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Novel *K*-best detection algorithms for MIMO system

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Abstract: Aiming at the optimum path excluding characteristics and the full constellation searching characteristics of the *K*-best detection algorithm, an improved-performance *K*-best detection algorithm and several reduced-complexity *K*-best detection algorithms are proposed. The improved-performance *K*-best detection algorithm deploys minimum mean square error (MMSE) filtering of a channel matrix before QR decomposition. This algorithm can decrease the probability of excluding the optimum path and achieve better performance. The reduced-complexity *K*-best detection algorithms utilize a sphere decoding method to reduce searching constellation points. Simulation results show that the improved performance *K*-best detection algorithm obtains a 1 dB performance gain compared to the *K*-best detection algorithm based on sorted QR decomposition (SQRD). Performance loss occurs when $K = 4$ in reduced complexity *K*-best detection algorithms. When $K = 8$, the reduced complexity *K*-best detection algorithms require less computational effort compared with traditional *K*-best detection algorithms and achieve the same performance.

Key words: sorted QR decomposition; *K*-best; sphere decoding; maximum-likelihood detection; minimum mean square error

Multiple-input multiple-output (MIMO) systems provide very high capacity compared to single-input single-output (SISO) systems in Rayleigh fading environments[1]. Signal detection in MIMO systems is more complex than in SISO channels because of the interference among multiple antennas. The maximum likelihood detection (MLD) algorithm is an optimal detection algorithm when the transmit symbols are equally probable, but it is not feasible in practice owing to its extremely high computational complexity.

The sphere decoding algorithm (SDA)[2-4] has been investigated in order to reach a near ML performance with reduced computational effort. The sphere decoding algorithm can be regarded as a depth-first tree search approach with pruning. The main disadvantage of the depth-first tree search approach is that the required computational effort varies with different signals and channels. Hence, the detection throughput is not fixed, which is not desirable for real time detection and hardware implementation. The *K*-best algorithm and the *K*-best-SQRD algorithm[5-7] deploy a breadth-first tree search method. These algorithms require less computational effort, have fixed throughput, and are suitable for hardware implementation.

In this paper, we introduce an improved-performance *K*-best algorithm based on the MMSE-SQRD[8-9] and several reduced-complexity *K*-best detection algorithms. The extended channel matrix deployed in the MMSE-SQRD algorithm is also applicable in the *K*-best algorithm. The performance of the *K*-best MMSE-SQRD (KB-MMSE-SQRD) algorithm is better than that of the traditional *K*-best algorithm. Complexity can be reduced by limiting the search points for each route, which is similar to the method used in a sphere decoding algorithm. The SQRD solution is used to calculate the initial value of squared Euclidean distance in order to reduce the number of candidate constellation points for each route. The number of routes in the route queue of the reduced-complexity *K*-best algorithms is not fixed to *K*. The maximum number of routes in route queue is *K* and the minimum is 0. The SQRD solution is used as default output when the route queue is empty.

1 System Description

Fig. 1 depicts a flat-fading layered space-time MIMO system with n_{T} transmit antennas and n_{R} receive antennas ($n_{\mathrm{T}} \leqslant n_{\mathrm{R}}$). Considering one time slot of the discrete complex baseband equivalent model of such a system, the received symbol vector $\boldsymbol{r} = \{r_1, r_2, \dots, r_{n_{\mathrm{R}}}\}^{\mathrm{T}}$ is given by

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{\eta} \tag{1}$$

where $\boldsymbol{s} = \{s_1, s_2, \dots, s_{n_{\mathrm{T}}}\}^{\mathrm{T}}$ denotes the transmit vector and the constellation size is M_{c}. The average transmit power of each antenna is normalized to one. $\boldsymbol{H}_{n_{\mathrm{R}} \times n_{\mathrm{T}}}$ is the channel matrix. $H_{i,j}$ represents the channel gain between the j-th transmit antenna and the i-th receive antenna. $\boldsymbol{H}$ is constant over a frame and changes independently from frame to frame. $\boldsymbol{\eta} = \{\eta_1, \eta_2, \dots, \eta_{n_{\mathrm{R}}}\}^{\mathrm{T}}$ is zero means that the uncorrelated complex additive Gaussian white noise with co-variance matrix $\mathrm{cov}(\boldsymbol{\eta}) = \sigma_\eta^2 \boldsymbol{I}$. We assume that channel matrix $\boldsymbol{H}$ is completely known at the receiver end.

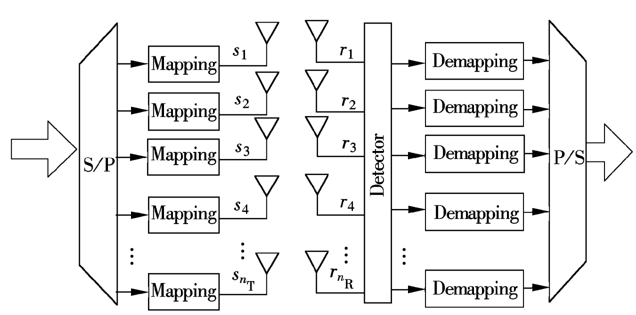

Fig. 1 A typical MIMO system with n_{T} transmit antennas and n_{R} receive antennas

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2 Sorted QR Decomposition and *K*-Best Algorithm

2.1 Sorted QR decomposition

Performing QR decomposition to channel matrix $\boldsymbol{H}$, we obtain

$$\boldsymbol{H} = \boldsymbol{QR} \tag{2}$$

where $\boldsymbol{R}$ is an up-triangular matrix and $\boldsymbol{Q}$ is a unitary matrix, $\boldsymbol{Q}^{\mathrm{H}}\boldsymbol{Q} = \boldsymbol{I}$. We multiplex $\boldsymbol{Q}^{\mathrm{H}}$ with the received vector

$$\tilde{\boldsymbol{r}} = \boldsymbol{Q}^{\mathrm{H}}\boldsymbol{r} = \boldsymbol{Rs} + \boldsymbol{Q}^{\mathrm{H}}\boldsymbol{\eta} = \boldsymbol{Rs} + \tilde{\boldsymbol{\eta}} \tag{3}$$

The co-variance matrix of $\tilde{\boldsymbol{\eta}}$ is identical to that of $\boldsymbol{\eta}$, i. e. $\mathrm{cov}(\boldsymbol{\eta}) = \sigma_{\eta}^{2}\boldsymbol{I} = \mathrm{cov}(\tilde{\boldsymbol{\eta}})$.

Sorted QR decomposition differs from QR decomposition in that the column of $\boldsymbol{H}$ is permuted, i. e. $\breve{\boldsymbol{Q}}\breve{\boldsymbol{R}} = \underline{\boldsymbol{HP}}$, where $\boldsymbol{P}$ is a permute matrix and $\boldsymbol{P}^{\mathrm{H}}\boldsymbol{P} = \boldsymbol{I}$. Multiplying $\breve{\boldsymbol{Q}}$ with the received vector, we obtain

$$\breve{\boldsymbol{r}} = \breve{\boldsymbol{Q}}^{\mathrm{H}}\boldsymbol{r} = \breve{\boldsymbol{R}}\boldsymbol{P}^{\mathrm{H}}\boldsymbol{s} + \breve{\boldsymbol{Q}}^{\mathrm{H}}\boldsymbol{\eta} = \breve{\boldsymbol{R}}\breve{\boldsymbol{s}} + \breve{\boldsymbol{\eta}} \tag{4}$$

where $\breve{\boldsymbol{\eta}}$, $\tilde{\boldsymbol{\eta}}$ and $\boldsymbol{\eta}$ have the same statistical characteristics.

2.2 *K*-best algorithm

According to the ML criterion, the maximum-likelihood solution $\boldsymbol{s}_{\mathrm{ML}}$ can be obtained by the following equation:

$$\boldsymbol{s}_{\mathrm{ML}} = \arg\min_{s} \|\boldsymbol{r} - \boldsymbol{Hs}\|^2 = \arg\min_{s} \|\tilde{\boldsymbol{r}} - \boldsymbol{Rs}\|^2 \tag{5}$$

Let ML metric E_{s_k} be the squared Euclidean distance between $\tilde{\boldsymbol{r}}$ and the candidate vector $\boldsymbol{Rs}_k$, where $\boldsymbol{s}_k = \{\underbrace{0, 0, \ldots,}_{k-1} s_k, s_{k+1}, \ldots, s_{n_{\mathrm{T}}}\}^{\mathrm{T}}$, s_k is candidate symbol chosen from constellation $\{C_1, C_2, \ldots, C_{M_c}\}$.

$$\begin{aligned} E_{s_k} &= \|\tilde{\boldsymbol{r}} - \boldsymbol{Rs}_k\|^2 = \sum_{i=n_{\mathrm{T}}}^{k} \left|\tilde{r}_i - \sum_{j=i}^{n_{\mathrm{T}}} R_{i,j}s_j\right|^2 = \\ &\sum_{i=n_{\mathrm{T}}}^{k+1} \left|\tilde{r}_i - \sum_{j=i}^{n_{\mathrm{T}}} \boldsymbol{R}_{i,j}s_j\right|^2 + \left|\tilde{r}_k - \sum_{j=k}^{n_{\mathrm{T}}} R_{k,j}s_j\right|^2 = \\ &E_{s_{k+1}} + \left|\tilde{r}_k - R_{k,k}s_k - \sum_{j=k+1}^{n_{\mathrm{T}}} R_{k,j}s_j\right|^2 \end{aligned} \tag{6}$$

According to Eq. (6), E_{s_k} is determined by $E_{s_{k+1}}$, s_{k+1} and s_k. In the *K*-best algorithm[5], detection starts from the last layer and proceeds layer by layer. In the *k*-th layer of this a lgorithm, E_{s_k} for all the search routes are calculated based on K best routes of the $(k+1)$-th layer and constellation. K best routes that have the least E_{s_k} are passed on to the next layer. The algorithms terminates when it reaches the first layer, and the output is generated based on the route which have the least metric in the route queue. $K \times M_{\mathrm{c}}$ candidates are to be searched in each layer.

3 Proposed MIMO Detection Algorithms

In this section, the sorted QR decomposition of the extended channel matrix is used to improve the performance of *K*-best algorithms and reduced-complexity *K*-best algorithms are proposed.

3.1 KB-MMSE-SQRD algorithm

In the linear MMSE detection algorithm[8], the MMSE filter matrix is given by

$$\boldsymbol{G}_{\mathrm{MMSE}} = (\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \sigma_{\eta}^{2}\boldsymbol{I})^{-1}\boldsymbol{H}^{\mathrm{H}} \tag{7}$$

The resulting filter output is

$$\tilde{\boldsymbol{s}}_{\mathrm{MMSE}} = \boldsymbol{G}_{\mathrm{MMSE}}\boldsymbol{r} = (\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \sigma_{\eta}^{2}\boldsymbol{I})^{-1}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{r} \tag{8}$$

Let $\underline{\boldsymbol{H}} = \begin{bmatrix} \boldsymbol{H} \\ \sigma_{\eta}\boldsymbol{I} \end{bmatrix}$ and $\underline{\boldsymbol{r}} = \begin{bmatrix} \boldsymbol{r} \\ 0_{n_{\mathrm{T}},1} \end{bmatrix}$, we rewrite Eq. (8) as

$$\tilde{\boldsymbol{s}}_{\mathrm{MMSE}} = (\underline{\boldsymbol{H}}^{\mathrm{H}}\underline{\boldsymbol{H}})^{-1}\underline{\boldsymbol{H}}^{\mathrm{H}}\underline{\boldsymbol{r}} = \underline{\boldsymbol{H}}^{+}\underline{\boldsymbol{r}} \tag{9}$$

The MMSE-SQRD algorithm[9] differs from the sorted QR detection in that the MMSE-SQRD deploys extended channel matrix $\underline{\boldsymbol{H}}$ instead of channel matrix $\boldsymbol{H}$. The MMSE-SQRD is also applicable to the *K*-best algorithm. Unitary matrix $\underline{\breve{\boldsymbol{Q}}}$ obtained from sorted QR decomposition $\underline{\breve{\boldsymbol{Q}}}\underline{\breve{\boldsymbol{R}}} = \underline{\boldsymbol{HP}}$ is a $(n_{\mathrm{T}} + n_{\mathrm{R}}) \times n_{\mathrm{T}}$ matrix. Up triangular matrix $\underline{\breve{\boldsymbol{R}}}$ and permute matrix $\underline{\boldsymbol{P}}$ are both $n_{\mathrm{T}} \times n_{\mathrm{T}}$ matrix. The received vector after filter $\breve{\boldsymbol{r}} = \underline{\breve{\boldsymbol{Q}}}^{\mathrm{H}}\underline{\boldsymbol{r}}$ is an n_{T} dimensional vector. Therefore, the search process of the KB-MMSE-SQRD algorithm is the same as that of the *K*-best algorithm. The implementation of the algorithm described above is described in Fig. 2.

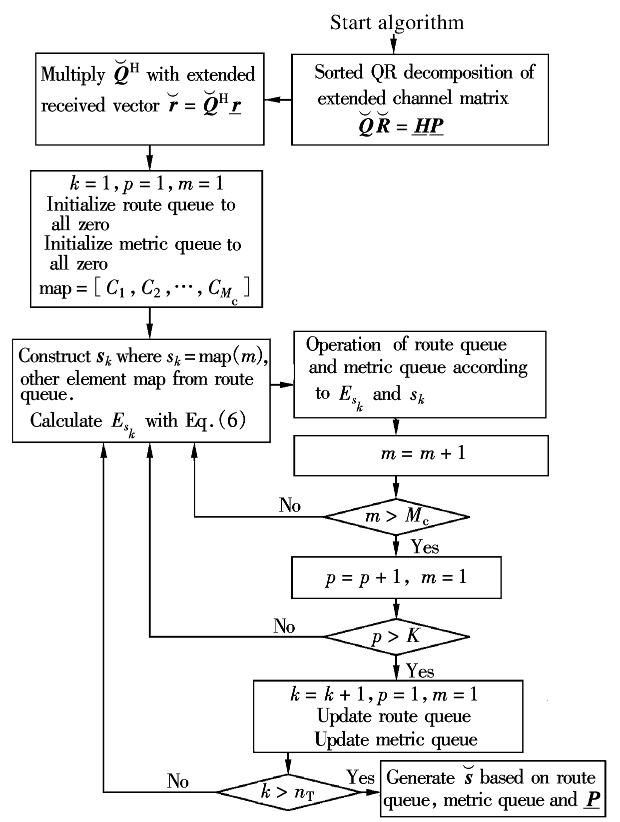

Fig. 2 KB-MMSE-SQRD algorithm

3.2 Reduced-complexity *K*-best algorithms

In *K*-best algorithms, all of the constellation points are searched for each route. We deploy a method that is similar to the method used in the sphere decoding algorithm[2] to reduce complexity. The core idea of this method is to reduce the number of constellation points to be searched for each route. The SQRD solution is used to calculate the initial squared Euclidean distance and it is also used as a default solution when no route is left in the route queue, the number of routes in the queue is a variable between 0 and n_T. This method is also applicable to the KB-MMSE-SQRD.

After sorted QR decomposition, SQRD solution $\hat{s}_{SQRD}$ is calculated and squared Euclidean distance d^2_{SQRD} is

$$d^2_{SQRD} = (\breve{r} - \breve{R}\breve{s}_{SQRD})^H(\breve{r} - \breve{R}\hat{s}_{SQRD}) \tag{10}$$

We choose the constellation points within the sphere

$$d^2_{SQRD} \geqslant E_{s_k} = E_{s_{k+1}} + \left| \breve{r}_k - \breve{R}_{k,k}s_k - \sum_{j=k+1}^{n_T} \breve{R}_{k,j}s_j \right|^2 \tag{11}$$

$E_{s_{k+1}}$ and $\sum_{j=k+1}^{n_T} \breve{R}_{k,j}s_j$ are known for each route at the k-th layer. Let $d'^2 = d^2_{SQRD} - E_{s_{k+1}}$ and $r'_k = \breve{r}_k - \sum_{j=k+1}^{n_T} \breve{R}_{k,j}s_j$, we have

$$d'^2 \geqslant |r'_k - \breve{R}_{k,k}s_k|^2 \tag{12}$$

So, the constellation points within the sphere are determined by

$$\left\lceil \frac{-d' + r'_k}{\breve{R}_{k,k}} \right\rceil \leqslant s_k \leqslant \left\lfloor \frac{d' + r'_k}{\breve{R}_{k,k}} \right\rfloor \tag{13}$$

The reduced-complexity K-best-SQRD algorithm described above is shown in Fig. 3. This reduced-complexity algorithm can be applied to the KB-MMSE-SQRD because the difference between the KB-MMSE-SQRD algorithm and the K-best-SQRD algorithm lies in the filter. In the RC-KB-MMSE-SQRD algorithm, the extended channel matrix is used and the MMSE-SQRD solution is deployed to calculate the initial squared Euclidean distance. The search process is the same as that of the RC-K-Best-SQRD algorithm.

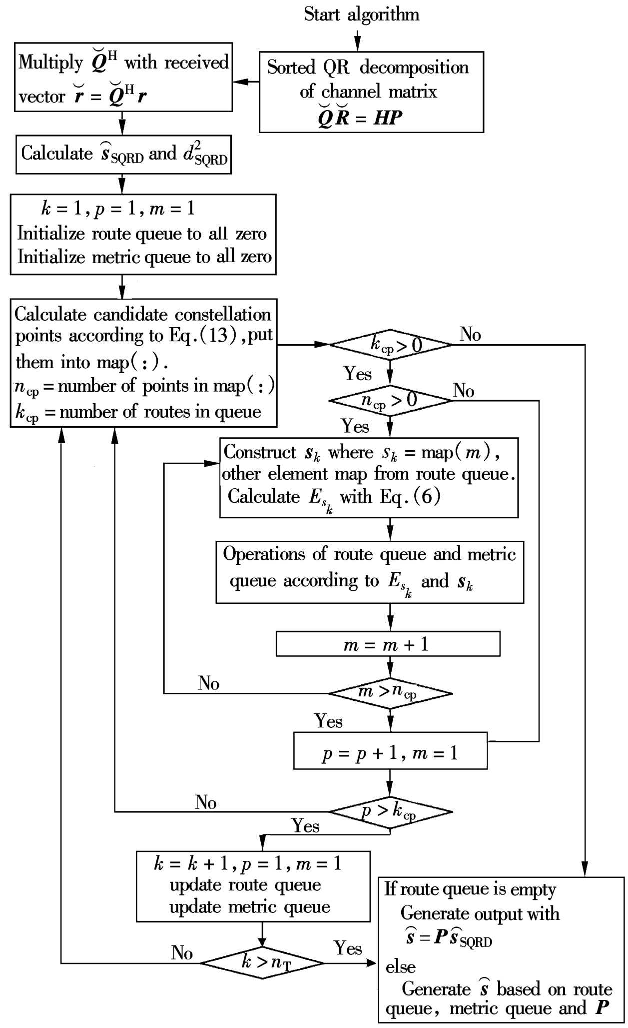

Fig. 3 RC-K-best-SQRD algorithm

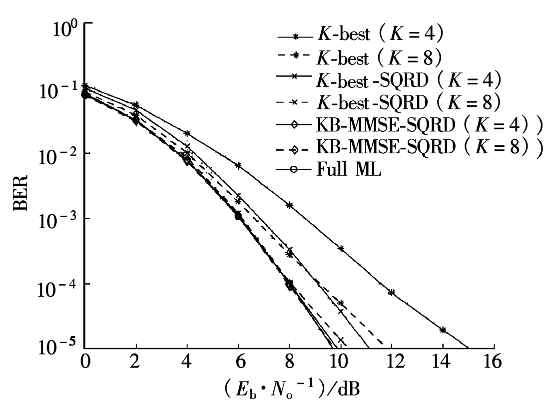

Fig. 4 Bit-error performance of K-best algorithms

4 Performance and Complexity Analysis

The performances of K-best detection algorithms and the full-ML algorithm are compared by means of MONTE CARLO simulations. Fig. 4 shows the bit error rate of an uncoded transmission of QPSK in a system with $n_T = 8$ and $n_R = 8$ antennas. The performance of the KB-MMSE-SQRD is very close to that of full-ML when $K = 4$ or $K = 8$. The K-best-SQRD obtains a 2 dB performance gain compared with K-best when $K = 4$. By using the extended channel matrix, we can obtain a 1 dB performance gain from the KB-MMSE-SQRD compared to that of the K-best-SQRD when $K = 4$.

Fig. 5 shows the performance of reduced-complexity K-best algorithms in an $n_T = 8$, $n_R = 8$ MIMO system. The RC-MMSE-SQRD achieves near ML performance when $K = 8$. The RC-KB-MMSE-SQRD algorithm still obtains about a 1 dB performance gain compared to the RC-K-best-SQRD algorithm when $K = 4$. The reduced-complexity algorithms have performance losses compared to K-best algorithms when $K = 4$. The optimal route which has a big metric in the lower layers may be excluded when the route queue is small. The performance of reduced-complexity K-best algorithms is identical to that of K-best algorithms when $K = 8$.

The complexity of a detection algorithm can be reflected by its required number of multiplications. As shown in Tab. 1, the number of multiplications required by ML is $O(n_T n_R M_c^{n_T})$. The complexity of K-best algorithms is correlated with K, M_c and n_T. In an 8×8 MIMO system, the K-best-SQRD requires 224 more complex multiplications than K-best and the difference of complexity between the KB-MMSE-SQRD and the K-best-SQRD is 672 complex multiplications.

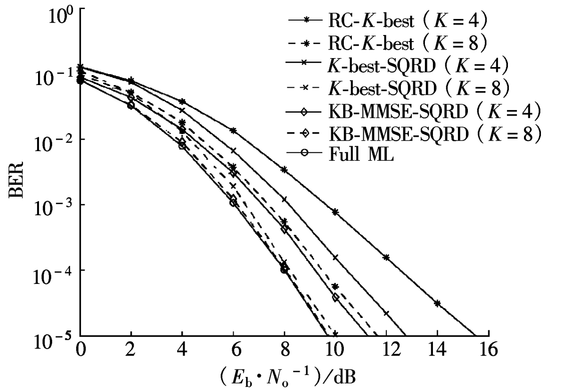

Fig. 5 Bit-error performance of reduced-complexity K-best algorithms

The required average multiplications for reduced-complexity algorithms in the 8×8 MIMO system is shown in Tab. 2. The complexity of reduced-complexity algorithms is correlated with E_b/N_o because there are more candidate constellation points for each route in low E_b/N_o than in high E_b/N_o. Average computational effort spent on searching is lower in the RC-KB-MMSE-SQRD than in the RC-K-best-SQRD. When $E_b/N_o=0$ dB, the RC-K-best-SQRD requires 358 more multiplications than the RC-KB-MMSE-SQRD in searching. In the RC-KB-MMSE-SQRD, the required computational effort drops very fast as E_b/N_o increases and it becomes stable when $E_b/N_o=8$ dB.

Tab. 1 Number of complex float multiplications required by detecting a signal vector

Algorithm	Steps	Required numbers of complex float multiplications per signal vector	Example of a QPSK, 8×8 system
Full maximum-likelihood	Generation of candidate vector $\boldsymbol{Hs}$	$n_T n_R M_c^{n_T}$	4 718 592
	Calculation of squared Euclidean distances	$n_R M_c^{n_T}$	
K-best	QR decomposition of matrix $\boldsymbol{H}$	$(n_T^2-n_T)n_R$	912($K=4$) 1 312($K=8$)
	Multiplication of $\boldsymbol{Q}^H$ to received signal vector	$n_R n_T$	
	Generation of routes	$n_T(n_T+1)K/2+n_T M_c K$	
	Calculation of squared Euclidean distances	$n_T M_c K$	
K-best-SQRD	Sorted QR decomposition of matrix $\boldsymbol{H}$	$3(n_T^2-n_T)n_R/2$	1 136($K=4$) 1 536($K=8$)
	Multiplication of $\boldsymbol{Q}^H$ to received signal vector	$n_R n_T$	
	Generation of routes	$n_T(n_T+1)K/2+n_T M_c K$	
	Calculation of squared Euclidean distances	$n_T M_c K$	
K-best-MMSE-SQRD	Sorted QR decomposition of matrix $\underline{\boldsymbol{H}}$	$3(n_T^2-n_T)(n_R+n_T)/2$	1 808($K=4$) 2 208($K=8$)
	Multiplication of $\underline{\boldsymbol{Q}}^H$ to received signal vector	$n_R n_T$	
	Generation of routes	$n_T(n_T+1)K/2+n_T M_c K$	
	Calculation of squared Euclidean distances	$n_T M_c K$	

Tab. 2 Average number of complex float multiplications required by detecting a signal vector ($K=8$)

$(E_b\cdot N_o^{-1})$/dB	RC-K-best	RC-K-best-SQRD	RC-KB-MMSE-SQRD
0	1 278.9	1 469.8	1 827.9
4	1 116.3	1 209.6	1 655.4
8	884.6	1 008.1	1 585.1
12	792.2	934.6	1 581.6
16	712.5	927.7	1 580.4
20	703.8	916.2	1 579.9

5 Conclusion

In this paper, an improved-performance K-best algorithm and reduced-complexity K-best algorithms are proposed, which include KB-MMSE-SQRD, RC-KB-MMSE-SQRD, RC-K-best-SQRD and RC-K-best. All these algorithms can improve the detection performance, or reduce the computational complexity compared to traditional K-best algorithms. Among these algorithms, the RC-KB-MMSE-SQRD is the most promising regarding performance and complexity. The KB-MMSE-SQRD algorithm can easily be applied to other tree search algorithms.

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MIMO 系统中的新型 *K*-best 检测算法

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摘要：针对 *K*-best 检测算法易将最优路径舍去的特点和 *K*-best 检测算法搜索星座图中所有点的特点，提出一种性能改进型 *K*-best 检测算法和几种降低复杂度 *K*-best 检测算法. 性能改进型 *K*-best 检测算法在进行 QR 分解之前对信道矩阵进行最小均方误差（MMSE）滤波，能有效减小最优路径被舍弃的概率，提高算法性能；降低复杂度 *K*-best 检测算法采用类似球形译码检测的方法减少搜索星座图中点的个数. 仿真结果显示，性能改进型 *K*-best 检测算法比基于排序 QR 分解（SQRD）的 *K*-best 检测算法有 1dB 的性能增益. 降低复杂度 *K*-best 检测算法在 $K=4$ 时有性能损失；当 $K=8$ 时，降低复杂度 *K*-best 检测算法和原 *K*-best 检测算法有同样的性能，同时前者比后者需要更少的计算量.

关键词：排序 QR 分解；*K*-best；球形译码；最大似然检测；最小均方误差

中图分类号：TN91