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Polynomial rooting based frequency offset estimation for MIMO OFDM systems

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Abstract: Based on the frequency domain training sequences, the polynomial-based carrier frequency offset (CFO) estimation in multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems is extensively investigated. By designing the training sequences to meet certain conditions and exploiting the Hermitian and real symmetric properties of the corresponding matrices, it is found that the roots of the polynomials corresponding to the cost functions are pairwise and that both integer CFO and fractional CFO can be estimated by the direct polynomial rooting approach. By analyzing the polynomials corresponding to the cost functions and their derivatives, it is shown that they have a common polynomial factor and the former can be expressed in a quadratic form of the common polynomial factor. Analytical results further reveal that the derivative polynomial rooting approach is equivalent to the direct one in estimation at the same signal-to-noise ratio (SNR) value and that the latter is superior to the former in complexity. Simulation results agree well with analytical results.

Key words: MIMO OFDM; frequency selective fading channels; frequency offset estimation; polynomial rooting

Orthogonal frequency division multiplexing (OFDM) is a leading modulation technique for wide-band wireless communications. Combining it with multiple-input multiple-output (MIMO) multi-antenna technique promises a significant increase in practically achievable throughput over wireless media. The performance of OFDM systems, however, is sensitive to carrier frequency offset (CFO) caused by Doppler effects or the mismatch between transmitter and receiver oscillators. Accurate estimation and compensation for CFO is, therefore, very important for realizing the advantages of MIMO OFDM.

CFO estimation is a well-studied problem for single antenna OFDM systems[1-4], but a relatively new one for MIMO or MIMO OFDM systems[5-9]. Numerical calculations of the CFO estimators in Refs. [5 – 6] required a large point discrete Fourier transform (DFT) operation and a time consuming line search. To reduce complexity, computationally efficient CFO estimators were introduced in Refs. [7 – 9]. Especially, for the training aided CFO estimators in Refs. [8 – 9], integer CFO (ICFO) was estimated first by a subcarrier-size DFT operation, and then fractional CFO (FCFO) was estimated by the roots of a complex-coefficient or real polynomial corresponding to the cost function. Besides, it has been shown recently in Ref. [10] that the CFO estimation via polynomial rooting indirectly from the first-order derivative of the cost function outperformed that via polynomial rooting directly from the cost function.

In this paper, grounded on the CFO estimators in Refs. [8 – 9] and by designing the training sequences properly, we propose to estimate ICFO and FCFO through the roots of the corresponding complex or real coefficient polynomials directly, and thus without the need of the subcarrier-size DFT operation. Moreover, we reveal the relationships between the polynomials corresponding to the cost function and their first-order derivatives, and show that the derivative rooting approach is equivalent to the direct rooting approach for our considered MIMO case, which is quite different from the blind single antenna case as shown in Ref. [10].

1 Signal Model

We consider a MIMO OFDM system with N subcarriers, N_{t} transmit antennas and N_{r} receive antennas. Let

$$M = \lfloor P/N_{\mathrm{t}} \rfloor$$

$$0 \leqslant i_0 < i_1 < \ldots < i_\mu < \ldots < i_{N_{\mathrm{t}}-1} < Q = N/P$$

where $\lfloor \ \rfloor$ denotes the floor operation, and i_μ is an integer. Define

$$\boldsymbol{\Theta}_q = [\boldsymbol{e}_N^q, \boldsymbol{e}_N^{q+Q}, \ldots, \boldsymbol{e}_N^{q+(P-1)Q}]$$

where $\boldsymbol{e}_N^q$ denotes the q-th column vector of the $N \times N$ identity matrix $\boldsymbol{I}_N$. Let s denote a length-P Chu sequence[11]. Define $\tilde{\boldsymbol{s}}_\mu = \sqrt{Q/N_{\mathrm{t}}}\boldsymbol{F}_P \boldsymbol{s}^{(\mu M)}$, where $\boldsymbol{F}_P$ denotes the $P \times P$ unitary DFT matrix, and $\boldsymbol{s}^{(\mu M)}$ denotes the μM-cyclic-down-shift version of $\boldsymbol{s}$ with $\mu M > 0$. Then, the training vector at the μ-th transmit antenna is constructed as follows[8-9]:

$$\tilde{\boldsymbol{t}}_\mu = \boldsymbol{\Theta}_{i_\mu} \tilde{\boldsymbol{s}}_\mu$$

For convenience, we henceforth refer to $\{\tilde{\boldsymbol{t}}_\mu\}_{\mu=0}^{N_{\mathrm{t}}-1}$ as the Chu sequence-based training sequences (CBTS).

Assume that all the transmit-receive antenna pairs are affected by the same CFO. Define

$$\boldsymbol{D}_N(\varepsilon) = \mathrm{diag}\{[1, \mathrm{e}^{\mathrm{j}2\pi\varepsilon/N}, \ldots, \mathrm{e}^{\mathrm{j}2\pi\varepsilon(N-1)/N}]^{\mathrm{T}}\}$$

where diag{} denotes a diagonal matrix with the elements of the vector within the brackets on its diagonal, and ε is the frequency offset normalized by the subcarrier spacing. Suppose that the length-L channel impulse response from the μ-th transmit antenna to the ν-th receive antenna is denoted by the $L \times 1$ vector $\boldsymbol{h}^{(\nu,\mu)}$. Define

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$$\tilde{\boldsymbol{h}}^{(\nu,\mu)} = \boldsymbol{F}_N[\boldsymbol{e}_N^0, \boldsymbol{e}_N^1, \dots, \boldsymbol{e}_N^{L-1}]\boldsymbol{h}^{(\nu,\mu)}$$

Then, after removing the cyclic prefix (CP) at the ν-th receive antenna, the $N \times 1$ received vector $\boldsymbol{y}_\nu$ can be written as[6]

$$\boldsymbol{y}_\nu = \sqrt{N}\mathrm{e}^{\mathrm{j}2\pi\varepsilon N_g/N}\boldsymbol{D}_N(\varepsilon)\sum_{\mu=0}^{N_t-1}\{\boldsymbol{F}_N^{\mathrm{H}}\mathrm{diag}\{\tilde{\boldsymbol{h}}^{(\nu,\mu)}\}\tilde{\boldsymbol{t}}_\mu\} + \boldsymbol{w}_\nu \quad (1)$$

where N_g denotes the length of the CP and is supposed to be longer than the length of the channel impulse response L, and $\boldsymbol{w}_\nu$ is an $N \times 1$ vector of additive white complex Gaussian noise (AWGN) samples with zero-means and equal variances of σ_w^2. Let $\boldsymbol{y} = [\boldsymbol{y}_0^{\mathrm{T}}, \boldsymbol{y}_1^{\mathrm{T}}, \dots, \boldsymbol{y}_\nu^{\mathrm{T}}, \dots, \boldsymbol{y}_{N_r-1}^{\mathrm{T}}]^{\mathrm{T}}$ denote the $N_rN \times 1$ cascaded vector from the N_r receive antennas. Then, $\boldsymbol{y}$ can be written as[8-9]

$$\boldsymbol{y} = \sqrt{N}\mathrm{e}^{\mathrm{j}2\pi\varepsilon N_g/N}\{\boldsymbol{I}_{N_r}\otimes[\boldsymbol{D}_N(\varepsilon)\boldsymbol{S}]\}\boldsymbol{h} + \boldsymbol{w} \quad (2)$$

where

$$\boldsymbol{h} = [\boldsymbol{h}_0^{\mathrm{T}}, \boldsymbol{h}_1^{\mathrm{T}}, \dots, \boldsymbol{h}_\nu^{\mathrm{T}}, \dots, \boldsymbol{h}_{N_r-1}^{\mathrm{T}}]^{\mathrm{T}}$$

$$\boldsymbol{h}_\nu = [(\boldsymbol{h}^{(\nu,0)})^{\mathrm{T}}, (\boldsymbol{h}^{(\nu,1)})^{\mathrm{T}}, \dots, (\boldsymbol{h}^{(\nu,\mu)})^{\mathrm{T}}, \dots, (\boldsymbol{h}^{(\nu,N_t-1)})^{\mathrm{T}}]^{\mathrm{T}}$$

$$\boldsymbol{S} = \{\boldsymbol{1}_{N_t}^{\mathrm{T}}\otimes\boldsymbol{F}_N^{\mathrm{H}}\}\mathrm{diag}\{[\tilde{\boldsymbol{t}}_0^{\mathrm{T}}, \tilde{\boldsymbol{t}}_1^{\mathrm{T}}, \dots, \tilde{\boldsymbol{t}}_{N_t-1}^{\mathrm{T}}]^{\mathrm{T}}\}\cdot \{\boldsymbol{I}_{N_t}\otimes[\boldsymbol{F}_N[\boldsymbol{e}_N^0, \boldsymbol{e}_N^1, \dots, \boldsymbol{e}_N^{L-1}]]\}$$

$$\boldsymbol{w} = [\boldsymbol{w}_0^{\mathrm{T}}, \boldsymbol{w}_1^{\mathrm{T}}, \dots, \boldsymbol{w}_\nu^{\mathrm{T}}, \dots, \boldsymbol{w}_{N_r-1}^{\mathrm{T}}]^{\mathrm{T}}$$

$\otimes$ denotes the Kronecker product operator, and $\boldsymbol{1}_{N_t}$ denotes the $N_t \times 1$ all-one vector.

In the following, unless otherwise stated, we assume $(N)_{2P} = 0, 0 \leqslant p \leqslant P-1, 0 \leqslant q \leqslant Q-1, 0 \leqslant \mu \leqslant N_t-1$ and $0 \leqslant \nu \leqslant N_r-1$.

2 Training Aided CFO Estimation for MIMO OFDM Systems via Polynomial Rooting

2.1 CFO estimation for MIMO OFDM via direct polynomial rooting

By exploiting the periodic property of the training sequences, the received vector $\boldsymbol{y}$ can be stacked into the $Q \times N_rP$ matrix $\boldsymbol{Y} = [\boldsymbol{Y}_0, \boldsymbol{Y}_1, \dots, \boldsymbol{Y}_\nu, \dots, \boldsymbol{Y}_{N_r-1}]$, where

$$[\boldsymbol{Y}_\nu]_{q,p} = [\boldsymbol{y}_\nu]_{qP+p}$$

Let $\beta_\mu = \varepsilon + i_\mu$. Then, $\boldsymbol{Y}$ can be expressed as

$$\boldsymbol{Y} = \boldsymbol{B}\boldsymbol{X} + \boldsymbol{W} \quad (3)$$

where

$$\boldsymbol{B} = [\boldsymbol{b}_0, \boldsymbol{b}_1, \dots, \boldsymbol{b}_\mu, \dots, \boldsymbol{b}_{N_t-1}]$$

$$\boldsymbol{b}_\mu = [1, \mathrm{e}^{\mathrm{j}2\pi\beta_\mu/Q}, \dots, \mathrm{e}^{\mathrm{j}2\pi\beta_\mu q/Q}, \dots, \mathrm{e}^{\mathrm{j}2\pi\beta_\mu(Q-1)/Q}]^{\mathrm{T}}$$

$$\boldsymbol{X} = [\boldsymbol{X}_0, \boldsymbol{X}_1, \dots, \boldsymbol{X}_\nu, \dots, \boldsymbol{X}_{N_r-1}]$$

$$\boldsymbol{X}_\nu = [\boldsymbol{x}^{(\nu,0)}, \boldsymbol{x}^{(\nu,1)}, \dots, \boldsymbol{x}^{(\nu,\mu)}, \dots, \boldsymbol{x}^{(\nu,N_t-1)}]^{\mathrm{T}}$$

$$\boldsymbol{x}^{(\nu,\mu)} = \sqrt{P}\mathrm{e}^{\mathrm{j}2\pi\varepsilon N_g/N}\boldsymbol{D}_P(\beta_\mu)\boldsymbol{F}_P^{\mathrm{H}}\mathrm{diag}\{\tilde{\boldsymbol{s}}_\mu\}\boldsymbol{\Theta}_{i_\mu}^{\mathrm{T}}\tilde{\boldsymbol{h}}^{(\nu,\mu)}$$

and $\boldsymbol{W}$ is the $Q \times N_rP$ matrix generated from $\boldsymbol{w}$ in the same way as $\boldsymbol{Y}$.

Let $\boldsymbol{L}$ denote a $Q \times Q$ unitary column conjugate symmetric matrix. Define

$$\hat{\boldsymbol{R}}_{YY} = \frac{R(\boldsymbol{L}^{\mathrm{H}}\boldsymbol{Y}\boldsymbol{Y}^{\mathrm{H}}\boldsymbol{L})}{N_rP}$$

where $R()$ denotes the real part of the enclosed parameter. Let $\boldsymbol{E}_X$ denote the $Q \times N_t$ real matrix which contains the unitary eigen-vectors spanning the signal subspace of $\hat{\boldsymbol{R}}_{YY}$. Define

$$z = \mathrm{e}^{\mathrm{j}2\pi\beta/Q}, \quad g = \cot\left(\frac{\pi\beta}{Q}\right)$$

$$\boldsymbol{a}(z) = \{1, z, \dots, z^{Q-1}\}^{\mathrm{T}}, \quad \boldsymbol{a}(g) = \{1, g, \dots, g^{Q-1}\}^{\mathrm{T}}$$

Then, we can estimate $\{\beta_\mu\}_{\mu=0}^{N_t-1}$ by the roots of either a complex-coefficient polynomial or a real-coefficient polynomial as follows[8-9]:

$$f^{\mathrm{c}}(z) = \boldsymbol{a}^{\mathrm{T}}(z)\boldsymbol{A}^{\mathrm{c}}\boldsymbol{a}(z) = 0 \quad (4)$$

$$f^{\mathrm{r}}(g) = \boldsymbol{a}^{\mathrm{T}}(g)\boldsymbol{A}^{\mathrm{r}}\boldsymbol{a}(g) = 0 \quad (5)$$

where

$$\boldsymbol{A}^{\mathrm{c}} = \boldsymbol{J}\boldsymbol{L}[\boldsymbol{I}_Q - \boldsymbol{E}_X\boldsymbol{E}_X^{\mathrm{T}}]\boldsymbol{L}^{\mathrm{H}}$$

$$\boldsymbol{A}^{\mathrm{r}} = \boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{L}[\boldsymbol{I}_Q - \boldsymbol{E}_X\boldsymbol{E}_X^{\mathrm{T}}]\boldsymbol{L}^{\mathrm{H}}\boldsymbol{\Phi}$$

$\boldsymbol{J}$ denotes the $Q \times Q$ exchange matrix with ones on its anti-diagonal and zeros elsewhere, and $\boldsymbol{\Phi}$ is the $Q \times Q$ column conjugate symmetric matrix with its elements given by

$$[\boldsymbol{\Phi}]_{q,q'} = \mathrm{j}^{Q-1-q'}\sum_{q''=\max\{0,q+q'-Q+1\}}^{\min\{q,q'\}}\{C_q^{q''}C_{Q-1-q}^{q'-q''}(-1)^{Q-1-q-q'+q''}\}$$

and $C_q^{q''} = q!/[(q-q'')!\,q''!]$. Note that $\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{L}$ is a real matrix due to the column conjugate symmetric property of both $\boldsymbol{\Phi}$ and $\boldsymbol{L}$. Let $\Lambda^{\mathrm{c}}(z)$ and $\Lambda^{\mathrm{r}}(g)$ denote the polynomials transformed from the cost function. Then, we can obtain[8]

$$\Lambda^{\mathrm{c}}(z) = z^{1-Q}\boldsymbol{a}^{\mathrm{T}}(z)\boldsymbol{A}^{\mathrm{c}}\boldsymbol{a}(z) \quad (6)$$

$$\Lambda^{\mathrm{r}}(g) = (g^2+1)^{1-Q}\boldsymbol{a}^{\mathrm{T}}(g)\boldsymbol{A}^{\mathrm{r}}\boldsymbol{a}(g) \quad (7)$$

Due to the Hermitian property of $\boldsymbol{J}\boldsymbol{A}^{\mathrm{c}}$, we establish from Eq. (4) that

$$f^{\mathrm{c}}(|z|^{-1}\mathrm{e}^{\mathrm{j}\arg(z)}) = (|z|^{-1}\mathrm{e}^{\mathrm{j}\arg(z)})^{2(Q-1)}(f^{\mathrm{c}}(z))^* \quad (8)$$

Hence, the roots of $f^{\mathrm{c}}(z) = 0$ are pairwise and in the form of $\{z, |z|^{-1}\mathrm{e}^{\mathrm{j}\arg(z)}\}$. Note that the roots of the polynomial equation $f^{\mathrm{c}}(z) = 0$ always exist no matter whether noise is absent or present, which is quite different from the analysis in Ref. [10]. Due to the real symmetric property of $\boldsymbol{A}^{\mathrm{r}}$, we immediately establish that the roots of $f^{\mathrm{r}}(g) = 0$ are also pairwise and in the form of $\{R(g) \pm \mathrm{j}I(g)\}$, where we have used the relationship $\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{L} = \boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{L}^*$ and $I()$ denotes the imaginary part of the enclosed parameter. In the following, the pairwise property of the roots of $f^{\mathrm{c}}(z) = 0$ and $f^{\mathrm{r}}(g) = 0$ are exploited for the CFO estimation.

To estimate the CFO, we first find the N_t pairwise roots of $f^{\mathrm{c}}(z) = 0$ which are the closest to the unit circle, $\{z_\mu, |z_\mu|^{-1}\mathrm{e}^{\mathrm{j}\arg(z_\mu)}\}_{\mu=0}^{N_t-1}$, or the N_t pairwise roots of $f^{\mathrm{r}}(g) = 0$ whose imaginary parts have the smallest values, $\{R(g_\mu) \pm \mathrm{j}I(g_\mu)\}_{\mu=0}^{N_t-1}$. Due to the influence of noise, the so-obtained N_t pairwise roots of $f^{\mathrm{c}}(z) = 0$ or $f^{\mathrm{r}}(g) = 0$ may not correspond to $\{\beta_\mu\}_{\mu=0}^{N_t-1}$ at a low signal-to-noise ratio (SNR). To reduce the influence of noise, we set a threshold λ_{th} and compare $\Lambda^c(z_\mu)$ or $\Lambda^r(R(g_\mu))$ with λ_{th}. If $\Lambda^c(z_\mu)$ or $\Lambda^r(R(g_\mu))$ exceeds λ_{th}, we find the N_t pairwise roots of $f^c(z)=0$ or $f^r(g)=0$ whose corresponding values of $\Lambda^c(z)$ or $\Lambda^r(R(g))$ are the smallest. Then, we can readily estimate $\{\beta_\mu\}_{\mu=0}^{N_t-1}$ from the N_t pairwise roots $\{z_\mu, |z_\mu|^{-1}e^{j\arg(z_\mu)}\}_{\mu=0}^{N_t-1}$ or $\{R(g_\mu)\pm jI(g_\mu)\}_{\mu=0}^{N_t-1}$ as follows:

$$\hat{\beta}_\mu = \left(\frac{Q/2}{\pi}\arg(z_\mu)\right)_Q \tag{9}$$

$$\hat{\beta}_\mu = \left(\frac{Q}{\pi}\operatorname{acot}[R(g_\mu)]\right)_Q \tag{10}$$

where $(\,)_Q$ denotes the remainder of the number within the brackets modulo Q.

Let ε_i and ε_f denote the ICFO and FCFO, respectively. Then, β_μ can be decomposed as $\beta_\mu=\varepsilon_i+\varepsilon_f+i_\mu$. Hence, by imposing proper design conditions on our CBTS training sequences, we can estimate ε_i and ε_f from $\{\hat{\beta}_\mu\}_{\mu=0}^{N_t-1}$. Define

$$\varepsilon_{f,th} = \frac{1}{N_t}\sum_{\mu=0}^{N_t-1}\{|\hat{\beta}_\mu - \operatorname{round}(\hat{\beta}_\mu)|\} \tag{11}$$

where round( ) denotes the round operator. To avoid the ambiguous estimation when ε_f is near 0 or $\pm 1/2$, we set $i'_\mu = (\lfloor \hat{\beta}_\mu \rfloor)_Q$ if $\varepsilon_{f,th}$ exceeds 1/4, or else we set $i'_\mu = (\operatorname{round}(\hat{\beta}_\mu))_Q$. Then, the FCFO can be immediately estimated as

$$\hat{\varepsilon}_f = \frac{1}{N_t}\sum_{\mu=0}^{N_t-1}\{\hat{\beta}_\mu - i'_\mu\} \tag{12}$$

Let $\boldsymbol{l}$ denote the pilot location vector which is given by $\boldsymbol{l} = \sum_{\mu=0}^{N_t-1}\boldsymbol{e}_Q^{i_\mu}$. To ensure the identifiability of the ICFO estimation, we impose the following conditions on $\boldsymbol{l}$:

$$(\boldsymbol{1}_Q - \boldsymbol{l})^T\boldsymbol{l}^{(q)} > 0 \qquad \forall q \in \{1,2,\dots,Q-1\} \tag{13}$$

Define $\boldsymbol{l}' = \sum_{\mu=0}^{N_t-1}\boldsymbol{e}_Q^{i'_\mu}$. Then, the ICFO can be uniquely estimated as

$$\hat{\varepsilon}_i = \underset{-Q/2<q\le Q/2}{\arg\max}\{(\boldsymbol{l}')^T\boldsymbol{l}^{(q)}\} \tag{14}$$

Note that the ICFO estimation approach in this paper can be applied to relatively benign channel environments. For bad channel environments where the SNR is often around or lower than 0 dB, we can still use the approach in Refs. [8 – 9] whose ICFO estimator is very robust against noise.

2.2 Analysis of the direct rooting approach and the derivative rooting approach

It has been shown recently in Ref. [10] that CFO estimation via polynomial rooting from the first-order derivative of the cost function is superior to that via polynomial rooting from the cost function in blind single-antenna OFDM systems. Note that the derivative rooting approach in Ref. [10] can be applied directly in our considered MIMO OFDM systems. In this paper, we will, however, show that these two approaches are equivalent in MIMO OFDM systems.

Let $\boldsymbol{E}_W$ denote the $Q(Q-N_t)$ real matrix which contains the unitary eigen-vectors spanning the noise subspace of $\hat{\boldsymbol{R}}_{YY}$. Then, we have

$$\boldsymbol{E}_X\boldsymbol{E}_X^T + \boldsymbol{E}_W\boldsymbol{E}_W^T = \boldsymbol{I}_Q \tag{15}$$

Besides, since $\boldsymbol{L}$ is a column conjugate symmetric matrix, we also have

$$\boldsymbol{JL} = \boldsymbol{L}^* \tag{16}$$

Hence, $f^c(z)$ and $f^r(g)$ can be further expressed as

$$f^c(z) = [\boldsymbol{k}^c(z)]^T\boldsymbol{k}^c(z) \tag{17}$$

$$f^r(g) = [\boldsymbol{k}^r(g)]^T\boldsymbol{k}^r(g) \tag{18}$$

where

$$\boldsymbol{k}^c(z) = \boldsymbol{E}_W^T\boldsymbol{L}^H\boldsymbol{a}(z)$$

$$\boldsymbol{k}^r(g) = \boldsymbol{E}_W^T\boldsymbol{L}^H\boldsymbol{\Phi a}(g)$$

Correspondingly, $\Lambda^c(z)$ and $\Lambda^r(g)$ can be expressed as

$$\Lambda^c(z) = z^{1-Q}[\boldsymbol{k}^c(z)]^T\boldsymbol{k}^c(z) \tag{19}$$

$$\Lambda^r(g) = (g^2+1)^{1-Q}[\boldsymbol{k}^r(g)]^T\boldsymbol{k}^r(g) \tag{20}$$

Taking the first-order derivative of $f^c(z)$ and $f^r(g)$ with respect to z and g respectively, we immediately establish

$$\frac{df^c(z)}{dz} = 2\frac{d[(\boldsymbol{k}^c(z))^T]}{dz}\boldsymbol{k}^c(z) \tag{21}$$

$$\frac{df^r(g)}{dg} = 2\frac{d[(\boldsymbol{k}^r(g))^T]}{dg}\boldsymbol{k}^r(g) \tag{22}$$

Note that $\Lambda^c(z)$, $f^c(z)$ and $\frac{df^c(z)}{dz}$ have the common polynomial factor $\boldsymbol{k}^c(z)$, while $\Lambda^r(g)$, $f^r(g)$ and $\frac{df^r(g)}{dg}$ have the common polynomial factor $\boldsymbol{k}^r(g)$.

Let $z_\mu = e^{j2\pi\beta_\mu/Q}$, $g_\mu = \cot(\pi\beta_\mu/Q)$. In the ideal case without considering noise, we have

$$f^c(z_\mu)=0, \quad f^r(g_\mu)=0 \tag{23}$$

Besides, $\Lambda^c(z)$ and $f^c(z)$ can also be written into the following equivalent forms[8]:

$$\Lambda^c(z) = [\boldsymbol{k}^c(z)]^H\boldsymbol{k}^c(z) \tag{24}$$

$$f^c(z) = z^{Q-1}[\boldsymbol{k}^c(z)]^H\boldsymbol{k}^c(z) \tag{25}$$

We can see from Eqs. (18) and (25) that $f^c(z_\mu)=0$ and $f^r(g_\mu)=0$ hold if and only if

$$\boldsymbol{k}^c(z_\mu)=\boldsymbol{0}_{Q-N_t}, \quad \boldsymbol{k}^r(g_\mu)=\boldsymbol{0}_{Q-N_t} \tag{26}$$

Due to the common polynomial factors $\boldsymbol{k}^c(z)$ and $\boldsymbol{k}^r(g)$, it follows immediately from Eq. (26) that

$$\left.\frac{df^c(z)}{dz}\right|_{z=z_\mu}=0, \quad \Lambda^c(z_\mu)=0 \tag{27}$$

$$\left.\frac{\mathrm{d}f^{\mathrm{r}}(g)}{\mathrm{d}g}\right|_{g=g_\mu}=0,\quad \Lambda^{\mathrm{r}}(g_\mu)=0 \tag{28}$$

From the above analyses, we can see that the roots from the direct rooting and derivative rooting approaches correspond to the same ones which make $\boldsymbol{k}^{\mathrm{c}}(z_\mu)$ or $\boldsymbol{k}^{\mathrm{r}}(g_\mu)$ equal $\mathbf{0}_{Q-N_{\mathrm{t}}}$ in the absence of noise or approximately equal $\mathbf{0}_{Q-N_{\mathrm{t}}}$ in the presence of noise, where $\mathbf{0}_{Q-N_{\mathrm{t}}}$ denotes the $(Q-N_{\mathrm{t}})\times 1$ all-zero vector. In this way, we can say that the derivative rooting approach is equivalent to the direct rooting approach for the considered MIMO case, which will be verified by simulation results in the next section.

2.3 Computational complexity

For description convenience, we only consider the real-coefficient polynomial case here. For the direct rooting approach, the N_{t} pairwise roots of $f^{\mathrm{r}}(g)=0$, whose imaginary parts have the smallest values, match well with the ones, whose corresponding values of $\Lambda^{\mathrm{r}}(R(g))$ are the smallest, at high SNR completely and at low SNR mostly, which can be easily verified through simulations. Therefore, the direct rooting approach only occasionally needs to calculate the values of the cost function $\Lambda^{\mathrm{r}}(R(g))$ with respect to the roots of $f^{\mathrm{r}}(g)=0$ at low SNR. Hence, the computational load of the direct rooting approach mainly involves the eigen-decomposition of $\hat{\boldsymbol{R}}_{YY}$ and the root-calculation of $f^{\mathrm{r}}(g)=0$, which require $9Q^3$ and $\frac{64}{3}(Q-1)^3$ real additions or multiplications[12-13], respectively. While for the derivative rooting approach, although its computation is decreased slightly due to the degree reduction of $\frac{\mathrm{d}f^{\mathrm{r}}(g)}{\mathrm{d}g}$ in comparison with $f^{\mathrm{r}}(g)$, it still needs an additional complicated calculation of the values of the cost function $\Lambda^{\mathrm{r}}(R(g))$ with respect to the roots of $\frac{\mathrm{d}f^{\mathrm{r}}(g)}{\mathrm{d}g}=0$, which require $4(Q-3/2)(Q-N_{\mathrm{t}})Q(Q+1)$ real additions or multiplications. Therefore, the direct rooting approach has lower computational complexity than the derivative rooting approach. Besides, since the direct rooting approach does not need the DFT operation, its complexity is also lower than that in Refs. [8-9].

3 Simulation Results

We provide simulations to validate the theoretical analysis and also to evaluate the performance of the proposed CFO estimators with the CBTS training sequences. In the simulations, the major parameters are set as follows: carrier frequency $f_{\mathrm{c}}=5$ GHz, bandwidth $B=20$ MHz, subcarrier number $N=1\,024$, CP length $N_{\mathrm{g}}=64$, the number of transmit antennas $N_{\mathrm{t}}=3$, the number of receive antennas $N_{\mathrm{r}}=2$. The channels are assumed to be independent and have four uncorrelated Rayleigh fading taps each. The relative propagation delays of the four taps are chosen to be equal to {0, 0.1, 0.2, 0.4} μs, and the variances of the taps are {0, −9.7, −19.2, −22.8}dB. The normalized CFO ε is generated within the range$(-Q/2, Q/2]$. For description convenience, we refer to the CFO estimator in Refs. [8-9], the CFO estimator via derivative rooting and the one via direct rooting proposed in this paper as estimator A, estimator B and estimator C, respectively.

As was pointed out in Ref. [14], the extended Miller and Chang bound (EMCB) can be tighter than the Cramer-Rao bound (CRB). In the following, we adopt the EMCB to benchmark the performance of the considered CFO estimators via polynomial rooting. The EMCB is obtained by averaging the snapshot CRB over independent channel realizations[14-15] as follows:

$$\mathrm{EMCB}_\varepsilon=E\left\{\frac{N\sigma_w^2}{8\pi^2\boldsymbol{h}^{\mathrm{H}}\boldsymbol{X}^{\mathrm{H}}\boldsymbol{B}[\boldsymbol{I}_{N_tN}-\boldsymbol{X}(\boldsymbol{X}^{\mathrm{H}}\boldsymbol{X})^{-1}\boldsymbol{X}^{\mathrm{H}}]\boldsymbol{B}\boldsymbol{X}\boldsymbol{h}}\right\} \tag{29}$$

where

$$\boldsymbol{X}=\boldsymbol{I}_{N_{\mathrm{t}}}\otimes\boldsymbol{S}$$

$$\boldsymbol{B}=\boldsymbol{I}_{N_{\mathrm{t}}}\otimes\mathrm{diag}\{[N_{\mathrm{g}},N_{\mathrm{g}}+1,\ldots,N_{\mathrm{g}}+N-1]^{\mathrm{T}}\}$$

and $E\{\cdot\}$ denotes the expectation operator. The $\mathrm{EMCB}_\varepsilon$ cannot be written in a closed form, and we resort to Monte Carlo simulations for its evaluation in this paper.

Fig. 1 shows the mean square error (MSE) performance of the considered three CFO estimators via complex-coefficient polynomial rooting with $P=128$, $Q=8$ and $P=64$, $Q=16$. Fig. 2 presents the MSE performance of the CFO estimators via real-coefficient polynomial rooting. The corresponding EMCBs are also included in the two figures. We can see that estimators A, B and C via complex-coefficient polynomial rooting have almost the same MSE performance. While for the real-coefficient polynomial case, estimators B and C outperform estimator A at all considered SNR values, and estimator C slightly outperforms estimator B at low SNR values, which is due to the influence of noise. Actually, in the ideal case without noise, the three estimators via real-coefficient polynomial rooting also have almost the same MSE performance. We can see that the simulation results support the theoretical results very well; i. e., the derivative rooting approach is equivalent to the direct one for the considered MIMO case, except for a slight mismatch at low SNR in Fig. 2.

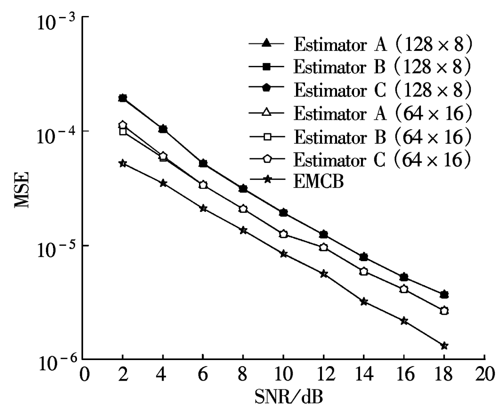

Fig. 1 MSE performance of the CFO estimators via complex-coefficient polynomial rooting

In Fig. 3, we show the MSE performance of the CFO estimator in Refs. [8-9] via complex-coefficient and real-coefficient polynomial rooting at low SNR. It can be seen that the estimation performance is satisfactory and acceptable even when the SNR is quite lower than 0 dB. Therefore, in environments which are affected greatly by noise, we can employ the estimator in Refs. [8-9].

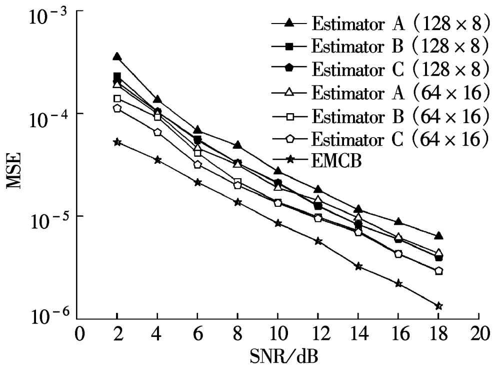

Fig. 2 MSE performance of the CFO estimators via real-coefficient polynomial rooting

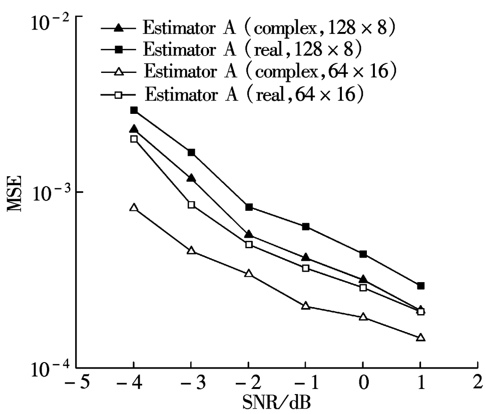

Fig. 3 MSE performance of the CFO estimator in Refs. [8 − 9] at low SNR

4 Conclusion

In this paper, we present a simplified CFO estimator via direct polynomial rooting for MIMO OFDM systems with properly designed training sequences. We analyze the CFO estimation via direct polynomial rooting and via derivative polynomial rooting. Our analytical results show that the two approaches are equivalent and they are well supported by the simulation results.

References

[1] Moose P. A technique for orthogonal frequency division multiplexing frequency offset correction [J]. *IEEE Trans Commun*, 1994, **42**(10): 2908 − 2914.

[2] Schmidl T M, Cox D C. Robust frequency and timing synchronization for OFDM [J]. *IEEE Trans Commun*, 1997, **45**(12): 1613 − 1621.

[3] Morelli M, Mengali U. An improved frequency offset estimator for OFDM applications [J]. *IEEE Commun Lett*, 1999, **3**(3): 75 − 77.

[4] Minn H, Bhargava V K, Letaief K B. A robust timing and frequency synchronization for OFDM systems [J]. *IEEE Trans Wireless Commun*, 2003, **2**(4): 822 − 839.

[5] Besson O, Stoica P. On parameter estimation of MIMO flat-fading channels with frequency offsets [J]. *IEEE Trans Signal Processing*, 2003, **51**(3): 602 − 613.

[6] Ma X, Oh M K, Giannakis G B, et al. Hopping pilots for estimation of frequency-offset and multi-antenna channels in MIMO OFDM [J]. *IEEE Trans Commun*, 2005, **53**(1): 162 − 172.

[7] Simoens F, Moeneclaey M. Reduced complexity data-aided and code-aided frequency offset estimation for flat-fading MIMO channels [J]. *IEEE Trans Wireless Commun*, 2006, **5**(6): 1558 − 1567.

[8] Jiang Y X, Gao X Q, You X H, et al. Training sequence assisted frequency offset estimation for MIMO OFDM [C]// *Proc of IEEE ICC'06*. Turkey, Istanbul, 2006: 5371 − 5376.

[9] Jiang Y X, Minn H, Gao X Q, et al. Frequency offset estimation and training sequence design for MIMO OFDM [J]. *IEEE Trans Wireless Commun*, 2008, **7**(4): 1244-1254.

[10] Gao F, Nallanathan A. Blind maximum likelihood CFO estimation for OFDM systems via polynomial rooting [J]. *IEEE Signal Processing Lett*, 2006, **13**(2): 73 − 76.

[11] Chu D. Polyphase codes with good periodic correlation properties [J]. *IEEE Trans Inform Theory*, 1972, **18**(4): 531 − 532.

[12] Golub G H, Van Loan C F. *Matrix computations* [M]. The John Hopkins University Press, 1996.

[13] Press W H. *Numerical recipes in C + +: the art of scientific computing* [M]. Cambridge: Cambridge University Press, 2002.

[14] Gini F, Reggiannini R. On the use of Cramer-Rao-like bounds in the presence of random nuisance parameters [J]. *IEEE Trans Commun*, 2000, **48**(12): 2120 − 2126.

[15] Kay S M. *Fundamentals of statistical signal processing: estimation theory* [M]. Prentical-Hall, 1993.

MIMO OFDM 系统中基于多项式求根的频偏估计算法

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摘要:基于频域训练序列,深入地分析了 MIMO-OFDM 系统中基于多项式建模的频偏估计问题. 设计训练序列使其结构满足适当的条件,根据相应矩阵的厄尔密特属性和实对称属性,分析出代价函数多项式方程根的成对性,进而提出整数频偏与小数频偏可同时通过直接多项式求根方法估计出来. 分析了导数多项式求根方法与直接多项式求根方法,研究出代价函数多项式与其导数多项式具有公共的多项式因子,且代价函数多项式可以表示成该公共多项式因子的二次型,并进一步揭示出二者在估计上的等效性以及后者在实现上的优越性. 计算机仿真结果验证了该理论分析结果.

关键词:多输入多输出正交频分复用;频率选择性衰落信道;频偏估计;多项式求根

中图分类号:TN91