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Cascade Kalman filter for gravity anomaly distortion correction based on second order potential model

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Abstract: Using a gravity anomaly covariance function based on the second-order Gaussian Markov gravity anomaly potential model, the state equation of a gravity anomaly signal is obtained in marine gravimetry. Combined with the system state equation and the measurement equation, a new method of the cascade Kalman filter is proposed and applied to the correction of gravity anomaly distortion. In the signal processing procedure, an inverse Kalman filter is used to restore the gravity anomaly signal and high frequency noises first. Then an adaptive Kalman filter, which uses the gravity anomaly state equation as the system equation, is set to estimate the actual gravity anomaly data. Emulations and experiments indicate that both the cascade Kalman filter method and the single inverse Kalman filter method are effective in alleviating the distortion of the gravity anomaly signal, but the performance of the cascade Kalman filter method is better than that of the single inverse Kalman filter method.

Key words: gravimeter; gravity anomaly; cascade Kalman filter; inverse Kalman filter; distortion correction

Gravity is one of the extremely important parameters for numerical calculations and controls in gravity/inertial navigation systems, so the accuracy of gravity determines the precision of the navigation system[1]. In such a system, a referenced ellipsoid model is commonly used to describe the gravity field[2]. For the whole shape of the geoid surface, the referenced ellipsoid model is an excellent approximation; but for some local regions, such as marine gravity with complex geological situations, the model introduces errors. Along with the development of the inertial instrument performance, the errors of inertial components are no longer the most crucial factors degrading the precision of the gravity/inertial navigation system. Gravity anomaly—the difference between real gravity and normal gravity—has been regarded as the greatest error resource in high precision gravity/inertial navigation systems. Further improvements on system precision rely on high accuracy of gravity information[3]. Thus, real-time measurement and error-correction techniques of gravity anomalies are essential in order to extend the system operation time in the gravity/inertial navigation system with satisfactory high precision.

Two common characteristics in a gravimeter are strong damping and a great time constant. They inhibit high-frequency interference, but result in distortion of the low-frequency gravity anomaly signals, such as amplitude attenuation and phase lag. Such distortions are fatal in the high precision gravity/inertial navigation system, so a real-time method to correct gravity anomaly distortion must be introduced to improve the accuracy of the gravity anomaly measurement.

Only a few papers have discussed the problem of the gravity anomaly distortion correction. An off-line correction algorithm was proposed in Ref. [4]. Besides, Ref. [5] proposed an inverse Kalman filter scheme to correct the gravity anomaly distortion and testified for the feasibility of the algorithm through simulations. However, the ideal situation was supposed in the simulation of the scheme, such as ignoring the influence of measurement noise and the high frequency disturbance in the original gravity anomaly signal. Thus, the performance under severe environmental noise and disturbances should be discussed through further researches. This paper proposes a gravity anomaly distortion correction method based on adaptive Kalman filter and inverse Kalman filter algorithms. Simulations and experiments indicate that the proposed method achieves better performance than the single inverse Kalman filter method.

1 Gravity Anomaly State Equation

Usually, the diversification of a geoid gravity field can be modeled as a stochastic procedure with mean zero which can be described as a lineal differential equation stimulated by white noise. Assuming that a signal $u(t)$ with the power spectral density function $\phi_u(\omega)$ is transferred through a linear system with the transfer function $H(\mathrm{j}\omega)$, the power spectral density function of the output signal of the system is $\phi_x(\omega) = |H(\mathrm{j}\omega)|^2\phi_u(\omega)$. Furthermore, if $u(t)$ is a white noise procedure, then $\phi_u(\omega) = 1$. Meanwhile, if the power spectral function of output signal $\phi_x(\omega)$ can be acquired, then $H(\mathrm{j}\omega)$ can be calculated. Thus, if the expectation of statistics and the covariance function of the stochastic procedure are known, the stochastic procedure can be expressed as a linear system stimulated by white noise. Assuming a stochastic procedure of one dimension with mean zero and variance $R(t)$, the state equation of this procedure is described as[5]

$$\left.\begin{aligned} \dot{x}(t) &= F(t)x(t) + u(t) \\ E[u(t)u'(t+\tau)] &= Q(t)\delta(t-\tau) \end{aligned}\right\} \tag{1}$$

If the procedure is a stationary process, then $F(t) = F(0) = F =$ const and F and Q can be calculated by

$$F = A'P^{-1}, \quad Q = -(A + A')$$

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$$A=\frac{\mathrm{d}}{\mathrm{d}t}[R(t)]\Big|_{t=0},\quad P=R(t)\Big|_{t=0}$$

Since the geoid gravity covariance model cannot be acquired accurately, the state equation cannot be obtained subsequently. To solve the problem, many researchers have proposed their empirical formulae of the gravity covariance model. In this paper, the gravity anomaly covariance function is used which is based on the second-order Gaussian Markov gravity anomaly potential model and expressed as

$$C_{\mathrm{TT}(r)}=\sigma_{\mathrm{T}}^2(1+\beta\gamma)\mathrm{e}^{-\beta\gamma} \tag{2}$$

where σ_{T}^2 and β are parameters of the model. From Eq. (2), the gravity anomaly covariance function can be calculated by

$$G_{\mathrm{TrTr}(r)}=2\sigma_{\mathrm{T}}^2\beta^2\left(1-\frac{\beta}{2}r\right)\mathrm{e}^{-\beta r}$$

Supposing the direction of a warship towards axis x with the velocity v and the direction far beyond the earth and vertical to the surface is y, then

$$x=vt,\quad y=0$$

So, the gravity anomaly covariance function is modified to

$$C_{\mathrm{TrTr}(t)}=2\sigma_{\mathrm{T}}^2\beta^2\left(1-\frac{\beta}{2}vt\right)\mathrm{e}^{-\beta vt}$$

Subsequently,

$$\left.\begin{aligned}
&A=\frac{\mathrm{d}[C(t)]}{\mathrm{d}t}\Big|_{t=0}=-3\sigma_{\mathrm{T}}^2\beta^3v\\
&P=C(t)\Big|_{t=0}\\
&F=A'P^{-1}=-1.5\beta v\\
&Q=-(A+A')=6\sigma_{\mathrm{T}}^2\beta^3v
\end{aligned}\right\}$$

Then the gravity anomaly state equation is

$$\left.\begin{aligned}
&\Delta\dot{g}(t)=-1.5\beta v\Delta g(t)+u(t)\\
&E[u(t)u'(t-\tau)]=6\sigma_{\mathrm{T}}^2\beta^3v\delta(t-\tau)
\end{aligned}\right\}$$

According to the supposed situation, the value of parameters can be set by[5]

$$\sigma_{\mathrm{T}}^2=2.356\,993(\mathrm{m/s^2})^2\times(\mathrm{km})^2$$

$$\beta=0.010\,298\,9\ \mathrm{km}$$

Here the velocity of the warship is 10 knots (18.52 km/h) and the sampling interval is 0.1 s. Then the discrete gravity anomaly state equation is modified as

$$x_{3(k)}=(1-0.794\,7\times10^{-5})x_{3(k-1)}+u_{(k)} \tag{3}$$

$$\Delta g_{(k)}=x_{3(k)} \tag{4}$$

2 Cascade Kalman Filter Method for Gravity Anomaly Distortion Correction

2.1 The principle of inverse Kalman filter distortion correction

Suppose that the discrete state equation of gravimeter is

$$\boldsymbol{X}_{1(k)}=\boldsymbol{A}_{1(k,k-1)}\boldsymbol{X}_{1(k-1)}+\boldsymbol{G}_{1(k)}\boldsymbol{S}_{(k)} \tag{5}$$

and the measurement equation is

$$\boldsymbol{Z}_{(k)}=\boldsymbol{H}_{1(k)}\boldsymbol{X}_{1(k)}+\boldsymbol{V}_{(k)} \tag{6}$$

According to the analysis above, the gravity anomaly signal can be regarded as the output of the linear system stimulated by white noise, so the discrete state equation of the anomaly signal is

$$\boldsymbol{X}_{2(k)}=\boldsymbol{A}_{2(k,k-1)}\boldsymbol{X}_{2(k-1)}+\boldsymbol{G}_{2(k)}\boldsymbol{u}_{(k)} \tag{7}$$

$$\boldsymbol{S}_{(k)}=\boldsymbol{H}_{2(k)}\boldsymbol{X}_{2(k)} \tag{8}$$

Combining with Eqs. (5) to (8), we conclude the matrix equation

$$\boldsymbol{X}_{(k)}=\boldsymbol{\Phi}_{(k,k-1)}\boldsymbol{X}_{(k-1)}+\boldsymbol{G}_{(k)}\boldsymbol{u}_{(k)} \tag{9}$$

$$\boldsymbol{Z}_{(k)}=\boldsymbol{H}_{(k)}\boldsymbol{X}_{(k)}+\boldsymbol{V}_{(k)} \tag{10}$$

where $\boldsymbol{S}_{(k)}$ is the gravity anomaly signal which is to be estimated, and

$$\boldsymbol{\Phi}_{(k,k-1)}=\begin{bmatrix}\boldsymbol{A}_{1(k,k-1)} & \boldsymbol{G}_{1(k)}\boldsymbol{H}_{2(k)}\\ \boldsymbol{0} & \boldsymbol{A}_{2(k,k-1)}\end{bmatrix},\quad \boldsymbol{G}_{(k)}=\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{G}_{2(k)}\end{bmatrix}$$

$$\boldsymbol{X}_{(k)}=\begin{bmatrix}\boldsymbol{X}_{1(k)}\\ \boldsymbol{X}_{2(k)}\end{bmatrix},\quad \boldsymbol{H}_{(k)}=[\boldsymbol{H}_{1(k)}\quad \boldsymbol{0}]$$

According to Eqs. (9) and (10), combining with Eqs. (11) to (13)[6] and using the iterative Kalman algorithm providing by Eq. (14), the state variable $\boldsymbol{X}_{2(k)}$ can be estimated first. Then the estimation of the gravity anomaly signal $\boldsymbol{S}_{(k)}$ is acquired using Eq. (8). The procedure of these two steps of estimation is called the inverse Kalman filter.

$$\hat{\boldsymbol{X}}_k=\boldsymbol{\varphi}_{k,k-1}\hat{\boldsymbol{X}}_{k-1}+\boldsymbol{K}_k(\boldsymbol{Y}_k-\boldsymbol{H}_k\boldsymbol{X}_{k/k-1}) \tag{11}$$

$$\boldsymbol{K}_k=\boldsymbol{P}_{k/k-1}\boldsymbol{H}_k^{\mathrm{T}}(\boldsymbol{H}_k\boldsymbol{P}_{k/k-1}\boldsymbol{H}_k^{\mathrm{T}}+\boldsymbol{R})^{-1} \tag{12}$$

$$\boldsymbol{P}_{k/k-1}=\boldsymbol{\varphi}_{k,k-1}\boldsymbol{P}_{k-1}\boldsymbol{\varphi}_{k,k-1}^{\mathrm{T}}+\boldsymbol{Q}_{k-1} \tag{13}$$

$$\boldsymbol{P}_k=(\boldsymbol{I}-\boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{k/k-1} \tag{14}$$

Using the inverse Kalman filter algorithm to estimate the gravity anomaly signal from the gravimeter output, the system is modeled as a uniform linear system combining the gravimeter system and the gravity anomaly description. In this model, the environmental noise and the turbulence are modeled as white noise, which introduces great errors sometimes because of the influence of complex non-stationary environmental noises in actual implementation. To overcome the problem, we propose the cascade Kalman filter method to avoid the system error introduced by the uniform model and improve the precision of the gravity anomaly correction.

2.2 Cascade Kalman filter method

According to the analysis above, it can be concluded that errors are introduced by the uniform model which is used to estimate the gravity anomaly directly. In order to avoid the errors, the Cascade Kalman filter method is proposed, which uses the inverse Kalman filter to estimate the input signal of the gravimeter by means of the state equation of gravimeter system first. Then the adaptive Kalman filter is used to estimate the gravity anomaly signal from complex noises by means of the state equation of the gravity anomaly signal.

For a gravimeter system in application, the accurate transfer function can be obtained by measurement, so the accurate systematic state equation can be acquired subsequently. Assuming the transfer function of the gravimeter system is

$$H(s) = \frac{8.861\,236 \times 10^{-3}}{s^2 + 0.133\,125\,8s + 8.861\,236 \times 10^{-3}} \qquad (15)$$

then the system can be described as the differential equation

$$\ddot{x} + 0.133\,125\,8\dot{x} + 8.861\,236 \times 10^{-3}x = 8.861\,23 \times 10^{-3}\Delta g(t)$$

Assuming that $x_1 = x$, $x_2 = \dot{x}_1$, and the sampling period is 0.1 s, the discrete systematic state equation is

$$\begin{bmatrix} x_{1(k)} \\ x_{2(k)} \end{bmatrix} = \begin{bmatrix} 1 & 0.1 \\ -8.861\,23 \times 10^{-4} & 1 - 0.013\,312\,58 \end{bmatrix} \cdot \begin{bmatrix} x_{1(k-1)} \\ x_{2(k-1)} \end{bmatrix} + \begin{bmatrix} 0 \\ 8.861\,23 \times 10^{-4}\Delta g(k) \end{bmatrix} \qquad (16)$$

and the measurement equation is

$$\boldsymbol{Z}_{(k)} = [1 \quad 0]\begin{bmatrix} x_{1(k)} \\ x_{2(k)} \end{bmatrix} + \boldsymbol{V}_{(k)} \qquad (17)$$

According to Eqs. (16) and (17) and the Kalman filter algorithm described by Eqs. (11) to (14), the input signal sequence of the gravimeter is estimated, which includes the actual gravity anomaly signal and all kinds of noises and disturbances.

Furthermore, besides the accurate state equation, an algorithm with high performance should be used to restore the gravity anomaly signal from complex noises and disturbances. The Sage-Husa adaptive filter is an improved Kalman filter, which estimates the statistics of noise in real time. Moreover, the Sage-Husa adaptive filter is suited to time varying systems and easy to use. In this paper, we use the Sage-Husa adaptive filter combined with the inverse Kalman filter to construct the cascade Kalman filter method.

Assuming that the gravity anomaly state equation is

$$\left.\begin{aligned} \boldsymbol{X}(k+1) &= \boldsymbol{\varphi}_{k,k-1}\boldsymbol{X}(k) + \boldsymbol{w}(k) \\ \boldsymbol{Y}(k) &= \boldsymbol{I}\boldsymbol{X}(k) \end{aligned}\right\} \qquad (18)$$

and the unknown measurement noise matrix is $\boldsymbol{R}$, then the simplified iterative algorithm of the Sage-Husa adaptive filter is described as follows[7-9]:

Calculating weighted coefficient

$$d_k = \frac{1-b}{1-b^k} \qquad (19)$$

Constructing one step prediction state equation

$$\boldsymbol{X}_{k/k-1} = \boldsymbol{\varphi}_{k,k-1}\hat{\boldsymbol{X}}_{k-1} \qquad (20)$$

Constructing one step prediction covariance equation

$$\boldsymbol{P}_{k/k-1} = \boldsymbol{\varphi}_{k,k-1}\boldsymbol{P}_{k-1}\boldsymbol{\varphi}_{k,k-1}^{\mathrm{T}} + \boldsymbol{Q}_{k-1} \qquad (21)$$

Constructing innovation sequence equation

$$\boldsymbol{v}_k = \boldsymbol{Y}_k - \boldsymbol{H}_k\boldsymbol{X}_{k/k-1} \qquad (22)$$

Estimating measurement noise

$$\boldsymbol{R}_k = (1-d_k)\boldsymbol{R}_{k-1} + d_k[\boldsymbol{v}_k\boldsymbol{v}_k^{\mathrm{T}} - \boldsymbol{H}_k\boldsymbol{P}_{k/k-1}\boldsymbol{H}_k^{\mathrm{T}}] \qquad (23)$$

Constructing filter gain equation

$$\boldsymbol{K}_k = \boldsymbol{P}_{k/k-1}\boldsymbol{H}_k^{\mathrm{T}}(\boldsymbol{H}_k\boldsymbol{P}_{k/k-1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R})^{-1} \qquad (24)$$

Estimating covariance equation

$$\boldsymbol{P}_k = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{k/k-1} \qquad (25)$$

Constructing estimation equation

$$\hat{\boldsymbol{X}}_k = \hat{\boldsymbol{X}}_{k/k-1} + \boldsymbol{K}_k\boldsymbol{v}_k \qquad (26)$$

In this algorithm, the forgetting factor b satisfies $0 < b < 1$. The forgetting factor is used to limit the memory length. Given $\boldsymbol{X}_0$, $\boldsymbol{P}_0$ and $\boldsymbol{R}_0$, the above iterative algorithm is used to estimate the gravity anomaly signal from the input signal of the gravimeter.

3 Simulations and Experiments of Gravity Anomaly Distortion Correction

Suppose that the gravity anomaly signal is $\Delta g = \sin(0.005t)$, and the acceleration of high frequency disturbance acting on the gravimeter is $\delta a = 1.5\sin(0.5t)$. Then the actual input signal of the gravimeter is

$$y = \sin(0.005t) + 1.5\sin(0.5t) \qquad (27)$$

which is shown in Fig. 1. Fig. 2 shows the ideal gravity anomaly signal and the output signal of the gravimeter system when the above signal is used as the input of the gravimeter.

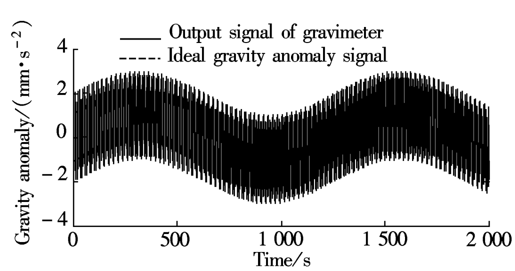

Fig. 1 The input signal of gravimeter

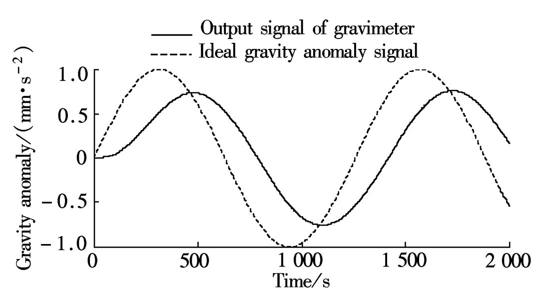

Fig. 2 The output signal of gravimeter and the ideal gravity anomaly signal

Fig. 2 indicates that the high frequency disturbance of the output signal of the gravimeter is well inhibited, but amplitude attenuation and phase lag still exist in contrast to the ideal gravity anomaly signal.

With the output signal of the gravimeter as a measurement signal, the inverse Kalman filter described in section 2. 1 is used to estimate the input gravity anomaly signal of the gravimeter and the result is shown in Fig. 3. According to Fig. 3, when the variance of measurement noise is zero, the inverse Kalman filter restores high frequency disturbance while restoring the gravity anomaly signal; when it is non-zero, the gravity anomaly signal is restored with great errors.

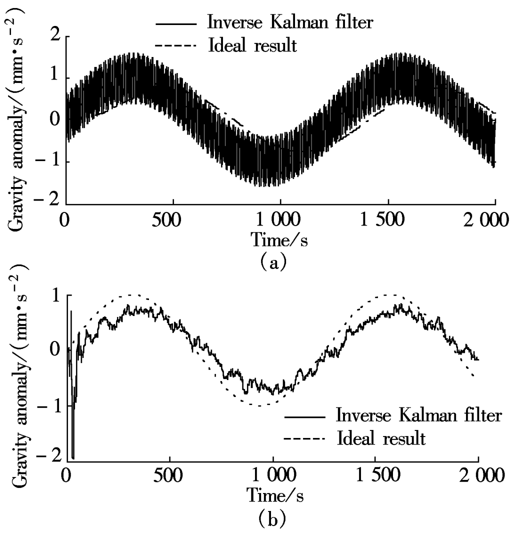

Fig. 3 The estimation of output signal of gravimeter using inverse Kalman filter. (a) The variance of measurement noise is zero; (b) The variance of measurement noise is non-zero

The estimation results of the proposed cascade Kalman filter method is shown in Fig. 4 with the output signal of the gravimeter shown in Fig. 2 as the measurement signal. From Fig. 3 and Fig. 4, it can be concluded that the cascade Kalman filter method achieves better performance of gravity anomaly distortion correction.

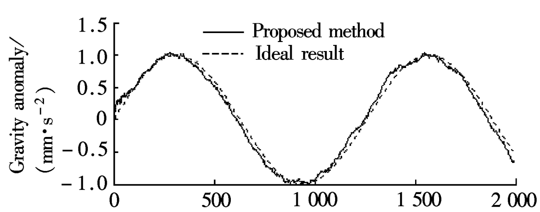

Fig. 4 The estimation of gravimeter output with cascade Kalman filter

In order to confirm the performance of the cascade Kalman filter, both the single Kalman filter method and the cascade Kalman filter method are used to deal with the real measurement data of marine gravity which is shown in Fig. 5. In Fig. 6(a), the results of the inverse Kalman filter are compared with the ideal result. In Fig. 6(b), the result of the proposed method is compared with the ideal result. From Fig. 6, the conclusion can be drawn that the performance of the cascade Kalman filter is greatly improved compared with that of the single inverse Kalman method.

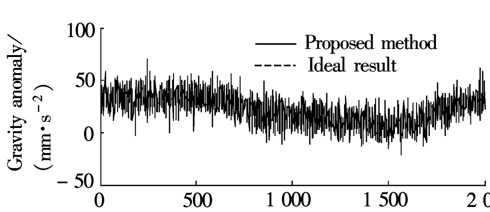

Fig. 5 The real measurement data of marine gravity

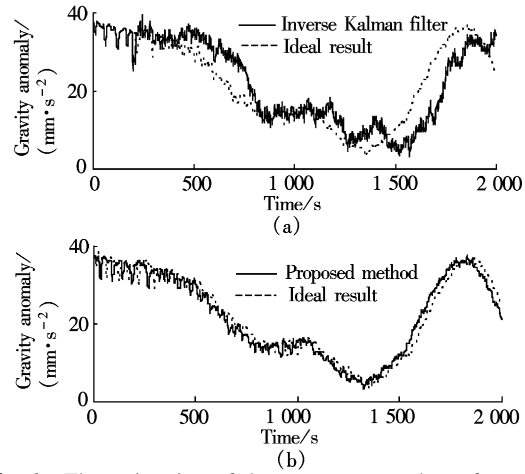

Fig. 6 The estimation of the measurement data of marine gravity. (a) The estimation of gravimeter output with inverse Kalman filter; (b) The estimation of gravimeter output with cascade Kalman filter

4 Conclusion

Amplitude attenuation and phase lag are the main kinds of distortions in a marine gravimeter with strong damping and a great time constant. In order to overcome the drawbacks and improve the precision of the gravity anomaly measurements, the real-time method of distortion correction should be applied. Based on the principle of distortion correction by the inverse Kalman filter algorithm, this paper analyzes the drawbacks of the inverse Kalman filter method introduced by the uniform model and proposes the cascade Kalman filter method to correct the distortion. Emulations and experiments indicate that both the cascade Kalman filter method and the single inverse Kalman filter method are effective in alleviating the distortion of the gravity anomaly signal, but the performance of the cascade Kalman filter method is better than that of single inverse Kalman filter method. The results of this research are valuable for the gravity/inertial navigation system by improving the precision of gravity anomaly information.

References

[1] Xu Zunyi, Yan Lei, Ning Shunian, et al. Situation and development of marine gravity aided navigation system[J]. *Progress in Geophysics*, 2007, **22**(1): 104 - 111.

[2] Hugh R, Steven K, Louis M. Geophysical navigation technologies and applications [C]//*IEEE Position Location and Navigation Symposium*. Las Vegas, NV, USA, 2004: 618 - 624.

[3] Clive A A, Albert J. Passive gravity gradiometer navigation system[C]//*IEEE Position Location and Navigation Symposium*. Monterey, CA, USA, 1990: 60 - 66.

[4] Haworth R T, Loncarevic B D. Inverse filter applied to the output of an askania gss-2 sea gravimeter[J]. *Geophysics*, 1974, **39**(6): 852 - 861.

[5] Wang Jiandi. Elimination of the distortion in dynamic gra-

vimetry using Kalman inverse filter[J]. *Acta Geodartica et Geophysica,* 1991, **12**: 57 – 69. (in Chinese)

[6] Qi Honghui, Moore J B. Direct Kalman filtering approach for GPS/INS integration[J]. *Aerospace and Electronic Systems*, 2002, **38**(2): 687 – 693.

[7] Grewal Mohinder S, Andrews Angus P. *Kalman filtering: theory and practice using Matlab*[M]. 2nd ed. John Wiley & Sons, Inc., 2001.

[8] Bozic S M. *Digital and Kalman filtering*[M]. Translated by Ling Yundan. Beijing: Science Press, 1984. (in Chinese)

[9] Weng Shiuh-Ku, Kuo Chung-Ming, Tu Shu-Kang. Video object tracking using adaptive Kalman filter[J]. *Journal of Visual Communication and Image Representation*, 2006, **17**(6): 1190 – 1208.

基于二阶异常位模型的重力异常畸变级联 Kalman 滤波校正方法

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摘要: 借助基于二阶高斯-马尔可夫异常位模型的重力异常协方差函数,得到了海洋重力测量中重力异常信号的状态方程. 结合实际重力仪的系统状态方程和系统量测方程,提出了级联卡尔曼滤波方法,并将其应用于重力异常畸变信号的校正处理中. 在信号处理过程中,首先采用卡尔曼逆滤波恢复含高频干扰的重力异常,然后采用自适应卡尔曼滤波,以重力异常状态方程为系统方程估计实际重力异常值,并与单一卡尔曼逆滤波器的处理结果进行了对比分析. 仿真和试验表明,级联卡尔曼滤波方法和单一卡尔曼逆滤波都能在一定程度上减小重力异常信号的畸变,但在相同背景条件下,级联卡尔曼滤波方法的性能优于单一逆卡尔曼滤波.

关键词: 重力仪;重力异常;级联卡尔曼滤波;卡尔曼逆滤波;畸变校正

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