient of variance of the $i$-th bond pricing residual.

Formula (4) is a heteroscedasticity model, whose coefficients can be estimated by the generalized least squares method. If $\boldsymbol{\Omega} = \operatorname{diag}(q_1^2, q_2^2, \ldots, q_n^2)$ and $\boldsymbol{X}$ is the coefficient matrix of formula (4), then $\hat{\boldsymbol{\alpha}} = (\boldsymbol{X}'\boldsymbol{\Omega}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{\Omega}^{-1}\boldsymbol{P}_{t_0}$.

In fact, the key to the whole optimized decision process is to determine heteroscedasticity $q_i^2$[12], which is the same as determining the weight of each bond in the generalized least squares method. There is one case in reality: the daily trade volume of some bonds is large, but the daily trade volume of other bonds is small. The former is more representative than the latter in reflecting the expected interest rate accepted by both seller and buyer on the bonds market. So the former has a heavy weight because of its big impact on the market. The interest rate curve obtained in this way can more truly reflect the term structure of interest rates implied in the market. Thus, we can let $q_i^2 = 1/Q_i$, where $Q_i$ is the daily trade volume of the $i$-th bond.

After obtaining the discount factor, translate it into the annualized spot rate by the following formula:

$$r(t_0, t) = \left[\frac{1}{E(t_0, t)}\right]^{1/(t-t_0)} - 1 \tag{5}$$

## 2 Theory of Discount Rate

The discount rate is a financial concept based on compound interest that calculates the present value of the future cash flow. It reflects the time value of the money. The essence of discount rates is a kind of investment return rate. The discount rate should be higher than the risk-free rate. In the normal capital market and property rights market, interest rates of government bonds and deposit rates are treated as returns on risk-free investment by investors. If the discount rate is lower than the return on a risk-free investment, investors will deposit their money or buy risk-free T-bonds instead of making risk investments. The discount rate is the sum of risk-free interest rates and risk premium rates. The formula is: discount rate = risk-free interest rate + return on risk investment + inflation rate[13]. The risk-free interest rate can be estimated by the static term structure model of interest rates mentioned above using data from the Shanghai Stock Exchange. Return on risk investment and inflation rates can be estimated by experiential judgment in macroeconomic environments, the developing prospects of industries, markets and competition among similar enterprises.

## 3 Real Option Pricing with Changing Interest Rate and Discount Rate

### 3.1 Option pricing model with dynamically changing risk-free interest rate

In 1973, two financial economists, Black and Scholes, derived a formula for the value of a European call option requiring that a stock pay no dividends. The formula is predicted under some assumptions, and it utilizes the no-arbitrage pricing method.

The Black-Scholes formula for the value of a European call option is

$$C = SN(d_1) - Xe^{-rT}N(d_2) \tag{6}$$

where $d_1 = \dfrac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$. $C$ is the current call option value; $S$ is the current stock price; $N(d)$ is the probability that a random draw from a standard normal distribution will be less than $d$; $X$ is the exercise price; e = 2.712 8, the base of the natural log function; $r$ is the risk free interest rate; $T$ is the time to maturity of the option in years; $\sigma$ is the standard deviation of the annualised continuously compounded rate of return of the stock.

The Black-Scholes pricing formula (6) requires that the risk-free interest rate be constant over the life of the option. If the risk-free interest rates are changing during the life cycle of the option, then $rT$ can be replaced by cumulative risk-free interest rates[14].

Using a basic dynamic model of interest rates:

$$\mathrm{d}r_t = (\alpha + \beta r_t)\,\mathrm{d}t + \sigma \mathrm{d}W_t \tag{7}$$

In this model, the change of interest rates is a random process. At an arbitrary time point $t$, the value of $r_t$ is a random variable, which means that $r_t$ is uncertain at an arbitrary time point $t$ over the life of the option. Instantaneous expected value $E(r_t)$ can be estimated as an instantaneous interest rate at an arbitrary time point $t$. According to Eq. (8)

$$\frac{1}{\beta}\mathrm{d}(\alpha + \beta r_t) = (\alpha + \beta r_t)\,\mathrm{d}t + \sigma \mathrm{d}W_t \tag{8}$$

Using the method of stochastic differential equations[15], we can obtain

$$E(\alpha + \beta r_t) = (\alpha + \beta r_0)\mathrm{e}^{\beta t} \tag{9}$$

Thus, the instantaneous expected value is

$$E(r_t) = \frac{(\alpha + \beta r_0)\mathrm{e}^{\beta t} - \alpha}{\beta} \tag{10}$$

According to Song[14], replacing $r_t$ by the cumulative risk-free interest rate, $\int_0^t E(r_t)\,\mathrm{d}t = \int_0^t \frac{(\alpha + \beta r_0)\mathrm{e}^{\beta t} - \alpha}{\beta}\mathrm{d}t = \frac{(\alpha + \beta r_0)\mathrm{e}^{\beta t}}{\beta^2} - \frac{\alpha}{\beta}t$, we can derive an option pricing formula whose change in interest rate obeys the basic dynamic model (7). The option pricing formula is

$$c(S_t, t) = S(t)N(d_1) - X\exp\left(\frac{\alpha}{\beta}t - \frac{(\alpha + \beta r_0)\mathrm{e}^{\beta t}}{\beta^2}\right)N(d_2) \tag{11}$$

where

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(\frac{\alpha + \beta r_0}{\beta^2}\mathrm{e}^{\beta t} - \frac{\alpha}{\beta}t + \frac{\sigma^2}{2}t\right)}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

### 3.2 Real option pricing with changing discount rates

A real option is the right to undertake some business decision. Its underlying assets are no longer stock, bonds, options or money, but typically some certain investment projects. The real option is a tool for decision-making on the non-financial assets investment with uncertain results. The pricing model of the real options is similar to the pricing model of financial options. The general pricing model of the real option can be derived by: replacing the current stock price $S$ with the current value of project income $P$; the exercise price $X$ with the investment cost $V$; the time to maturity of option in years $T$ with the duration of investment opportunity $T$; and the standard deviation $\sigma$ with the fluctuating rate of a project's value $\theta$.

When pricing the investment project by real option theory, project income and investment cost do not occur as soon as the project begins, but happens in the next several years. So the cash flow should be discounted at a certain discount rate. The real option pricing model on the condition of changing discount rates is the one to derive the term structure of discount rates, calculate different discount rates in the corresponding terms and discount the cash flow.

## 4 Empirical Analysis

Company A plans to invest in a research and development project on Sep 20th, 2006. The project is treated as a simplified typical model. The term of the cash flow is divided into two stages. The first stage, the first and second years, is about research and development of application technology. The second stage, from the third year to the seventh year, is about commercialization. So the products have a five-year life cycle. There is no residual value. The volatility is $\theta = 30\%$, the discount rate $\mu$ and the risk free interest rate $r$ are unknown. Cash flow of the project is shown in Tab. 1. The investment occurs at the beginning of each year, and return is obtained at the end of the year).

**Tab. 1** Cash flow of the two stages of the project $10^4$ dollars

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Investment cost | 50 | 40 | 600 | 450 | 400 | | |
| Return | 0 | 0 | 150 | 300 | 450 | 460 | 300 |

The traditional method and the improved method mentioned above are used separately to assess the value of the real option as follows.

### 4.1 When risk-free rate and discount rate are constant

Let the overnight repurchase rates of Interbank Market on Sep 20th, 2006, $r = 0.019\,5$, be the risk-free rate. Let one and a half time of a one-year government bond yield in the Shanghai Stock Exchange on Sep 20th, 2006, $\mu = 0.038\,2$, be the discount rate. So the investment amount in the second stage $= 600 + 450/1.038\,2 + 400/1.038\,2^2 = 1\,404.5$ ($10^4$ dollars). The return in the second stage $= 150/1.038\,2 + 300/1.038\,2^2 + 450/1.038\,2^3 + 460/1.038\,2^4 + 300/1.038\,2^5 = 1\,469.6$ ($10^4$ dollars). Using data: the exercise price $X = 1\,404.5$, the current stock price $S = 1\,469.6/1.038\,2^2 = 1\,363.4$ ($10^4$ dollars), $r = 0.019\,5$, $\theta = 30\%$. Substituting them into the Black-Scholes formula to value the real option, we can obtain

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{1\,363.4}{1\,404.5}\right) + \left(0.019\,5 + \frac{0.3^2}{2}\right)\times 2}{0.3\sqrt{2}} = 0.234\,1$$

$$d_2 = 0.234\,1 - 0.3\sqrt{2} = -0.190\,2$$

$$C = SN(d_1) - X\mathrm{e}^{-rT}N(d_2) = 1\,363.4N(0.234\,1) - 1\,404.5\mathrm{e}^{-0.019\,5\times 2}N(-0.190\,2) = 232.1$$

Thus, the investment value of the project is $232.1 - 50 - 40/1.038\,2 = 143.6$ ($10^4$ dollars).

### 4.2 When risk-free rate and discount rate are changing

1) Derive the term structure of discount rates on that day

Select 22 government bonds in the Shanghai Stock Exchange on Sep 20th, 2006. The polynomial spline method mentioned in section 1 is used to fit the term structure of the interest rates on Sep 20th, 2006.

Discount factor $E(t)$ is obtained as follows:

$$E(t)=\begin{cases} E_0(t)=1-0.026\,901\,584\,468\,67t+ \\ 0.002\,803\,739\,176\,78t^2-0.000\,767\,427\,128\,78t^3 & t\in[0,2] \\ E_2(t)=B_0(t)+0.000\,953\,293\,285\,52(t-2)^3 & t\in[2,5] \\ E_5(t)=B_2(t)-0.000\,125\,202\,716\,73(t-5)^3 & t\in[5,10] \\ E_{10}(t)=B_5(t)-0.000\,115\,027\,128\,52(t-10)^3 & t\in[10,20] \end{cases} \tag{12}$$

Convert $E(t)$ to an annual risk free rate by

$$r(t)=[1/E(t)]^{1/t}-1 \tag{13}$$

Estimate return on risk investment and inflation rate according to macroeconomic environments, the developing prospect of the industry, and the market and competition among similar enterprises. Suppose that both returns on risk investment and inflation rate are half of the risk free rate. The term structure of discount rates is

$$\mu(t)=1.5r(t)=1.5\{[1/E(t)]^{1/t}-1\} \tag{14}$$

Long-term discount rate ${}_{t_i}\mu_{t_j}$ from $t_i$ to $t_j$ can be calculated on the basis of expectation theory.

$${}_{t_i}\mu_{t_j}=\left\{\frac{[1+\mu(t_j)]^{t_j}}{[1+\mu(t_i)]^{t_i}}\right\}^{1/(t_j-t_i)}-1 \tag{15}$$

2) Calculate the investment amount and return in the second stage

According to Eqs. (14) and (15), we can obtain

$$\mu(1)=0.038\,2,\ \mu(2)=0.037\,9,\ \mu(3)=0.039\,6$$
$$\mu(4)=0.041\,9,\ \mu(5)=0.044\,0,\ \mu(6)=0.045\,9$$
$$\mu(7)=0.047\,5,\ {}_2\mu_3=0.043\,0,\ {}_2\mu_4=0.045\,8$$
$${}_2\mu_5=0.048\,1,\ {}_2\mu_6=0.049\,9,\ {}_2\mu_7=0.051\,3$$

Thus, the investment amount in the second stage equals $600+450/1.043\,0+400/1.045\,8^2=1\,397.2$ ($10^4$ dollars). The return in the second stage equals $150/1.043\,0+300/1.045\,8^2+450/1.048\,1^3+460/1.049\,9^4+300/1.051\,3^5=1\,421.2$ ($10^4$ dollars).

3) Parameter estimation in the dynamic model of interest rates

On the basis of the second sector, assume the dynamic process of China's risk free rate is

$$\mathrm{d}r_t=(\alpha+\beta r_t)\mathrm{d}t+\sigma\mathrm{d}W_t \tag{16}$$

Using repurchase rates of the interbank market, estimate parameters in the dynamic model of interest rates. Selecting 527 overnight repurchase rates of the interbank market from China's money market from Aug 2nd, 2004 to Sep 20th, 2006, parameter values are estimated: $\alpha=0.350\,788$, $\beta=-22.437\,69$.

4) Substitute parameter values into the Black-Scholes formula to value the real option

The enterprise has a call option. Its exercise price $X=1\,397.2$ ($10^4$ dollars) and the current stock price $S=1\,421.2/1.037\,9^2=1\,319.3$ ($10^4$ dollars). Substituting them into Eq. (11), we can obtain

$$d_1=\frac{\ln\left(\frac{S}{X}\right)+\left(\frac{\alpha+\beta r_0}{\beta^2}\mathrm{e}^{\beta t}-\frac{\alpha}{\beta}t+\frac{\sigma^2}{2}t\right)}{\sigma\sqrt{t}}=\frac{\ln\left(\frac{1\,319.3}{1\,397.2}\right)+\left(\frac{0.350\,788-22.437\,69\times0.019\,5}{(-22.437\,69)^2}\mathrm{e}^{-22.437\,69\times2}-\frac{0.350\,788}{-22.437\,69}\times2+\frac{0.3^2}{2}\times2\right)}{0.3\sqrt{2}}=0.150\,6$$

$$d_2=d_1-\sigma\sqrt{T}=0.150\,6-0.3\sqrt{2}=-0.273\,7$$

$$c(S_t,t)=S(t)N(d_1)-X\exp\left(\frac{\alpha}{\beta}t-\frac{(\alpha+\beta r_0)\mathrm{e}^{\beta t}}{\beta^2}\right)N(d_2)=1\,319.3N(0.150\,6)-$$
$$1\,397.2\exp\left(\frac{0.350\,788}{-22.437\,69}\times2-\frac{(0.350\,788-22.437\,69\times0.019\,5)\mathrm{e}^{-22.437\,69\times2}}{(-22.437\,69)^2}\right)N(-0.273\,7)=205.3(10^4\text{ dollars})$$

Thus, the value of the investment project $=205.3-50-40/1.038\,2=116.8$ ($10^4$ dollars).

The estimated outcome indicates that if the changes in risk-free rates and discount rates are not considered, the value of the investment project is overvalued $143.6-116.8=26.8$ ($10^4$ dollars). The improved method, taking those changes into consideration, makes an estimation of the project better based on the real option.

## 5 Conclusion

At present, China's marketing process of interest rates has just begun. The researches on the application of the term structure of interest rates and discount rates, based on Chinese financial facts, are relatively lagging behind. A simplification is often made when using real option theory to estimate the investment value of a project, in which risk-free interest rates and discount rates are regarded as constants and not distinguished in many cases. But in many cases, especially some multi-stage projects on a large scale, the real option has a long term and there are many uncertain factors affecting it. So the reality is that the risk-free interest rate and the discount rate may change. On the basis of former researches, the dynamic and static term structure models of interest rates are studied and applied to real option pricing. But this paper only researches and applies the simple term structure models which are the polynomial spline model and a simple basic dynamic model. In future researches, we will study and apply a complex model which more approaches the facts or can fit the term structure of interest rates of China more accurately.

## References

[1] Kester W C. Today's options for tomorrow's growth [J]. *Journal of Business*, 1984, **62**(2): 153 − 160.

[2] Trigeorgis L, Mason S P. Valuing managerial flexibility [J]. *Midland Corporate Financial Journal*, 1987, **5**(1): 14 − 21.

[3] Kulatilaka N, Marcus A J. Project valuation under uncertainty: when does DCF fail [J]. *Journal of Applied Corporate Finance*, 1992, **5**(3): 92 − 100.

[4] Hu Wenxiu, Liu Xiangfang. On real option pricing method under changing risk-free rate [J]. *Journal of Industrial Engineering Management*, 2006, **20**(3): 46 − 51. (in Chinese)

[5] He Qizhi, He Jianmin. Analysis on R& D investment decision based on term structure and game theory [J]. *Chinese Journal of Management*, 2007, **4**(5): 580 − 583. (in Chinese)

[6] Yu Jin. Latest development of modern term structure model of interest rate empirical and its application in China [J]. *Studies of International Finance*, 2004(10): 61 − 67. (in Chinese)

[7] Chan K C, Karolyi G A, Longstaff F A, et al. An empirical comparison of alternative models of the short-term interest rate [J]. *The Journal of Finance*, 1992, **47**(3): 1209 − 1227.

[8] Ma Xiaolan, Pan Guanzhong. GMM estimation for single factor term structure of interest rate models and the empirical test of Chinese money market [J]. *The Journal of Quantitative & Technical Economics*, 2006(1): 107 − 116. (in Chinese)

[9] Peng Huafei, Ren Zhaozhang. Term structure of China's short term interest rate [C]//*Proceedings of ICSSSM'05*. Piscataway: IEEE, 2005: 1273 − 1277.

[10] Zhu Shiwu, Chen Jianheng. Empirical research of the term structure of interest rates of stock exchange [J]. *Journal of Financial Research*, 2003, **280**(10): 63 − 73. (in Chinese)

[11] He Jianmin, He Qizhi. Research on term structure of treasury interest rates using a new model [C]//*International Conference on Wireless Communications, Networking and Mobile Computing*. Piscataway: IEEE, 2007: 4097 − 4100.

[12] He Qizhi. A new method for heteroscedasticity of term structure model using exponential splines [C]//*International Conference on Wireless Communications, Networking and Mobile Computing*. Piscataway: IEEE, 2007: 4068 − 4071.

[13] Chen Jingfeng, Ji Li. Real option to discount rate—expected yield rate to maturity [J]. *China Management Information*, 2005(12): 33 − 34. (in Chinese)

[14] Song Pengming. *Financial engineering principles*[M]. Beijing: Tsinghua University Press, 1999: 94 − 95. (in Chinese)

[15] Liu Jinfu, Li Xuewei. *Using stochastic process* [M]. Beijing: China Railway Press, 2000: 137 − 140. (in Chinese)

# 无风险利率和折现率变化时的R&D投资实物期权方法

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**摘要:**研究了利率期限结构静态模型中的多项式样条模型,并在此基础上得到折现率估计模型,从而对项目各期现金流进行相对准确的折现计算. 研究了利率期限结构动态模型中的一个简单的基本模型,并将其纳入到期权定价公式中,得到利率动态变化下的期权定价公式. 分别利用我国银行间回购利率数据、上海证券交易所数据估计出我国利率动态基本模型和静态多项式样条模型,给出一个实例对传统实物期权定价方法和考虑无风险利率、折现率变化时的实物期权定价方法进行了比较.

**关键词:**无风险利率;折现率;多项式样条;实物期权

**中图分类号:**F830