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Design of rate compatible punctured IRA codes for IR-HARQ schemes

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Abstract: An incremental redundancy hybrid automatic repeat-request (IR-HARQ) scheme based on irregular repeat-accumulate (IRA) codes is proposed. The design of rate compatible punctured IRA codes suitable for an IR-HARQ scheme is well formulated and efficiently solved by a linear-programming method, along with a one-dimensional approach for density evolution. Compared to IR-HARQ schemes based on turbo codes, simulation shows that the proposed IR-HARQ schemes based on IRA codes may achieve almost the same performance at a block size of 1 024, but better throughput at a block size of 4 096. The advantages of the proposed scheme in implementation, including decoding complexity and parallelism, make it more attractive in practice than the IR-HARQ schemes based on both turbo and LDPC codes.

Key words: hybrid automatic repeat-request protocol; incremental redundancy; IRA code; turbo code

The application of recently introduced capacity-approaching codes[1-3] in hybrid automatic repeat-request (HARQ) schemes seems encouraging. Indeed, rate compatible punctured turbo (RCPT) codes and LDPC codes have been proposed to achieve high throughput for IR-HARQ schemes[4-5]. RCPT codes have been shown to outperform rate compatible punctured convolutional (RCPC) codes introduced by Hagenauer[6] for medium to large block sizes. With iterative message-passing decoding, punctured LDPC codes can yield almost the optimal throughput by assuming infinite block size[5].

Irregular repeat-accumulate (IRA) codes are a class of serially concatenated codes[7], and therefore have a rather simple encoding algorithm. IRA codes, again as a special class of LDPC codes, can be decoded by the sum-product algorithm (SPA) in a fully parallel form. In this paper, we introduce an IR-HARQ scheme based on IRA codes. The well-designed IRA codes are competitive with the best-known LDPC codes, and outperform turbo codes for medium to large block sizes. Hopefully, the use of punctured IRA codes in IR-HARQ schemes may achieve better performance than that of RCPT codes at least for large block sizes.

The main contribution of this paper is to design RCPIRA codes suitable for an IR-HARQ scheme. By observing an interesting check-node merging property for puncturing the parity bits of IRA codes, we formulate the code design problem by employing a simplified one-dimensional density evolution (DE) approach[8].

1 Rate Compatible Punctured IRA Codes and HARQ Scheme

1.1 RCPIRA codes

Fig. 1 shows the Tanner graph representation of an IRA code. The encoding process of an IRA code can be explained as follows. A block of binary information bits $\boldsymbol{b} = \{b_1, b_2, \dots, b_{N_I}\}$ is encoded by an irregular repetition code of rate N_I/N_E. Among N_I binary bits, the fraction of information bits which repeat exactly i times is denoted as f_i, where $f_i \geqslant 0$, $\sum_i f_i = 1$. The block of repeated bits is interleaved, and the resulting block $\boldsymbol{v} = \{v_1, v_2, \dots, v_{N_E}\}$ is further encoded by an accumulator, defined by the recursion

$$x_{j+1} = x_j + \sum_{i=0}^{a-1} v_{aj+i} \qquad j = 1, 2, \dots, N_P$$

with initial condition $x_0 = 0$, where a is a positive integer (the left degree of check nodes), and $\boldsymbol{x} = \{x_1, x_2, \dots, x_{N_P}\}$ denotes the accumulator output block. For a systematic IRA code, the codeword is finally given by $\boldsymbol{c} = \{\boldsymbol{b}, \boldsymbol{x}\}$.

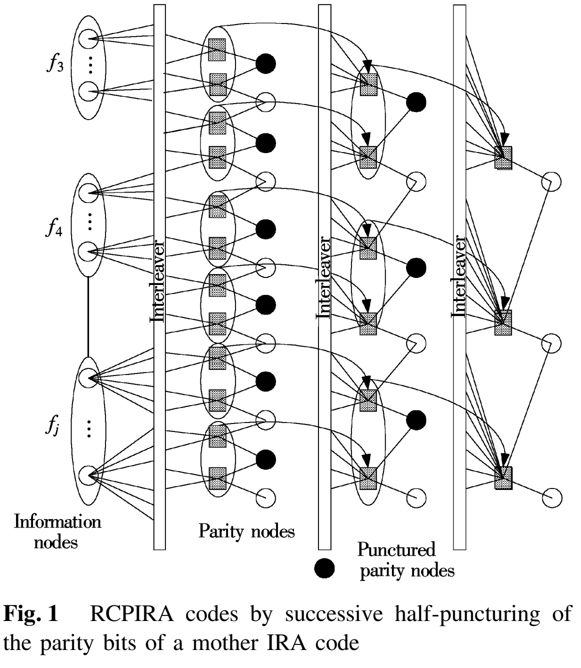

Fig. 1 RCPIRA codes by successive half-puncturing of the parity bits of a mother IRA code

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Essentially, a general IRA code ensemble can be specified by parameters $(f_1, f_2, \ldots, f_J; a)$ [7]. Another equivalent description adopts the concept from irregular LDPC codes, where the parameters $(\lambda_1, \lambda_2, \ldots, \lambda_J; a)$ are employed to represent an IRA code ensemble and $\lambda_i = if_i / \sum_j jf_j$ is the fraction of edges between the information and the check nodes that are adjacent to an information node of a degree of i in a Tanner graph representation of an IRA code. Let $b = \sum_i if_i$ denote the average repetition degree (or the average edge degree) of information nodes in the Tanner graph, and then the coding rate of a systematic IRA code is given by

$$R_c = \frac{1}{1 + \frac{b}{a}} = \frac{a}{a + \sum_i if_i} = \frac{\sum_i \frac{\lambda_i}{i}}{\frac{1}{a} + \sum_i \frac{\lambda_i}{i}} \tag{1}$$

Rate compatible IRA codes can be obtained by puncturing the parity bits (nodes) as shown in Fig. 1. By successively puncturing a low-rate mother IRA code, a family of IRA codes of a variety of rates can be obtained.

A useful property, a check-node merging property, can be observed from Fig. 1 and summarized as follows: Puncturing a parity node can be equivalently viewed as two neighboring check nodes being merged into one check node, whose left degree equals the sum of left degrees of two respective check nodes.

With the above property, puncturing half of the parity bits may result in another well-defined IRA code. For a mother $(\lambda_1, \lambda_2, \ldots, \lambda_J; a)$ IRA code, puncturing half of the parity bits results in an $(\lambda_1, \lambda_2, \ldots, \lambda_J; 2a)$ IRA code. By iteratively puncturing half of the parity bits as shown in Fig. 1, a class of IRA codes can be generated, where the code after the m-th half-puncturing has the parameters $(\lambda_1, \lambda_2, \ldots, \lambda_J; a_m)$ and $a_0 = a$, $a_m = 2a_{m-1}$, $m = 1, 2, \ldots, M$. The rate of the code after the m-th half-puncturing is

$$R_m = \frac{2^m R_0}{1 + (2^m - 1) R_0} \qquad m = 1, 2, \ldots, M \tag{2}$$

For example, we consider a mother IRA code of rate $R_0 = 1/3$ and $a_0 = 4$. By successively puncturing half of the parity bits as shown in Fig. 1, a family of IRA codes of rates 1/3, 1/2, 2/3, 4/5 can be generated. For the purpose of the ARQ protocol, the above rates may still not be enough. More rates can be obtained by adjusting the puncturing patterns over the parity bits. Generally speaking, the check-node merging property indicates that punctured IRA codes may have check nodes of different left degrees.

1.2 RCPIRA-HARQ protocol

In this paper, we focus on the IR-HARQ schemes over AWGN channels. Just as in Refs. [5-6], we do not consider the effects of finite memory or unreliable feedback on the performance of the system.

By successively puncturing a mother IRA code, the parity bits of the mother IRA code can be naturally divided into $Q + 1$ groups. Let Pnd_0 denote the set comprising all parity bits of the mother IRA code, Pu_q, $q = 1, 2, \ldots, Q$ as the set comprising the punctured parity bits at the q-th puncturing, and Pnd_q, $q = 1, 2, \ldots, Q$ as the set comprising all parity bits of the q-th punctured IRA code. Obviously, the highest rate is achieved by the Q-th punctured IRA code. A successive puncturing operation ensures the following relationships:

$$\mathrm{Pnd}_0 \supset \mathrm{Pnd}_1 \supset \mathrm{Pnd}_2 \supset \ldots \supset \mathrm{Pnd}_Q \tag{3}$$

$$\mathrm{Pnd}_0 = \mathrm{Pu}_1 \cup \mathrm{Pu}_2 \cup \ldots \cup \mathrm{Pu}_Q \cup \mathrm{Pnd}_Q \tag{4}$$

$$\mathrm{Pnd}_j = \mathrm{Pu}_{j+1} \cup \mathrm{Pnd}_{j+1} \qquad j = 0, 1, \ldots, Q-1 \tag{5}$$

With an IR-scheme, the ARQ protocol performs the following steps:

1) Add N_C parity bits to N_I information bits to form an error detection code C_d with a codeword length of $N = N_I + N_C$;

2) Input the N bit coded information block into the rate-R_0 mother IRA encoder, and all the parity bits belong to the set Pnd_0, which can be divided into $Q + 1$ disjoint sets according to Eq. (4) in a predetermined way;

3) At the 0-th retransmission (i.e., the first transmission), transmit the N bit coded information block and all the parity bits belonging to the set Pnd_Q;

4) Transmit additional parity bits belonging to Pu_{Q-q+1} for the q-th retransmission ($q = 1, 2, \ldots, Q$);

5) Decode the IRA code by the SPA in an iterative manner;

6) Check the syndrome of code C_d after a complete decoding iteration. If the syndrome is zero, output N_I information bits and send an ACK to the transmitter. Otherwise, continue the decoding loop until the maximum allowable iteration number is reached, upon which, if the syndrome is still nonzero, exit the loop, send an NAK to the transmitter and the system increases m and repeats steps 4) to 6).

If decoding is still not successful, several possibilities exist as discussed in Ref. [6]. The simplest option is to give up and accept a given bit- or frame-error rate upon failure at the lowest code rate. In simulations, we only consider the simplest option.

1.3 Throughput analysis

Generally, the performance of IR-HARQ schemes is measured by two figures of merit: the average throughput R_{AV} and the probability that a frame is finally not correctly decoded, the so-called frame error rate[6]. In this paper, the average throughput is considered as the primary figure of merit for the RCPIRA-HARQ system, which can be computed directly as

$$R_{AV} = \frac{N_I}{N} \frac{N}{N + P_{AV}} \tag{6}$$

where P_{AV} is the average number of additionally transmitted parity bits. It is always assumed that the error detection code C_d has a negligible probability of undetected frame errors.

2 Design of RCPIRA Codes Suitable for IR-HARQ Schemes

For a practical RCPIRA-HARQ system, various system parameters should be determined. In fact, there are three remaining issues to be investigated:

1) Design of a mother IRA code;
2) Finite length interleaver design;
3) Puncturing pattern identification.

In this section, we focus on the first problem. An apparent object is to maximize the system throughput averaged over the channel. Instead of maximizing the system throughput averaged over the SNR variation, we consider designing a mother IRA code, based on which the RCPIRA codes perform well for a variety of rates required for the IR-HARQ scheme.

2.1 Design of IRA codes

In order to formulate the problem mathematically, we first give a brief review of density evolution and its application on the design of IRA codes.

Let $p_v^{(l)}$ (resp., $\tilde{p}_v^{(l)}$) denote the average probability density function (pdf) of a log-likelihood ratio (LLR) message from an information node (resp., parity node) to a check node, at the l-th iteration. Let $p_u^{(l)}$ (resp., $\tilde{p}_u^{(l)}$) denote the pdf of the LLR message from a check node to an information node (resp., parity node) at the l-th iteration. By borrowing some concepts from quantized density evolution[3], it is not difficult to show[8]

$$p_v^{(l)} = p_{u0} * \lambda(p_u^{(l)}), \quad \tilde{p}_v^{(l)} = p_{u0} * \tilde{p}_u^{(l)} \tag{7}$$

$$p_u^{(l+1)} = \mathscr{R}(\mathscr{R}^{a-1} p_v^{(l)}, \mathscr{R}^2 \tilde{p}_v^{(l)}), \quad \tilde{p}_u^{(l+1)} = \mathscr{R}(\mathscr{R}^a p_v^{(l)}, \tilde{p}_v^{(l)}) \tag{8}$$

where p_{u_0} is the pdf from the channel, $*$ is the discrete convolution, $\lambda(p) = \sum_{i=1}^{d_l} \lambda_i \otimes^{i-1} p$ with $\otimes^i p = (\otimes^{i-1} p) * p$, $\mathscr{R}^j p = \mathscr{R}(\mathscr{R}^{j-1} p, p)$ and $p_c = \mathscr{R}(p_a, p_b)$ means that one can compute it numerically as

$$p_c(k\Delta) = \mathscr{R}(p_a, p_b)(k\Delta) = \sum_{i,j:\, k\Delta = \lfloor 2\tanh^{-1}(\tanh(i\Delta/2)\tanh(j\Delta/2)) \rfloor} p_a(i\Delta) p_b(j\Delta) \tag{9}$$

where Δ denotes the quantization step.

Let BER(σ) denote the limiting bit error rate (BER) of SPA decoding averaged over the IRA code ensemble for a binary-input AWGN channel defined by $x = s + z$, where $s \in \{+1, -1\}$ and z is the zero-mean white Gaussian noise with variance σ^2. With quantized density evolution (7) and (8), it follows that

$$\mathrm{BER}(\sigma) = \lim_{l \to \infty} \int_{-\infty}^{0} p_u^{(l)}(\sigma, x)\,\mathrm{d}x \tag{10}$$

Thus, given the threshold σ and the parameter a, the optimal IRA ensemble parameters $(\lambda_1, \lambda_2, \ldots, \lambda_J)$ maximize the coding rate R subject to vanishing BER(σ); i.e., they are solutions of the optimization problem.

$$\begin{aligned} &\max \sum_{j=1}^{J} \frac{\lambda_j}{j} \\ &\text{s.t.} \sum_{j=1}^{J} \lambda_j = 1 \qquad \lambda_j \geqslant 0,\ \forall j \\ &\qquad \mathrm{BER}(\sigma) = 0 \end{aligned} \tag{11}$$

2.2 Design of RCPIRA codes suitable for an IR-HARQ scheme

To design RCPIRA codes for an IR-HARQ scheme, it is tractable to design a mother IRA code, upon which the punctured RCPIRA codes of required rates are still good. In order to simplify the problem, a special class of RCPIRA codes, namely, HP-RCPIRA codes, is considered by successively half-puncturing of the parity bits of a mother IRA code as shown in Fig. 1.

Given a class of thresholds $\{\sigma_m; m = 0, 1, \ldots, M\}$ and the parameter $a_0 = a$ (then $a_m = 2a_{m-1}; m = 1, 2, \ldots, M$ for HP-RCPIRA codes), the optimal HP-RCPIRA ensemble parameters $\{(\lambda_1, \lambda_2, \ldots, \lambda_J; a_m); m = 0, 1, \ldots, M\}$ maximize the rates $\{R_m; m = 0, 1, \ldots, M\}$ subject to vanishing $\{\mathrm{BER}_m(\sigma_m); m = 0, 1, \ldots, M\}$; i.e., they are solutions of the optimization problem.

$$\begin{aligned} &\max \sum_{j=1}^{J} \frac{\lambda_j}{j} \\ &\text{s.t.} \sum_{j=1}^{J} \lambda_j = 1 \qquad \lambda_j \geqslant 0,\ \forall j \\ &\qquad \mathrm{BER}_m(\sigma_m) = 0 \qquad m = 0, 1, \ldots, M \end{aligned} \tag{12}$$

The above formulation is exact. Indeed, due to Eq. (2), it can be well understood that maximizing $M+1$ rates $\{R_m; m = 0, 1, 2, \ldots, M\}$ is just equivalent to maximizing the rate of the mother IRA code R_0.

Clearly, a nonlinear search is required to find a good degree profile for HP-RCPIRA codes, which is, indeed, a time-consuming task. A further simplification is possible by introducing a one-dimensional approach for density evolution. Here, we adopt the method in Ref. [8] and upon which, to ensure $\mathrm{BER}_m(\sigma_m) = 0$, it is equivalent to forcing the following condition being satisfied

$$\phi_{a_m}(x, \sigma_m) = \sum_{j=1}^{J} \lambda_j J\left(\frac{2}{\sigma_m^2} + (j-1) J^{-1}(1 - J((a_m - 1) \cdot J^{-1}(1-x) + 2J^{-1}(1 - \tilde{x}(x))))\right) > x \tag{13}$$

$$x = \tilde{x}^{-1}(\tilde{x}) = 1 - J\left(\frac{1}{a_m}\left(J^{-1}\left(1 - J\left(J^{-1}(\tilde{x}) - \frac{2}{\sigma_m^2}\right)\right) - J^{-1}(1-\tilde{x})\right)\right) \qquad m = 0, 1, \ldots, M$$

for any $x \in [0, 1)$, where $J(u) \triangleq \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-z^2} \log_2(1 + \mathrm{e}^{-2z\sqrt{u} - u})\,\mathrm{d}z$, and $x = \tilde{x}^{-1}(\tilde{x})$ denotes the inverse function of $\tilde{x} = \tilde{x}(x)$ (We refer readers to Eq. (38) of Ref. [8] for more details about Eq. (13)). Since Eq. (13) is linear in λ_j's, an effective linear programming approach can be employed to find a good degree profile for HP-RCPIRA codes.

$$\begin{aligned} &\max \sum_{j=1}^{J} \frac{\lambda_j}{j} \\ &\text{s.t.} \sum_{j=1}^{J} \lambda_j = 1 \qquad \lambda_j \geqslant 0,\ \forall j \\ &\qquad \phi_{a_m}(x, \sigma_m) > x \qquad \forall x \in [0, 1);\ m = 0, 1, \ldots, M \end{aligned} \tag{14}$$

For a practical design purpose, a remaining problem is to choose multiple thresholds of σ_m, $m = 0, 1, \ldots, M$. By assuming that the performance of the designed HP-RCPIRA codes is close to channel capacity limit, the number of degrees of freedom in choosing multiple σ_m can be greatly reduced. Indeed, let $\sigma_m^{\lim}$ be the binary-input AWGN capacity limit for the m-th rate of R_m, and σ_0 be the only variable threshold corresponding to the mother IRA code. By defining $\Delta\delta_m = \sigma_m^{\lim} - \sigma_0^{\lim}$, $m = 0, 1, \ldots, M$, we can choose other parameters as

$$\sigma_m = \sigma_0 + \Delta\delta_m \qquad m = 1, 2, \ldots, M \tag{15}$$

With the above constraints, the only freedom remaining is to vary σ_0, which greatly simplifies the design procedure shown in (12).

Design Example In order to obtain 8 ($Q = 7$) RCPIRA codes of rates 1/3, 4/11, 2/5, 4/9, 1/2, 4/7, 2/3, 4/5 (considered in Ref. [5]), we consider a mother IRA code of $a = 4$ and $R_0 = 1/3$.

As indicated in (2), the aimed RCPIRA codes of rates 1/3, 1/2, 2/3, 4/5 can be generated by successively half puncturing of the parity bits of the mother IRA code. The other RCPIRA codes of rates 4/11, 2/5, 4/9 can be obtained by adjusting the puncturing pattern over the parity bits of the mother IRA code. We will consider it later in the numerical section. At this point, we can formulate the design problem as shown in (14) by optimizing the HP-RCPIRA codes of rates 1/3, 1/2, 2/3, 4/5.

By restricting both the maximum and minimum repetition degrees (denoted as $d_{v,\max}$, $d_{v,\min}$ respectively), the degree profile of the mother IRA code can be obtained by solving the linear programming problem formulated in (14). In order to ensure a good FER performance, it is essential to have $\lambda_2 = 0$ as discussed in Ref. [7], which means $d_{v,\min} = 3$. The degree profiles for different $d_{v,\max}$'s are listed in Tab. 1, which is found to have just four kinds of repetition degrees and therefore desirable for an implementation purpose.

Tab. 1 Designed degree profiles of the mother IRA codes

a	R	SL	$d_{v,\max} = 40$		$d_{v,\max} = 70$	
			i	λ_i	i	λ_i
			3	0.2370	3	0.2422
			9	0.0275	11	0.0820
			10	0.3272	12	0.3932
			40	0.4083	70	0.2826
4	1/3	1.2966	1.2910 *		1.2956 *	
8	1/2	0.9787	0.9557 *		0.9540 *	
16	2/3	0.7666	0.7304 *		0.7343 *	
32	4/5	0.6251	0.5932 *		0.5965 *	

Notes: SL is the Shannon limit; * is the threshold predicted by quantized density evolution.

Under the SPA decoding, the performance of our designed codes in the limit of large block size can be well predicted by density evolution. The thresholds predicated by quantized density evolution are summarized in Tab. 1. Clearly, the designed RCPIRA codes are approaching the Shannon limits in the limits of large block sizes.

3 Numerical Results

3.1 Interleaver design based on loop-removal approach

The practical RCPIRA-HARQ system always assumes a finite block size, typically a block size of 100 to 1 000. For an LDPC code, the interleaver mentioned here is referred to as the edge inteleaver between the variable and check nodes.

Indeed, we proposed a semi-construction approach, which is an extension of "avoiding short cycles in the bipartite graph of the IRA code". The technical details about the proposed algorithm are summarized in Ref. [9].

3.2 Simulation conditions and puncturing pattern identification

In simulations, we consider the mother IRA codes in Tab. 1 of rate $R_0 = 1/3$ and $a = 4$. By successively puncturing half of the parity bits as shown in Fig. 1, a family of IRA codes of rates 1/3, 1/2, 2/3, 4/5, 8/9 can be straightforwardly constructed. For the purpose of the ARQ protocol, the above rates may still not be enough. General rate compatible IRA codes of more rates can be obtained by adjusting the puncturing patterns. In simulations, in order to obtain 8 ($Q = 7$) RCPIRA codes of rates 1/3, 4/11, 2/5, 4/9, 1/2, 4/7, 2/3, 4/5, all the parity bits of the mother IRA code are divided into P subblocks of the equal subblock size of $B = 8$ (the number of all parity bits is $P \times B$). Here, we consider periodic puncturing patterns with a period of B. Hence, the puncturing patterns can be represented by a binary $M \times B$ puncturing matrix $\Xi = \{\psi(m)\}$, $m = 0, 1, \ldots, M - 1$. In simulations, the following puncturing patterns are adopted:

$$\psi(0) = [11111111], \quad \psi(1) = [01111111]$$

$$\psi(2) = [01110111], \quad \psi(3) = [01010111]$$

$$\psi(4) = [01010101], \quad \psi(5) = [00010101]$$

$$\psi(6) = [00010001], \quad \psi(7) = [00000001]$$

where "0" means that the corresponding bit is punctured while "1" means that the bit is preserved. It is clear that the RCPIRA codes of puncturing patterns $\psi(0)$, $\psi(4)$, $\psi(6)$, $\psi(7)$ are of rates 1/3, 1/2, 2/3, 4/5, which can be well defined by the half-puncturing process as shown in Fig. 1.

Theoretically, it is still possible to optimize the system performance by varying the puncturing patterns of $\psi(1)$, $\psi(2)$, $\psi(3)$, $\psi(5)$. However, simulations show that our choice is reasonable and there is no practical room for performance improvement by adopting other puncturing patterns.

In simulations, the ARQ protocol terminated upon reaching the lowest code rate; i. e., no repetitions of the protocol were allowed. The maximum number of iterations allowed at any given decoding attempt was fixed at 40. For a comparison purpose, we run an RCPT-HARQ scheme in Ref. [4]. The RCPT-HARQ system employs the encoder B (see Tab. 2 of Ref. [5]). The maximum number of iterations allowed at any given decoding attempt is fixed at 12 for the RCPT-ARQ system. For simplicity, we assume a genie-aided error detection scheme for both RCPT- and RCPIRA-HARQ schemes.

3.3 Decoding algorithms and their complexity

In general, there are two decoding schemes for an RCPIRA-HARQ system, one of which is to run the iterative SPA over the graph resulting from the check-node merging property, while the other is over the original graph of the mother IRA code with some of the parity-nodes erased.

As far as decoding complexity is concerned, it is suitable to estimate the number of additions and lookups required for iterative decoding of both IRA and turbo codes. At a check node of degree K, the implementation of $\sum_{k=1, k\neq j}^{K} \oplus L_k$ is required, where the so-called check operation $\oplus$ is defined by $c = a \oplus b \Leftrightarrow c = 2\tanh^{-1}\left(\tanh\frac{a}{2}\tanh\frac{b}{2}\right)$. Let $\phi(x) = -\log\left(\tanh\left(\frac{x}{2}\right)\right)$, then it is not difficult to show that $L = \sum_{k=1}^{K} \oplus L_k = \left(\prod_{k=1}^{K}\mathrm{sign}(L_k)\right)\phi\left(\sum_{k=1}^{K}\phi(|L_k|)\right)$. Hence, we assume that the implementation of $\sum_{k=1}^{K} \oplus L_k$ may require as many as $K-1$ additions and $K+1$ table lookups (assuming $\phi(x)$ are implemented via table lookups). At a variable node of degree K, it is required to compute $A_j = \sum_{k=1, k\neq j}^{K} L_k, j = 1, 2, \ldots, K$. Here, the computation of a list of $A_j = \sum_{k=1, k\neq j}^{K} L_k, j = 1, 2, \ldots, K$ is assumed to be efficiently implemented by obtaining $A = \sum_{k=1}^{K} L_k$ at first and computing $A_j = A - L_j, j = 1, 2, \ldots, K$ subsequently. Thus, it requires $2K$ table lookups and $(2K-1)$ additions for each check node of degree K. When the above rule is applied to an $(f_1, f_2, \ldots, f_J; a)$ IRA code with parameters, it requires an average of $2b = 2\sum_i if_i$ additions for each information node. At a parity node, we require two additions. In summary, for a rate $-1/3$ IRA code with the left degree $a = 4$, the total number of additions per information bit is $(2(a+2)-1)\times 2 + 2b + 2\times 2 = 42$, and the total number of table lookups per information bit is $2(a+2)\times 2 = 24$. Then, the total number of operations (counting additions and lookups as the same) per information bit is 66. For the turbo code used in Ref. [4], the same complexity can be evaluated, and it is about 400-operations/information bit/iteration. Even taking into account the fact that IRA codes use 40 iterations, whereas only 12 iterations were used with turbo codes in Ref. [4], the decoding complexity is still lower for the IRA codes.

3.4 Simulation results and discussion

To investigate the choice of decoding scheme on system performance, we simulate the RCPIRA-HARQ system of a block size of 1 024 and $d_{v,\max} = 40$ under two decoding schemes. As shown in Fig. 2, the first decoding scheme has better performance than the second one. Due to this performance advantage, the first decoding scheme may find its application in software implementation of the decoding algorithm. The second one, however, may be more preferred in hardware implementation, as only one decoder is required for all RCPIRA codes. In what follows, the first decoding scheme is always assumed.

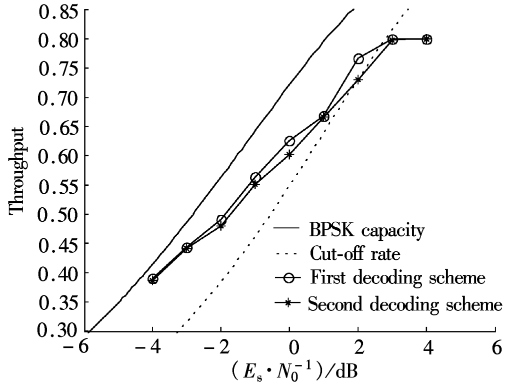

Fig. 2 Performance comparison of two decoding schemes for an RCPIRA-HARQ system

The throughput of the RCPT-HARQ system (under the optimal puncturing) in Ref. [4] is given and compared with the RCPIRA-HARQ system at a block size of 1 024 in Fig. 3. The performance of the proposed RCPIRA-HARQ system is almost the same as that of the RCPT-HARQ system at a block size of 1 024. At a larger block size of 4 096, the results are given in Fig. 4. We see that the performance of RCPIRA-HARQ system is slightly better than that of the RCPT-HARQ system at a block size of 4 096, and the throughput of both RCPIRA- and RCPT-HARQ systems are above the cutoff rate. Our scheme shows its advantage on the

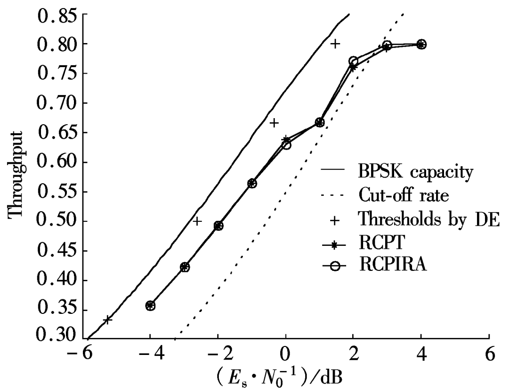

Fig. 3 Performance comparison between RCPIRA and RCPT codes at a block size of 1 024

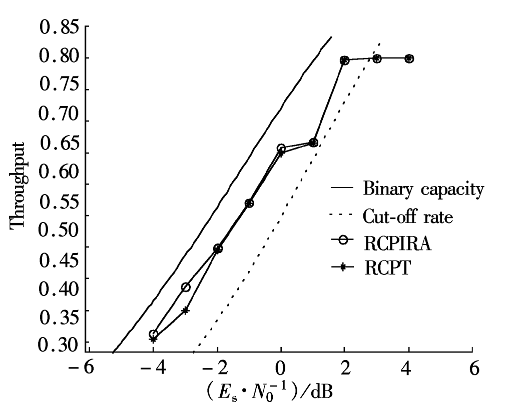

Fig. 4 Performance comparison between RCPIRA and RCPT codes at a block size of 4 096

encoding and decoding complexities compared to other schemes based on turbo codes[4] and LDPC codes[5]. The encoding of IRA codes is competitive to that of turbo codes, and is often lower than that of LDPC codes, while the decoding complexity of IRA codes is shown to be just the same as that of LDPC codes and often lower than that of turbo codes. Therefore, the proposed RCPIRA-HARQ system is more attractive in practice.

4 Conclusion

In this paper, a novel application of IRA codes to an IR-based hybrid ARQ scheme is proposed. By exploiting an interesting check-node merging property for puncturing the parity bits of an IRA code, an efficient linear-programming method is proposed to design RCPIRA codes suitable for an IR-HARQ protocol. The performance of the proposed RCPIRA-HARQ system is slightly better than that of the RCPT-HARQ system at large block sizes. Furthermore, the proposed RCPIRA-HARQ systems have low encoding and decoding complexity, which makes it more attractive than previously introduced HARQ schemes based on both turbo and LDPC codes.

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适于 IR-HARQ 协议的速率匹配删截 IRA 码设计

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摘要:基于非规则重复累加码(IRA)提出一种冗余递增自动重传协议(IR-HARQ)且该自动重传协议的性能取决于速率匹配删截 IRA 码的性能. 在此基础上提出用线性规划结合一维概率密度演化算法解决核心的速率匹配删截 IRA 码的设计问题,并给出了设计结果. 仿真表明:相比于基于 turbo 码的相应方案,所提方案的性能在短帧(帧长 1 024)下大体相仿,而在长帧(帧长 4 096)下表现出一定的优势. 相比于 turbo 和低密度校验码,基于 IRA 码的方案在解码的复杂性以及并行化方面具有独到的优势.

关键词:自动重传协议;冗余递增;非规则重复累加码;turbo 码

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