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Some results on circular chromatic number of a graph

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Abstract: For two integers k and d with $(k, d) = 1$ and $k \geqslant 2d$, let G_k^d be the graph with vertex set $\{0, 1, \ldots, k-1\}$ in which ij is an edge if and only if $d \leqslant |i-j| \leqslant k-d$. The circular chromatic number $\chi_c(G)$ of a graph G is the minimum of k/d for which G admits a homomorphism to G_k^d. The relationship between $\chi_c(G-v)$ and $\chi_c(G)$ is investigated. In particular, the circular chromatic number of $G_k^d - v$ for any vertex v is determined. Some graphs with $\chi_c(G-v) = \chi_c(G) - 1$ for any vertex v and with certain properties are presented. Some lower bounds for the circular chromatic number of a graph are studied, and a necessary and sufficient condition under which the circular chromatic number of a graph attains the lower bound $\chi - 1 + 1/\alpha$ is proved, where χ is the chromatic number of G and α is its independence number.

Key words: (k, d)-coloring; r-circular-coloring; circular chromatic number; Mycielski's graph

All graphs in this paper are simple, finite and undirected. The circular chromatic number of a graph is a natural generalization of the ordinary chromatic number of a graph, introduced by Vince[1] in 1988, under the name star chromatic number of a graph. Let k and d be two integers with $0 < 2d \leqslant k$. A (k, d)-coloring of a graph G is a coloring c of vertices of G with k colors $0, 1, \ldots, k-1$ such that for any edge xy, $d \leqslant |c(x) - c(y)| \leqslant k-d$. The circular chromatic number $\chi_c(G)$ of G is defined as

$$\chi_c(G) = \inf\left\{\frac{k}{d}: \text{there is a } (k, d)\text{-coloring of } G\right\}$$

It was proved[1-2] that for finite graphs, the infimum is always attained, hence it can be replaced by the minimum. The following is an equivalent definition of the circular chromatic number of a graph given in Ref. [3].

Let C^r be a circle of (Euclidean) circumference r. An r-circular-coloring of a graph G is a mapping c which assigns an open unit arc $c(x)$ of C^r to each vertex x of G, such that for every edge xy of G, $c(x) \cap c(y) = \varnothing$. We say that a graph G is r-circular-colorable if there is an r-circular-coloring of G. The circular chromatic number of G is equal to

$$\chi_c(G) = \inf\{r: G \text{ is } r\text{-circular-colorable}\}$$

Given two graphs G and H, a homomorphism from G to H is a mapping f from $V(G)$ to $V(H)$ such that $f(x)f(y) \in E(H)$ whenever $xy \in E(G)$. If there is a homomorphism from G to H, we say G is homomorphic to H. Two graphs G and H are homo-equivalent if G is homomorphic to H and H is homomorphic to G. Homomorphism can be viewed as a generalization of graph coloring. It is easy to see that

$$\chi(G) = \min\{n: G \text{ admits a homomorphism to } K_n\}$$

For two integers k and d with $(k, d) = 1$ and $k \geqslant 2d$, let G_k^d be the graph with a vertex set $\{0, 1, \ldots, k-1\}$ in which ij is an edge if and only if $d \leqslant |i-j| \leqslant k-d$. It is not difficult to see that a (k, d)-coloring of a graph G is a homomorphism from G to G_k^d. Thus, we have[2]

$$\chi_c(G) = \inf\left\{\frac{k}{d}: G \text{ admits a homomorphism to } G_k^d\right\}$$

In the study of circular chromatic numbers, graphs G_k^d play the same role as complete graphs in the study of chromatic numbers.

In this paper, we first determine the exact value of the circular chromatic number of $G_k^d - v$ for any vertex v in G_k^d. Then we prove that for any graph G with at least 3 vertices, there exist two vertices u and v of G such that $\chi_c(G-u-v) \geqslant \chi_c(G) - 2$. We also investigate graphs G for which $\chi_c(G-v) = \chi_c(G) - 1$ for each vertex v of G.

The lower bounds of circular chromatic numbers of a graph G were also studied[4]. Here we investigate the graphs whose circular chromatic numbers attain the lower bounds. We establish a necessary and sufficient condition under which the circular chromatic numbers of a graph G attain the lower bound $\chi - 1 + 1/\alpha$, where χ is the chromatic number of G and α is the independent number.

1 About $\chi_c(G-v)$

In this section, we present some results concerning the circular chromatic numbers of $G-v$ and $G-e$.

Theorem 1 Let k and d be two positive integers with $k \geqslant 2d$. If $(k, d) \neq 1$, then $\chi_c(G_k^d - v) = \chi_c(G_k^d) = k/d$ for any vertex v of G_k^d. If $(k, d) = 1$, then $\chi_c(G_k^d - v) = \dfrac{k-\alpha}{d-\beta}$, where α is the smallest positive integer such that there is some integer β with $\alpha d = \beta k + 1$.

Proof Let $[k]$ denote the set $\{0, 1, \ldots, k-1\}$. Then $V(G_k^d) = [k]$. Since $G_k^d - v$ is the subgraph of G_k^d, $\chi_c(G_k^d - v) \leqslant \chi_c(G_k^d) = k/d$. Since G_k^d is vertex transitive, without loss of generality we may assume that $v = d$.

Case 1 $(k, d) \neq 1$.

Suppose that $(k, d) = t > 1$. Then it is easy to see that $G_{k/t}^{d/t}$ is homomorphic to $G_k^d - d$, therefore $k/d = \chi_c(G_{k/t}^{d/t}) \leqslant \chi_c(G_k^d - d)$. It follows that $\chi_c(G_k^d - v) = \chi_c(G_k^d) = k/d$.

Case 2 $(k, d) = 1$.

Let α be the smallest positive integer such that there is some integer β with $\alpha d = \beta k + 1$. In Ref. [5], Zhu defined a mapping c: $V(G_k^d) \setminus \{d\} \mapsto Z_{k-\alpha}$ as

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$$c(i) = i - |\{t \mid 0 < t \leqslant \alpha: td \bmod k \leqslant i, t \in Z\}|$$

where the multiplication td is in the field Z_k and the order $td \leqslant i$ is the order of natural numbers. It is easy to verify that c is a $(k-\alpha, d-\beta)$-coloring of $G_k^d - d$. Thus $\chi_c(G_k^d - v) \leqslant (k-\alpha)/(d-\beta)$.

Let $S_\alpha = \{td \bmod k \mid t = 1, 2, \ldots, \alpha\}$. Then S_α is a subset of $[k]$ with cardinality α. Let $G_k^d - S_\alpha$ be the subgraph of G_k^d induced by $[k] \setminus S_\alpha$. We prove that $G_k^d - S_\alpha$ is isomorphic to $G_{k-\alpha}^{d-\beta}$. For any $j \in [k] \setminus S_\alpha$, let x be the minimum positive integer such that $j + x$ is in $[k] \setminus S_\alpha$ and $j + x$ is adjacent to j in G_k^d. Then it is easy to see that if $j \neq 0$ then $x = d$, and if $j = 0$ then $x = d + 1$. Furthermore, it is not difficult to check that for any $j \in [k] \setminus S_\alpha$, $|[0, d+1] \cap S_\alpha| = \beta + 1$ and if $j \neq 0$ then $|[j, j+d]_k \cap S_\alpha| = \beta$ (where $[a, b]_k = \{a, a+1, a+2, \ldots, b\}$ and the additions are taken modulo k). It follows that $G_k^d - S_\alpha$ is isomorphic to $G_{k-\alpha}^{d-\beta}$. Since $G_k^d - S_\alpha$ is a subgraph of $G_k^d - d$, we have

$$\chi_c(G_k^d - d) \geqslant \chi_c(G_k^d - S_\alpha) = \chi_c(G_{k-\alpha}^{d-\beta}) = \frac{k-\alpha}{d-\beta}$$

Therefore, $\chi_c(G_k^d - v) = (k-\alpha)/(d-\beta)$.

An r-interval-coloring of a graph G is a mapping g which sends each vertex x of G to a unit length open sub-interval $g(x)$ of the interval $[0, r]$, such that adjacent vertices are sent to disjoint sub-intervals. It is well-known that the chromatic number $\chi(G)$ of G is the least real number such that there is an r-interval-coloring of G[6]. An r-interval-coloring of G corresponds to a mapping f from $V(G)$ to $[0, r)$ such that $1 \leqslant |f(x) - f(y)| \leqslant r - 1$ for every edge xy of G and $f(x) \leqslant r - 1$ for all $x \in V(G)$. Therefore any r-interval-coloring of G corresponds to an r-circular-coloring of G.

Theorem 2[7] Let G be a graph and e be any edge of G, then $\chi_c(G - e) \geqslant \chi_c(G) - 1$.

Proof Let $e = xy$ be any edge of G and let $G_1 = G - e$. Suppose that $\chi_c(G_1) = r$ and let c be an r-circular-coloring of G_1. For each vertex v of G_1, $c(v)$ is an open interval on C^r. We denote $c(v)$ by $(s_v, s_v + 1)_r$. Without loss of generality, we may assume that $s_x = 0$. We then construct an $(r+1)$-interval-coloring c' of G as follows: $c'(x) = c(x)$, $c'(v) = c(v)$ if $s_v \notin [r-1, 0]_r$, $c'(v) = (s_v, s_v + 1)$ if $s_v \in [r-1, 0)_r$, and $c'(v) = (r, r+1)$ if $s_v = 0$ and $v \neq x$. This implies that $\chi_c(G) \leqslant r + 1$. Thus $r = \chi_c(G - e) \geqslant \chi_c(G) - 1$. This proves the theorem.

The equality in theorem 2 is attainable. An example with $\chi_c(G - e) = \chi_c(G) - 1$ for any edge e of G will be presented later.

A graph G is k-vertex-critical if $\chi(G) = k$ and $\chi(G - v) = k - 1$ for any vertex v of G.

Theorem 3 For any graph G with $|V(G)| \geqslant 2$, there exist two vertices u and v of G such that $\chi_c(G - u - v) \geqslant \chi_c(G) - 2$. Furthermore, if G is not isomorphic to K_n then the inequality is strict.

Proof Suppose that $\chi(G) = n$. If G is not n-vertex-critical, then there exists a vertex u of G such that $\chi(G - u) = n$. Let v be any vertex of $G - u$, then clearly $\chi(G - u - v) \geqslant n - 1$, which implies $\chi_c(G - u - v) > n - 2$. So we may assume that G is n-vertex-critical.

Let u be an arbitrary vertex of G, since G is n-vertex-critical, we have $\chi(G - u) = n - 1$. If $G - u$ is not $(n-1)$-vertex-critical, then there exists a vertex v of G such that $\chi(G - u - v) = n - 1$ which implies $\chi_c(G - u - v) > n - 2$. Thus we assume that $G - u$ is $(n-1)$-vertex-critical. That is for any vertex v of $G - u$, $\chi(G - u - v) = n - 2$. Given an $(n-2)$-coloring of $G - u - v$, by coloring the vertex v with a new color $n - 1$, we obtain an $(n-1)$-coloring of $G - u$. If $uv \notin E(G)$, then we can color the vertex u with color $n - 1$ and obtain an $(n-1)$-coloring of G. This contradicts $\chi(G) = n$. As v is an arbitrary vertex of $G - u$, u is adjacent to all vertices of $G - u$. Again, since u is an arbitrary vertex of G, we conclude that G is exactly the graph K_n. It is obvious that $\chi_c(K_n - u - v) = n - 2$. This completes the proof of theorem 3.

An interesting problem involving the deletion of a vertex was raised by Zhu[5] as follows: Which graphs have the property that the deletion of any vertex will decrease its circular chromatic number by exactly 1? Suppose G is a graph. An extremely stable set of G is a nonempty subset S of $V = V(G)$ such that for any $v \in V \setminus S$, v is either adjacent to every vertex of S or adjacent to no vertex of S. Zhu then posed the following question: Is it true that if $|V(G)| > 2$ and $\chi_c(G - v) = \chi_c(G) - 1$ for each vertex v of G, then G has a nontrivial extremely stable set S (i. e., $2 \leqslant |S| < |V(G)|$)? Here we present two classes of graphs which have the property that the deletion of any vertex will decrease their circular chromatic number by exactly 1 but they will have no nontrivial extremely stable set.

For a graph G with vertex set V and edge set E, the Mycielskian of G, which was first introduced by Mycielski[8], is the graph $\mu(G)$ with vertex set $V \cup V' \cup \{u\}$, where $V' = \{x' : x \in V\}$, and edge set $E \cup \{xy' : xy \in E\} \cup \{y'u : y' \in V'\}$.

The following theorem was proved in Ref. [9].

Theorem 4 For $n \geqslant 3$, $\mu(K_n)$ is $(n+1)$-critical and $\chi_c(\mu(K_n)) = n + 1$.

One can easily verify that $\chi_c(\mu(K_n) - v) = \chi_c(\mu(K_n)) - 1 = n$ for any v of $\mu(K_n)$ and $\chi_c(\mu(K_n) - e) = \chi_c(\mu(K_n)) - 1 = n$ for any e of $\mu(K_n)$. Following that we prove that $\mu(K_n)$ has no nontrivial extremely stable set. This gives a counterexample to Zhu's question.

Suppose, to the contrary, that S is a nontrivial extremely stable set of $\mu(K_n)$. Let $V(K_n) = \{v_1, v_2, \ldots, v_n\}$. We consider three cases.

Case 1 There is some i such that both x_i and x_i' are in S. Since u is adjacent to x_i but not to x_i', according to the definition of an extremely stable set, u must be in S. For each j $(\neq i)$, since v_j is adjacent to v_i and not to u, all v_js are in S. And since v_j' is not adjacent to v_j but adjacent to u, we have all v_j's are in S. Therefore, $S = V(\mu(K_n))$. A contradiction.

Case 2 There is some i such that $v_i \in S$ but $v_i' \notin S$. If $u \in S$, then similar to case 1 we can prove that $S = V(\mu(K_n))$, therefore, a contradiction. Thus $u \notin S$. Since S is a nontrivial extremely stable set, there is some $j(\neq i)$ such that v_j or v_j' is in S. If $v_j \in S$ then v_j' should be in S and it should be reduced to case 1. So we assume that $v_j' \in S$. But then we should have $v_j \in S$ because v_j is adjacent to v_i and not to v_j'. This is a contradiction.

Case 3 There is some i such that $v_i' \in S$ but $v_i \notin S$. If u

$\in S$ then all v_is are in S. And it follows that $S = V(\mu(K_n))$, a contradiction. Thus $u \notin S$. Then there is some j such that v_j or v_j' is in S. Similarly as in case 2, one can reach contradictions.

This proves that $\mu(K_n)$ has no nontrivial extremely stable set.

Given a graph G with vertex set $V_0 = \{v_1^0, v_2^0, \ldots, v_\nu^0\}$ and edge set E_0 and an integer $m \geqslant 0$, the generalized Mycielskian of G is the graph $\mu_m(G)$ with vertex set $V_0 \cup V_1 \cup V_2 \cup \ldots \cup V_m \cup \{u\}$, where $V_i = \{v_j^i : v_j^0 \in V_0\}(i = 1, 2, \ldots, m)$, and edge set $E_0 \cup \left(\bigcup_{i=0}^{m-1} \{v_j^i v_{j'}^{i+1} : v_j^0 v_{j'}^0 \in E_0\} \cup \{v_j^m u : v_j^m \in V_m\}\right)$. Clearly when $m = 0$, $\mu_0(G)$ is the graph obtained from G by adding a universal vertex u. And $\mu_1(G)$ is just the Mycielskian $\mu(G)$ of G. The following theorem was proved in Ref. [10].

Theorem 5 For $n \geqslant 3$ odd and any integer $m \geqslant 0$, $\mu_m(K_n)$ is $(k+1)$-critical and $\chi_c(\mu_m(K_n)) = n+1$.

It is easy to check that $\chi_c(\mu_m(K_n) - v) = \chi_c(\mu_m(K_n)) - 1 = n$ for each vertex v of $\mu_m(K_n)$. And similarly as above, one can show that $\mu_m(K_n)$ has no nontrivial extremely stable set.

2 Lower Bounds for $\chi_c(G)$

In Ref. [1], Vince proved that for any graph G,

$$\chi_c(G) > \chi(G) - 1 \tag{1}$$

Since there are graphs G whose circular chromatic number can be arbitrarily close to $\chi(G) - 1$, in this sense, this lower bound for $\chi_c(G)$ is sharp. However it can still be improved in some sense. Zhou[4] proved that,

$$\chi_c(G) \geqslant \chi(G) - 1 + \frac{1}{\alpha(G)} \tag{2}$$

and

$$\chi_c(G) \geqslant \chi(G) - 1 + \frac{\chi(G) - 1}{c(G) - 1} \tag{3}$$

where $\alpha(G)$ is the independence number of G and $c(G)$ is the length of a longest cycle of G. Using a result of Ref. [3] below, we shall give a simple proof of the two above lower bounds.

Lemma 1 For any graph G, $\chi_c(G)$ is equal to some p/q, where p is at most the length of a longest cycle of G and q is at most the independence number of G.

By lemma 1, suppose that $\chi_c(G) = p/q$ for some p and q, where $p \leqslant c(G)$ and $q \leqslant \alpha(G)$. By Eq. (1),

$$\chi_c(G) = \frac{p}{q} > \chi(G) - 1, \quad p \geqslant (\chi(G) - 1)q + 1$$

$$\frac{p}{q} \geqslant \chi(G) - 1 + \frac{1}{q} \geqslant \chi(G) - 1 + \frac{1}{\alpha(G)} \tag{4}$$

By Eq. (4) we have

$$q \leqslant \frac{1}{\chi(G) - 1} p - \frac{1}{\chi(G) - 1}$$

$$\frac{q}{p} \leqslant \frac{1}{\chi(G) - 1} - \frac{1}{\chi(G) - 1} \frac{1}{c(G)}$$

Hence,

$$\frac{p}{q} \geqslant \chi(G) - 1 + \frac{\chi(G) - 1}{c(G) - 1}$$

For graphs G and K, let $\nu(G)$ denote the number of vertices of G, and $\nu(G, K)$ the maximum number of vertices in a subgraph of G that admits a homomorphism to K. Bondy and Hell[2] proved the following two lemmas.

Lemma 2 Let G, H and K be graphs, where H is vertex-transitive. If there is a homomorphism $f: G \to H$, then

$$\frac{\nu(G, K)}{\nu(G)} \geqslant \frac{\nu(H, K)}{\nu(H)}$$

Lemma 3 $\chi_c(G) \leqslant k/d$ if and only if G is homomorphic to G_k^d.

Theorem 6 For a graph G, let $\alpha_i(G) = \frac{1}{i}\max(|S_1| + |S_2| + \ldots + |S_i|)$ where "max" takes over all i pairwise disjoint independent sets $S_1, S_2, \ldots, S_i$ in G. Then

$$\chi_c(G) \geqslant \frac{\nu(G)}{\alpha_i(G)} \tag{5}$$

Proof Suppose that $\chi_c(G) = k/d$, by lemma 3, G is homomorphic to G_k^d. Consider the three graphs G, G_k^d and K_i, by lemma 2, we have

$$\frac{\nu(G, K_i)}{\nu(G)} \geqslant \frac{\nu(G_k^d, K_i)}{\nu(G_k^d)}$$

hence

$$\frac{\max(|S_1| + |S_2| + \ldots + |S_i|)}{\nu(G)} \geqslant \frac{id}{k}$$

where "max" takes over all i pairwise disjoint independent sets $S_1, S_2, \ldots, S_i$ in G. And it follows that (5) is true.

Corollary 1 For any graph G,

$$\chi_c(G) \geqslant \frac{\nu(G)}{\alpha(G)} \tag{6}$$

Note that $\alpha_1 \leqslant \alpha_2 \leqslant \ldots \leqslant \alpha_{\chi-1}$, $\dfrac{\nu(G)}{\alpha_{\chi-1}(G)}$ is the biggest among these lower bounds.

The following lemma was proved in Ref. [2].

Lemma 4 If G is a graph on n vertices that has a (k, d)-coloring with $(k, d) = 1$ and $k > n$, then G has a (k', d')-coloring with $k' < k$ and $\dfrac{k'}{d'} < \dfrac{k}{d}$.

The next lemma can be found in Ref. [5].

Lemma 5 Let G be a graph with $\chi_c(G) = \dfrac{k}{d}$, if $(k, d) = 1$, then any homomorphism from G to G_k^d is surjective.

Theorem 7 Let G be a graph with chromatic number χ and independence number α, then $\chi_c(G) = \chi - 1 + 1/\alpha$ if and only if G is a spanning subgraph of $G_{(\chi-1)\alpha+1}^{\alpha}$ with independence number α.

Proof If $\chi_c(G) = \chi - 1 + 1/\alpha$, then by lemma 4, $\nu(G) \geqslant (\chi - 1)\alpha + 1$. On the other hand, by corollary 1, $\chi_c(G) \geqslant \nu(G)/\alpha$. This means $\nu(G) \leqslant (\chi - 1)\alpha + 1$. Hence $\nu(G) = (\chi$

$-1)\alpha+1$. By lemma 5, any homomorphism from G to $G^{\alpha}_{(\chi-1)\alpha+1}$ is surjective. Thus G is a spanning subgraph of $G^{\alpha}_{(\chi-1)\alpha+1}$ with independence number α. On the other hand, if G is a spanning subgraph of $G^{\alpha}_{(\chi-1)\alpha+1}$ with independence number α and chromatic number χ, then $\chi_c(G) \leqslant \chi_c(G^{\alpha}_{(\chi-1)\alpha+1}) = \chi-1+1/\alpha$. By corollary 1, $\chi_c(G) \geqslant \nu(G)/\alpha(G) = \chi-1+1/\alpha$. Thus $\chi_c(G) = \chi - 1 + \frac{1}{\alpha}$. This completes the proof of theorem 7.

Let

$$G^* = \left\{G \mid \chi_c(G) = \chi(G) - 1 + \frac{1}{\alpha(G)}\right\}$$

$$H^* = \left\{H \mid \chi_c(H) = \frac{\nu(H)}{\alpha(H)}\right\}$$

$$R^* = \left\{R \mid \chi_c(R) = \chi(R) - 1 + \frac{\chi(R)-1}{c(R)-1}\right\}$$

By theorem 7, $G^* \subseteq H^*$. Since there are many graphs G whose circular chromatic numbers equal $\frac{\nu(G)}{\alpha(G)}$, but do not equal $\chi(G) - 1 + \frac{1}{\alpha(G)}$, for example, $G^{d}_{td+i}(2 \leqslant i < d)$, we have $G^* \subset H^*$. From the proof of theorem 7, if $\chi_c(G) = \chi - 1 + 1/\alpha$ then $\nu(G) = c(G) = (\chi-1)\alpha+1$. Thus $G^* \subseteq R^*$. And since there are many graphs G whose circular chromatic numbers equal $\chi-1+(\chi-1)/(c-1)$ but do not equal $\chi-1+1/\alpha$. Hence we have $G^* \subset R^*$.

Theorem 8 $G^* \subseteq H^* \cap R^*$.

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图的圆色数的一些结果

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摘要:设 k 和 d 是 2 个互素的正整数且 $k \geqslant 2d$. G^{d}_{k}是一个图,它的顶点集合为$\{0,1,\cdots,k-1\}$,边集合为$\{ij \mid d \leqslant |i-j| \leqslant k-d, i,j=0,1,\cdots,k-1\}$. 图 G 的圆色数$\chi_c(G)$定义为使得图 G 与 G^{d}_{k}同态的 2 个正整数 k 和 d 的最小比值 k/d. 研究了$\chi_c(G)$和$\chi_c(G-v)$之间的关系,对任意顶点 v 求出了$\chi_c(G^{d}_{k}-v)$的精确值,给出了具有对任意顶点$\chi_c(G-v) = \chi_c(G)-1$ 和其他特定性质的图类;并对图的圆色数的一些下界进行了探讨,给出了图的圆色数达到下界$\chi-1+1/\alpha$ 的充要条件,这里χ 和 α 分别是图 G 的点色数和独立数.

关键词:(k,d)-着色;r-圆着色; 圆色数; Mycielski 图
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