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Multiple attribute decision making method based on trapezoid fuzzy linguistic variables

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Abstract: The problem of multiple attribute decision making under fuzzy linguistic environments, in which decision makers can only provide their preferences (attribute values) in the form of trapezoid fuzzy linguistic variables (TFLV), is studied. The formula of the degree of possibility between two TFLVs is defined, and some of its characteristics are studied. Based on the degree of possibility of fuzzy linguistic variables, an approach to ranking the decision alternatives in multiple attribute decision making with TFLV is developed. The trapezoid fuzzy linguistic weighted averaging (TFLWA) operator method is utilized to aggregate the decision information, and then all the alternatives are ranked by comparing the degree of possibility of TFLV. The method can carry out linguistic computation processes easily without loss of linguistic information, and thus makes the decision results reasonable and effective. Finally, the implementation process of the proposed method is illustrated and analyzed by a practical example.

Key words: trapezoid fuzzy linguistic variables; degree of possibility; multiple attribute decision making

Multiple attribute decision making under a linguistic environment is an interesting research topic having received more and more attention from researchers during the past several years[1-6]. In the process of multiple attribute decision making, the linguistic decision information needs to be aggregated by means of some proper approaches so as to rank the given decision alternatives and then to select the most desirable one. Bordogna et al. [1] developed a model within fuzzy set theory by linguistic ordered weighted average (OWA) operators for group decision making in a linguistic context. Herrera and Martínez[2] established a linguistic 2-tuple computational model for dealing with linguistic information. Li and Yang[3] developed a linear programming technique for a multidimensional analysis of preferences in multiple attribute group decision making under fuzzy environments, in which all the linguistic information and real numbers are transformed into triangular fuzzy numbers. Xu et al. [4-7] proposed some methods, which directly compute with words.

Recently, Xu[8] introduced the concept of trapezoid fuzzy linguistic variable (TFLV) first and developed a similarity measure between two TFLVs. Based on the similarity measure and the ideal points of attribute values, Xu investigated the multiple attribute decision making problems under a fuzzy linguistic environment. In this paper, we further discuss this problem, and introduce a degree of possibility between two TFLVs. Based on the degree of possibility, an approach to ranking the decision alternatives in multiple attribute decision making with TFLVs is developed. The practical example shows that it is easier than that of Xu's method.

1 Degree of Possibility between Two Trapezoid Fuzzy Linguistic Variables

Let $S=\{s_i \mid i=0, \dots, t\}$ be a linguistic term set with odd cardinality. Any label s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics[7]:

1) The set is ordered: $s_i<s_j$ if $i<j$;

2) There is a negation operator: $\mathrm{neg}(s_i)=s_j$ such that $i+j=t$;

3) Maximum operator: $\max(s_i, s_j)=s_i$ if $s_i \geqslant s_j$;

4) Minimum operator: $\min(s_i, s_j)=s_i$ if $s_i \leqslant s_j$. For example, a set of nine labels S could be

$$\begin{aligned} S=\{&s_0=\text{extremely poor},\ s_1=\text{very poor},\\ &s_2=\text{poor},\ s_3=\text{slightly poor},\ s_4=\text{fair},\\ &s_5=\text{slightly good},\ s_6=\text{good},\\ &s_7=\text{very good},\ s_8=\text{extremely good}\}\end{aligned}$$

To preserve all the given information, we extend the discrete term set S to a continuous term set $\bar{S}=\{s_a \mid s_0 \leqslant s_a \leqslant s_q, a \in [0, q]\}$, whose elements also meet all the characteristics above. If $s_a \in S$, then we call s_a the original linguistic term; otherwise, we call s_a the virtual linguistic term; q is a large positive integer. In general, the decision maker uses the original linguistic terms to evaluate attributes and alternatives, and the virtual linguistic terms can only appear in the calculations.

Since the decision maker is characterized by his own personal background and experience, in some situations, the decision maker may provide fuzzy linguistic information because of time pressure, lack of knowledge, and his limited expertise related to the problem domain. So, Xu defined the concept of TFLV.

Definition 1[8] Let $\tilde{s}=[s_\alpha, s_\beta, s_\gamma, s_\eta] \in \tilde{S}$, where $s_\alpha, s_\beta, s_\gamma, s_\eta \in \bar{S}$, s_β and s_γ indicate the intervals in which the membership value is 1, with s_α and s_η indicating the lower and upper values of $\tilde{S}$, respectively, then $\tilde{S}$ is called a trapezoid fuzzy linguistic variable. It is characterized by the following member function (see Fig. 1):

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$$\mu_{\tilde{S}}(\theta)=\begin{cases}0 & s_{-q}\leqslant s_\theta\leqslant s_\alpha\\ \dfrac{d(s_\theta,s_\alpha)}{d(s_\beta,s_\alpha)} & s_\alpha\leqslant s_\theta\leqslant s_\beta\\ 1 & s_\beta\leqslant s_\theta\leqslant s_\gamma\\ \dfrac{d(s_\theta,s_\eta)}{d(s_\gamma,s_\eta)} & s_\gamma\leqslant s_\theta\leqslant s_\eta\\ 0 & s_\eta\leqslant s_\theta\leqslant s_q\end{cases}$$

where $\tilde{S}$ is the set of all trapezoid fuzzy linguistic variables. Especially, if any two of $\alpha, \beta, \gamma, \eta$ are equal, then $\tilde{S}$ is reduced to a triangular fuzzy linguistic variable; if any three of $\alpha, \beta, \gamma, \eta$ are equal, then $\tilde{S}$ is reduced to an uncertain linguistic variable.

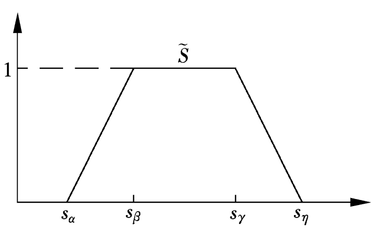

Fig. 1 A trapezoid fuzzy linguistic variable $\tilde{S}$

Consider any three trapezoid fuzzy linguistic variables $\tilde{s}=[s_\alpha, s_\beta, s_\gamma, s_\eta]$, $\tilde{s}_1=[s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}, s_{\eta_1}]$ and $\tilde{s}_2=[s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}, s_{\eta_2}]\in\tilde{S}$, and suppose that $\lambda\in[0,1]$, then their operational laws are as follows:

$$\tilde{s}_1\oplus\tilde{s}_2=[s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}, s_{\eta_1}]\oplus[s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}, s_{\eta_2}]=[s_{\alpha_1+\alpha_2}, s_{\beta_1+\beta_2}, s_{\gamma_1+\gamma_2}, s_{\eta_1+\eta_2}]$$

$$\lambda\tilde{s}=\lambda[s_\alpha, s_\beta, s_\gamma, s_\eta]=[s_{\lambda\alpha}, s_{\lambda\beta}, s_{\lambda\gamma}, s_{\lambda\eta}]$$

Definition 2 Let $\tilde{s}_1=[s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}, s_{\eta_1}]$ and $\tilde{s}_2=[s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}, s_{\eta_2}]$ be two trapezoid fuzzy linguistic variables, and let $\text{len}(\tilde{s}_1)=(\gamma_1+\eta_1)-(\alpha_1+\beta_1)$ and $\text{len}(\tilde{s}_2)=(\gamma_2+\eta_2)-(\alpha_2+\beta_2)$, then the degree of possibility of $\tilde{s}_1\geqslant\tilde{s}_2$ is defined as

$$p(\tilde{s}_1\geqslant\tilde{s}_2)=\min\left\{\max\left\{\frac{(\gamma_1+\eta_1)-(\alpha_2+\beta_2)}{\text{len}(\tilde{s}_1)+\text{len}(\tilde{s}_2)},0\right\},1\right\} \quad (1)$$

From definition 2, we can easily obtain the following results:

1) $0\leqslant p(\tilde{s}_1\geqslant\tilde{s}_2)\leqslant 1, 0\leqslant p(\tilde{s}_2\geqslant\tilde{s}_1)\leqslant 1$;

2) $p(\tilde{s}_1\geqslant\tilde{s}_2)+p(\tilde{s}_2\geqslant\tilde{s}_1)=1$. Especially, $p(\tilde{s}_1\geqslant\tilde{s}_1)=p(\tilde{s}_2\geqslant\tilde{s}_2)=\dfrac{1}{2}$;

3) Let $p(\tilde{s}_1\geqslant\tilde{s}_2)\geqslant\dfrac{1}{2}$ and $p(\tilde{s}_2\geqslant\tilde{s}_3)\geqslant\dfrac{1}{2}$, then, $p(\tilde{s}_1\geqslant\tilde{s}_3)\geqslant\dfrac{1}{2}$;

4) Let $p(\tilde{s}_1\geqslant\tilde{s}_2)\geqslant\dfrac{1}{2}$ and $p(\tilde{s}_2\geqslant\tilde{s}_3)\geqslant\dfrac{1}{2}$, then, $p(\tilde{s}_1\geqslant\tilde{s}_2)+p(\tilde{s}_2\geqslant\tilde{s}_3)\geqslant p(\tilde{s}_1\geqslant\tilde{s}_3)$.

Definition 3 Let TFLWA: $\tilde{S}^n\to\tilde{S}$, if

$$\text{TFLWA}_w(\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_n)=w_1\tilde{s}_1\oplus w_2\tilde{s}_2\oplus\ldots\oplus w_n\tilde{s}_n \quad (2)$$

where $\boldsymbol{w}=\{w_1, w_2, \ldots, w_n\}$ is the weighting vector of the $\tilde{s}_i\in\tilde{S}$, $w_i\geqslant 0, i=1,2,\ldots,n$, $\sum_{i=1}^{n}w_i=1$ then TFLWA is called a trapezoid fuzzy linguistic weighted averaging (TFLWA) operator.

Example 1 Assume $\tilde{s}_1=[s_0, s_1, s_3, s_5]$, $\tilde{s}_2=[s_2, s_3, s_4, s_5]$, $\tilde{s}_3=[s_3, s_4, s_5, s_7]$, $\tilde{s}_4=[s_2, s_4, s_5, s_6]$, $\boldsymbol{w}=\{0.3, 0.1, 0.2, 0.4\}$. Then by the operational laws of TFLVs, we have

$$\text{TFLWA}_w(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3, \tilde{s}_4)=0.3\times[s_0, s_1, s_3, s_5]\oplus 0.1\times[s_2, s_3, s_4, s_5]\oplus 0.2\times[s_3, s_4, s_5, s_7]\oplus 0.4\times[s_2, s_4, s_5, s_6]=[s_{1.6}, s_3, s_{4.3}, s_{5.8}]$$

2 Degree of Possibility Based Approach

A multiple attribute decision making problem under a fuzzy linguistic environment is represented as follows:

Let $X=\{x_1, x_2, \ldots, x_n\}$ be the set of alternatives, and $U=\{u_1, u_2, \ldots, u_m\}$ be the set of attributes. Let $\boldsymbol{w}=\{w_1, w_2, \ldots, w_m\}$ be the weight vector of attributes, where $w_i\geqslant 0, i=1, 2, \ldots, m$, $\sum_{i=1}^{m}w_i=1$. Suppose that $\tilde{\boldsymbol{A}}=(\tilde{a}_{ij})_{n\times m}$ is the fuzzy linguistic decision matrix, where $\tilde{a}_{ij}=[a_{ij}^{(\alpha)}, a_{ij}^{(\beta)}, a_{ij}^{(\gamma)}, a_{ij}^{(\eta)}]\in\tilde{S}$ is the attribute value, which takes the form of TFLV, given by the decision maker, for the alternative $x_i\in X$ with respect to the attribute $u_j\in U$. Let $\tilde{\boldsymbol{a}}_i=\{\tilde{a}_{i1}, \tilde{a}_{i2}, \ldots, \tilde{a}_{im}\}$ be the vector of the attribute values corresponding to the alternative x_i, $i=1,2,\ldots,n$.

In the following, we develop an approach to ranking the decision alternatives based on the degree of possibility of TFLVs.

Step 1 Utilize the TFLWA operator

$$\tilde{z}_i=\text{TFLWA}_w(\tilde{a}_{i1}, \tilde{a}_{i2}, \ldots, \tilde{a}_{im})=w_1\tilde{a}_{i1}\oplus w_2\tilde{a}_{i2}\oplus\ldots\oplus w_m\tilde{a}_{im} \qquad i=1,2,\ldots,n \quad (3)$$

to derive the overall values $\tilde{z}_i(i=1,2,\ldots,n)$ of the alternatives $x_i(i=1,2,\ldots,n)$.

Step 2 To rank these collective overall preference values $\tilde{z}_i$, we first compare each $\tilde{z}_i$ with all the $\tilde{z}_i$ by using Eq. (1). For simplicity, we let $p_{ij}=p(\tilde{z}_i\geqslant\tilde{z}_j)$; then we develop a complementary matrix as $\boldsymbol{P}=(p_{ij})_{n\times n}$, where $p_{ij}\geqslant 0, p_{ij}+p_{ji}=1$.

Step 3 Summing all the elements in each line of matrix $\boldsymbol{P}$, we have $p_i=\sum_{j=1}^{n}p_{ij}, \quad i=1,2,\ldots,n$. Then we rank the overall preference values $\tilde{z}_i$ in descending order in accordance with the values of p_i.

Step 4 Rank all the alternatives x_i and select the best one(s) in accordance with the collective overall preference values $\tilde{z}_i$.

3 Numerical Examples

In this section, a decision making problem of assessing cars for buying (adapted from Refs. [2, 8]) is used to illustrate the developed approach.

Let us consider a customer who intends to buy a car. Four types of cars $x_i(i=1,2,3,4)$ are available. The customer takes into account four attributes to decide which car to buy: 1) G_1: economy, 2) G_2: comfort, 3) G_3: design, and 4) G_4: safety. The decision maker evaluates these four types of cars $x_i(i=1,2,3,4)$ under the attributes $G_i(i=1,2,3,4)$ (whose weight vector is $\boldsymbol{w}=\{0.3, 0.2, 0.1, 0.4\}$) by using the linguistic scale

$$\begin{aligned} S=\{&s_0=\text{extremely poor}, s_1=\text{very poor},\\ &s_2=\text{poor}, s_3=\text{slightly poor}, s_4=\text{fair},\\ &s_5=\text{slightly good}, s_6=\text{good},\\ &s_7=\text{very good}, s_8=\text{extremely good}\}\end{aligned}$$

and gives a fuzzy linguistic decision matrix as listed in Tab. 1.

Tab. 1 Fuzzy linguistic decision matrix $\tilde{\boldsymbol{A}}$

G_i	G_1	G_2	G_3	G_4
x_1	$[s_1,s_2,s_4,s_5]$	$[s_3,s_4,s_7,s_8]$	$[s_4,s_5,s_6,s_8]$	$[s_2,s_3,s_4,s_6]$
x_2	$[s_2,s_4,s_5,s_6]$	$[s_4,s_5,s_6,s_7]$	$[s_3,s_4,s_7,s_8]$	$[s_3,s_4,s_6,s_7]$
x_3	$[s_3,s_5,s_7,s_8]$	$[s_0,s_1,s_3,s_5]$	$[s_5,s_6,s_7,s_8]$	$[s_2,s_3,s_4,s_5]$
x_4	$[s_4,s_5,s_6,s_8]$	$[s_3,s_6,s_7,s_8]$	$[s_2,s_4,s_5,s_6]$	$[s_5,s_6,s_7,s_8]$

In the following, we utilize the approach developed in this paper to obtain the most desirable car. From Tab. 1, we obtain the vector of the attribute values corresponding to the alternative $x_i(i=1,2,3,4)$ as follows:

1) $\tilde{\boldsymbol{a}}_1=\{\tilde{a}_{11},\tilde{a}_{12},\tilde{a}_{13},\tilde{a}_{14}\}$, where

$$\tilde{a}_{11}=[s_1,s_2,s_4,s_5],\quad \tilde{a}_{12}=[s_3,s_4,s_7,s_8]$$
$$\tilde{a}_{13}=[s_4,s_5,s_6,s_8],\quad \tilde{a}_{14}=[s_2,s_3,s_4,s_6]$$

2) $\tilde{\boldsymbol{a}}_2=\{\tilde{a}_{21},\tilde{a}_{22},\tilde{a}_{23},\tilde{a}_{24}\}$, where

$$\tilde{a}_{21}=[s_2,s_4,s_5,s_6],\quad \tilde{a}_{22}=[s_4,s_5,s_6,s_7]$$
$$\tilde{a}_{23}=[s_3,s_4,s_7,s_8],\quad \tilde{a}_{24}=[s_3,s_4,s_6,s_7]$$

3) $\tilde{\boldsymbol{a}}_3=\{\tilde{a}_{31},\tilde{a}_{32},\tilde{a}_{33},\tilde{a}_{34}\}$, where

$$\tilde{a}_{31}=[s_3,s_5,s_7,s_8],\quad \tilde{a}_{32}=[s_0,s_1,s_3,s_5]$$
$$\tilde{a}_{33}=[s_5,s_6,s_7,s_8],\quad \tilde{a}_{34}=[s_2,s_3,s_4,s_5]$$

4) $\tilde{\boldsymbol{a}}_4=\{\tilde{a}_{41},\tilde{a}_{42},\tilde{a}_{43},\tilde{a}_{44}\}$, where

$$\tilde{a}_{41}=[s_4,s_5,s_6,s_8],\quad \tilde{a}_{42}=[s_3,s_6,s_7,s_8]$$
$$\tilde{a}_{43}=[s_2,s_4,s_5,s_6],\quad \tilde{a}_{44}=[s_5,s_6,s_7,s_8]$$

Step 1 Utilize Eq. (3) to derive the overall values $\tilde{z}_i(i=1,2,3,4)$ of the alternatives $x_i(i=1,2,3,4)$.

$$\tilde{z}_1=[s_{2.1},s_{3.1},s_{4.8},s_{6.3}],\quad \tilde{z}_2=[s_{2.9},s_{4.2},s_{5.8},s_{6.8}]$$
$$\tilde{z}_3=[s_{2.2},s_{3.5},s_{5.0},s_{6.2}],\quad \tilde{z}_4=[s_{4.0},s_{5.5},s_{6.5},s_{7.8}]$$

Step 2 To rank these overall preference values $\tilde{z}_i$, we first compare each $\tilde{z}_i$ with all the $\tilde{z}_i$ by using Eq. (1), and then develop a complementary matrix:

$$\boldsymbol{P}=\begin{bmatrix}0.5000 & 0.3509 & 0.4737 & 0.1495\\ 0.6491 & 0.5000 & 0.6273 & 0.3010\\ 0.5263 & 0.3727 & 0.5000 & 0.1650\\ 0.8505 & 0.6990 & 0.8350 & 0.5000\end{bmatrix}$$

Step 3 Summing all the elements in each line of matrix $\boldsymbol{P}$, we have

$$p_1=1.4741,\ p_2=2.0774,\ p_3=1.5641,\ p_4=2.8844$$

Then we rank the overall preference values $\tilde{z}_i$ in descending order in accordance with the values of $p_i(i=1,2,3,4)$: $\tilde{z}_4>\tilde{z}_2>\tilde{z}_3>\tilde{z}_1$.

Step 4 Rank all the alternatives x_i in accordance with the overall preference values $\tilde{z}_i$: $x_4 \succ x_2 \succ x_3 \succ x_1$, and thus the best car is x_4.

In this example, our approach produces the same ranking as Xu's approach, but it is simpler.

4 Conclusion

In this paper, we investigate the multiple attribute decision making problems under a fuzzy linguistic environment, in which the attribute values are in the form of TFLVs. The formula of the degree of possibility between two TFLVs is defined. Based on the degree of possibility of fuzzy linguistic variables, we develop an approach to multiple attribute decision making under a fuzzy linguistic environment, which can carry out fuzzy linguistic computation processes easily without loss of information.

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基于梯形模糊语言变量的多属性决策方法

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摘要: 研究了决策者的偏好信息(属性值)为梯形模糊语言变量(TFLV)的模糊语言多属性决策问题. 定义了2个梯形模糊语言变量比较的可能度公式并研究了它的一些基本性质,给出了基于可能度公式的梯形模糊语言变量多属性决策的方案排序方法. 这种方法通过对决策信息进行TFLWA算子加权集结,并通过比较梯形模糊语言变量的可能度得到所有方案的排序结果. 在不损失模糊语言信息的条件下,该方法计算过程简单,决策结果合理且有效. 最后,通过一个实例说明了该方法的操作过程.

关键词: 梯形模糊语言变量;可能度;多属性决策

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