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Achievability of Chong-Motani-Garg relay channel capacity bounds using forward decoding

Zhang Yuan

(National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China)

Abstract: To reduce decoding delay of a communication scheme which is backward-decoding-based and achievable Chong-Motani-Garg capacity bounds, a novel forward-sliding-window decoding-based communication scheme is proposed. In this scheme, if $w=(w_1, w_2)$ is the message to be sent in block b, the relay will decode message w_1 and generate a new message z at the end of block b, and the receiver will decode message w_1 at the end of block $b+1$ and decode message z and w_2 at the end of block $b+2$. Analysis results show that this new communication scheme can achieve the same Chong-Motani-Garg bounds and the decoding delay is only two blocks which is much shorter than that of backward decoding. Therefore, Chong-Motani-Garg bounds can be achieved by a forward decoding-based communication scheme with short decoding delay.

Key words: achievable rate; capacity; backward decoding; forward decoding; mixed strategy; relay channel

The discrete memoryless three-node relay channel denoted by $(S_{X1}\times S_{X2}, p(y_2, y_3 \mid x_1, x_2), S_{Y2}\times S_{Y3})$ consists of a sender $X_1 \in S_{X1}$, a receiver $Y_3 \in S_{Y3}$, a relay sender $X_2 \in S_{X2}$, a relay receiver $Y_2 \in S_{Y2}$, and a family of conditional probability mass functions $p(y_2, y_3 \mid x_1, x_2)$ on $S_{Y2}\times S_{Y3}$, one for each $(x_1, x_2)\in S_{X1}\times S_{X2}$, as illustrated in Fig. 1. For the relay channel, a $(2^{nR}, n)$ code consists of a set of integers $S_W=\{1, 2, \ldots, 2^{nR}\}$, an encoding function that maps each message $w\in S_W$ into a codeword $\boldsymbol{x}_1(w)$ of length n, a set of relay encoding functions $x_2^i = f_i(y_2^1, y_2^2, \ldots, y_2^{i-1})$ for $1\leqslant i\leqslant n$, and a decoding function that maps each received n-sequence $\boldsymbol{y}_3$ into an estimate $\hat{w}(\boldsymbol{y}_3)$. A rate R is achievable if there exists a sequence of $(2^{nR}, n)$ codes with $P_{\text{error}}^{(n)}=\Pr\{\hat{W}\neq W\}\rightarrow 0$ as n approaches infinity, and the supremum over all achievable rates is defined as the channel capacity C.

The relay channel was first introduced by van der Meulen[1-2].

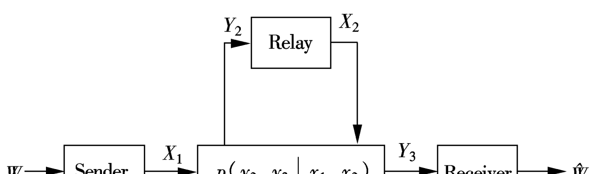

Fig. 1 The three-node relay channel

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Biography: Zhang Yuan (1977—), male, doctor, lecturer, y. zhang@seu.edu.cn.

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Substantial progress was made by Cover and Gamal who developed three classes of coding strategies: the decode-and-forward (DF) strategy, the compress-and-forward (CF) strategy, and the mixed strategy which combines DF and CF[3-4]. Guided by this classification, many different coding and decoding schemes for the relay channel have been developed in the literature and a recent survey on the relay channel can be found in Refs. [5 – 6]. This paper focuses on the mixed strategy for the general discrete memoryless three-node relay channel.

For three-node relay channel using mixed strategy, the capacity remains unknown to date. But some capacity lower bounds have been established in the literature. The first capacity lower bound was proposed by Cover and Gamal in Ref. [3]. Recently, Chong, Motani and Garg proposed two alternative bounds in Ref. [7]. It is shown in Ref. [7] that these two Chong-Motani-Garg bounds always include the Cover-Gamal bound. However, these two better bounds are achieved by a backward decoding scheme[8], which can incur a substantial decoding delay. The contribution of this paper is that, we show that the Chong-Motani-Garg bounds can also be achieved by a forward decoding scheme, whose decoding delay is negligible.

1 Chong-Motani-Garg Lower Bounds

For a three-node relay channel using a mixed strategy, some capacity lower bounds have been established in the literature. The first lower bound was proposed by Cover and Gamal in Ref. [3], which is

$$R_{\text{CG}} = \sup \min\{I(U; Y_2 \mid V, X_2) + I(X_1; \hat{Y}_2, Y_3 \mid U, X_2), \\ I(X_1, X_2; Y_3) - I(Y_2; \hat{Y}_2 \mid U, X_1, X_2, Y_3)\} \tag{1}$$

where the supremum is taken over all joint probability density functions of the form

$$p(u, v, x_1, x_2, y_2, \hat{y}_2, y_3) = p(v)p(u \mid v)p(x_1 \mid u)\cdot \\ p(x_2 \mid v)p(y_2, y_3 \mid x_1, x_2)p(\hat{y}_2 \mid x_2, y_2, u) \tag{2}$$

subject to the constraint

$$I(\hat{Y}_2; Y_2 \mid U, X_2, Y_3) \leqslant I(X_2; Y_3 \mid V) \tag{3}$$

Recently, Chong, Motani, and Garg proposed two alternative lower bounds. The first one is achieved by sequential backward (SeqBack) decoding in Ref. [7], which is

$$R_{\text{CMG}}^{\text{SeqBack}} = \sup \min\{I(U; Y_2 \mid V, X_2) + I(X_1; \hat{Y}_2, Y_3 \mid U, X_2), \\ I(U, V; Y_3) + I(X_1; \hat{Y}_2, Y_3 \mid U, X_2)\} \tag{4}$$

where the supremum is taken over all joint probability density functions of the form (2) subject to the constraint

$$I(\hat{Y}_2; Y_2 \mid U, X_2, Y_3) \leqslant I(X_2; Y_3 \mid U, V) \tag{5}$$

The second one is achieved by simultaneous backward (SimBack) decoding in Ref. [7], which is

$$R_{\mathrm{CMG}}^{\mathrm{SimBack}} = \sup \min\{I(U; Y_2 \mid V, X_2) + I(X_1; \hat{Y}_2, Y_3 \mid U, X_2), \quad I(X_1, X_2; Y_3) - I(Y_2; \hat{Y}_2 \mid U, X_1, X_2, Y_3)\} \tag{6}$$

where the supremum is taken over all joint probability density functions of the form (2) subject to the constraint (5).

It is shown in Ref. [7] that $R_{\mathrm{CMG}}^{\mathrm{SeqBack}}$ and $R_{\mathrm{CMG}}^{\mathrm{SimBack}}$ always include R_{CG} and are therefore two better bounds. However, in Ref. [7], backward decoding is used to prove the achievability of these two better bounds. It is well known that, backward decoding can incur a substantial decoding delay which may be unacceptable in practice.

2 Forward Instead of Backward

Forward decoding does not incur a substantial decoding delay. In this section, we prove that lower bounds $R_{\mathrm{CMG}}^{\mathrm{SeqBack}}$ and $R_{\mathrm{CMG}}^{\mathrm{SimBack}}$ can be achievable under forward instead of backward decoding.

We first consider $R_{\mathrm{CMG}}^{\mathrm{SeqBack}}$ and the main result can be summarized in the following theorem.

Theorem 1 Forward decoding can achieve the capacity lower bound $R_{\mathrm{CMG}}^{\mathrm{SeqBack}}$ for the general discrete memoryless three-node relay channel using mixed strategy.

Proof (Outline) As in Refs. [3, 7], a block Markov coding argument is used. Therefore, we consider B blocks, each of which contains n symbols, and a sequence of messages $w'_b \times w''_b \in [1, 2^{nR'}] \times [1, 2^{nR''}]$, $b = 1, 2, \ldots, B-1$, will be sent over the channel in nB transmissions.

We first generate a codebook, which consists of five steps.

1) Generate $2^{nR'}$ i. i. d. n-sequences $\boldsymbol{v}$ with

$$p(\boldsymbol{v}) = \prod_{i=1}^{n} p(v_i)$$

Label these $\boldsymbol{v}(m)$, $m \in [1, 2^{nR'}]$.

2) For each $\boldsymbol{v}(m)$, generate $2^{nR'}$ i. i. d. n-sequences $\boldsymbol{u}$ with

$$p(\boldsymbol{u} \mid \boldsymbol{v}(m)) = \prod_{i=1}^{n} p(u_i \mid v_i(m))$$

Label these $\boldsymbol{u}(w' \mid m)$, $w' \in [1, 2^{nR'}]$.

3) For each $\boldsymbol{u}(w' \mid m)$, generate $2^{nR''}$ i. i. d. n-sequences $\boldsymbol{x}_1$ with

$$p(\boldsymbol{x}_1 \mid \boldsymbol{u}(w' \mid m)) = \prod_{i=1}^{n} p(x_{1,i} \mid u_i(w' \mid m))$$

Label these $\boldsymbol{x}_1(w'' \mid w', m)$, $w'' \in [1, 2^{nR''}]$.

4) For each $\boldsymbol{v}(m)$, generate $2^{n\hat{R}}$ i. i. d. n-sequences $\boldsymbol{x}_2$ with

$$p(\boldsymbol{x}_2 \mid \boldsymbol{v}(m)) = \prod_{i=1}^{n} p(x_{2,i} \mid v_i(m))$$

Label these $\boldsymbol{x}_2(s \mid m)$, $s \in [1, 2^{n\hat{R}}]$.

5) For each $(\boldsymbol{u}(w' \mid m), \boldsymbol{x}_2(s \mid m))$, generate $2^{n\hat{R}}$ i. i. d. n-sequences $\hat{\boldsymbol{y}}_2$ with

$$p(\hat{\boldsymbol{y}}_2 \mid \boldsymbol{u}(w' \mid m), \boldsymbol{x}_2(s \mid m)) = \prod_{i=1}^{n} p(\hat{y}_{2,i} \mid u_i(w' \mid m), x_{2,i}(s \mid m))$$

Label these $\hat{\boldsymbol{y}}_2(z \mid w', s, m)$, $z \in [1, 2^{n\hat{R}}]$.

Given the generated codebook and the message $w_b = (w'_b, w''_b)$ which is to be sent in block b, the encoding operation consists of two steps.

1) The source sends $\boldsymbol{x}_1(w''_b \mid w'_b, w'_{b-1})$ in block b.

2) The relay at the end of block $b-1$ has an estimate $\hat{w}'_{b-1}$ of w'_{b-1} from decoding step 1) and generates z_{b-1} from decoding step 2), and then sends $\boldsymbol{x}_2(z_{b-1} \mid \hat{w}'_{b-1})$ in block b.

Finally, the decoding operation also consists of five steps:

1) The relay determines the unique $\hat{w}'_b$ such that $(\boldsymbol{u}(\hat{w}'_b \mid w'_{b-1}), \boldsymbol{y}_{2,b}, \boldsymbol{v}(w'_{b-1}), \boldsymbol{x}_2(z_{b-1} \mid w'_{b-1}))$ is jointly ε-typical at the end of block b. For sufficiently large n, $\hat{w}'_b = w'_b$ with high probability if

$$R' < I(U; Y_2 \mid V, X_2) \tag{7}$$

2) The relay determines the unique z_b such that $(\hat{\boldsymbol{y}}_2(z_b \mid w'_b, z_{b-1}, w'_{b-1}), \boldsymbol{y}_{2,b}, \boldsymbol{u}(w'_b \mid w'_{b-1}), \boldsymbol{x}_2(z_{b-1} \mid w'_{b-1}))$ is jointly ε-typical at the end of block b. For sufficiently large n, such a z_b will exist with high probability if

$$\hat{R} > I(\hat{Y}_2; Y_2 \mid U, X_2) \tag{8}$$

3) The receiver determines the unique $\hat{w}'_{b-1}$ such that $(\boldsymbol{v}(\hat{w}'_{b-1}), \boldsymbol{y}_{3,b})$ is jointly ε-typical at the end of block b and $(\boldsymbol{u}(\hat{w}'_{b-1} \mid w'_{b-2}), \boldsymbol{y}_{3,b-1}, \boldsymbol{v}(w'_{b-2}))$ is jointly ε-typical at the end of block $b-1$. For sufficiently large n, $\hat{w}'_{b-1} = w'_{b-1}$ with high probability if

$$R' < I(V; Y_3) + I(U; Y_3 \mid V) \tag{9}$$

4) The receiver determines the unique $\hat{z}_{b-1}$ such that $(\boldsymbol{x}_2(\hat{z}_{b-1} \mid w'_{b-1}), \boldsymbol{y}_{3,b}, \boldsymbol{u}(w'_b \mid w'_{b-1}), \boldsymbol{v}(w'_{b-1}))$ is jointly ε-typical at the end of block $b+1$ and $(\hat{\boldsymbol{y}}_2(\hat{z}_{b-1} \mid w'_{b-1}, z_{b-2}, w'_{b-2}), \boldsymbol{y}_{3,b-1}, \boldsymbol{u}(w'_{b-1} \mid w'_{b-2}), \boldsymbol{x}_2(z_{b-2} \mid w'_{b-2}))$ is jointly ε-typical at the end of block b. For sufficiently large n, $\hat{z}_{b-1} = z_{b-1}$ with high probability if

$$\hat{R} < I(X_2; Y_3 \mid U, V) + I(\hat{Y}_2; Y_3 \mid U, X_2) \tag{10}$$

5) The receiver determines the unique $\hat{w}''_{b-1}$ such that $(\boldsymbol{x}_1(\hat{w}''_{b-1} \mid w'_{b-1}, z_{b-2}), \hat{\boldsymbol{y}}_2(z_{b-1} \mid w'_{b-1}, z_{b-2}, w'_{b-2}), \boldsymbol{y}_{3,b-1}, \boldsymbol{u}(w'_{b-1} \mid w'_{b-2}), \boldsymbol{x}_2(z_{b-2} \mid w'_{b-2}))$ is jointly ε-typical at the end of block $b+1$. For sufficiently large n, $\hat{w}''_{b-1} = w''_{b-1}$ with high probability if

$$R'' < I(X_1; \hat{Y}_2, Y_3 \mid U, X_2) \tag{11}$$

From (7) and (11), we obtain the first term of (4). From (9) and (11), we obtain the second term of (4). From (8) and (10), we obtain the constraint (5).

Remark 1 This proof uses the regular encoding and sliding window forward decoding and does not use the backward decoding technique. Simply speaking, the receiver first uses $\boldsymbol{y}_{3,b-1}$ and $\boldsymbol{y}_{3,b}$ to decode w'_{b-1} at the end of block b, then uses w'_{b-1}, w'_b, $\boldsymbol{y}_{3,b-1}$, and $\boldsymbol{y}_{3,b}$ to decode z_{b-1} at the end of block $b+1$, and finally uses z_{b-1}, w'_{b-1}, and $\boldsymbol{y}_{3,b-1}$ to decode w''_{b-1} at the end of block $b+1$. Therefore, the decoding delays of w'_{b-1} and w''_{b-1} are one block and two blocks, respectively.

Remark 2 The decoding scheme used in this proof is just a small modification of that of Cover and Gamal. Exactly speaking, our proof modifies the decoding order of that of Cover and Gamal. Actually, for the decoding scheme of Cover and Gamal in Ref. [2], considering the usage of irregular encoding, the receiver will first decode partial w'_{b-1} at the end of block b, then decode partial z_{b-1} at the end of block b, then decode complete w'_{b-1} at the end of block b, then decode complete z_{b-1} at the end of block b, and finally decode w''_{b-1} at the end of block b. Therefore, the decoding order of Cover and Gamal is

$$\{\text{partial } w'_{b-1}, \text{partial } z_{b-1}, w'_{b-1}, z_{b-1}, w''_{b-1}\}$$

In our proof, the decoding order has been modified to be

$$\{\text{partial } w'_{b-1}, w'_{b-1}, \text{partial } z_{b-1}, z_{b-1}, w''_{b-1}\}$$

Considering the usage of regular encoding, the actual decoding order used in our proof is

$$\{w'_{b-1}, z_{b-1}, w''_{b-1}\}$$

After modifying the decoding order, the achievable rate can be increased from R_{CG} to $R_{\mathrm{CMG}}^{\mathrm{SeqBack}}$ at the cost of one more block decoding delay.

Next consider $R_{\mathrm{CMG}}^{\mathrm{SimBack}}$, and the main result can be summarized in the following theorem.

Theorem 2 Forward decoding can achieve the capacity lower bound $R_{\mathrm{CMG}}^{\mathrm{SimBack}}$ for the general discrete memoryless three-node relay channel using mixed strategy.

Proof The achievability of $R_{\mathrm{CMG}}^{\mathrm{SimBack}}$ is a simple corollary of the achievability of $R_{\mathrm{CMG}}^{\mathrm{SeqBack}}$. The codebook generations, encoding and decoding are exactly the same as in the last proof. Therefore, we obtain the first term of (6) from (7) and (11), and obtain the constraint (5) from (8) and (10). Finally, from (9) and (11), the rate of transmission from sender to receiver is bounded by

$$\begin{aligned} I(U,V;Y_3)+I(X_1;\hat{Y}_2,Y_3\mid U,X_2) &= I(U,V;Y_3)+ \\ &I(X_1;Y_3\mid U,X_2)+I(X_1;\hat{Y}_2\mid U,X_2,Y_3)= \\ &I(X_1,X_2;Y_3)-I(X_2;Y_3\mid U,V)+I(X_1;\hat{Y}_2\mid U,X_2,Y_3) \end{aligned}$$

Substituting the constraint $I(\hat{Y}_2;Y_2\mid U,X_2,Y_3)\leqslant I(X_2;Y_3\mid U,V)$ from (5) and considering $I(\hat{Y}_2;Y_2\mid U,X_2,Y_3)=I(X_1;\hat{Y}_2\mid U,X_2,Y_3)+I(Y_2;\hat{Y}_2\mid U,X_1,X_2,Y_3)$, we obtain the second term of (6).

3 Conclusion

This paper studies the Chong-Motani-Garg capacity lower bounds of the general three-node relay channel. In the literature, the Chong-Motani-Garg bounds are achieved under backward decoding, which can cause long decoding delay. In this paper, we propose a forward sliding window decoding scheme to achieve the same bounds with negligible decoding delay. Additionally, the decoding complexity of our forward scheme is comparable with that of the backward scheme.

References

[1] Meulen E C. Three-terminal communication channels [J]. *Advances in Applied Probability*, 1971, **3**(3): 120 − 154.

[2] Meulen E C. A survey of multiway channels in information theory: 1961-1976 [J]. *IEEE Transactions on Information Theory*, 1977, **23**(1): 1 − 37.

[3] Cover T M, Gamal A A. Capacity theorems for the relay channel [J]. *IEEE Transactions on Information Theory*, 1979, **25**(5): 572 − 584.

[4] Cover T M, Thomas J A. *Elements of information theory* [M]. New York: Wiley, 1991.

[5] Xie L, Kumar P R. An achievable rate for the multiple-level relay channel [J]. *IEEE Transactions on Information Theory*, 2005, **51**(4): 1348 − 1358.

[6] Kramer G, Gastpar M, Gupta P. Cooperative strategies and capacity theorems for relay networks[J]. *IEEE Transactions on Information Theory*, 2005, **51**(9): 3037 − 3063.

[7] Chong H, Motani M, Garg H K. Generalized backward decoding strategies for the relay channel[J]. *IEEE Transactions on Information Theory*, 2007, **53**(1): 394 − 401.

[8] Zeng C, Kuhlmann F, Buzo A. Achievability proof of some multiuser channel coding theorems using backward decoding [J]. *IEEE Transactions on Information Theory*, 1989, **35**(6): 1160 − 1165.

中继信道 Chong-Motani-Garg 容量界的前向解码可达性

张　源

（东南大学移动通信国家重点实验室，南京 210096）

摘要：为了解决基于后向解码的三节点中继信道 Chong-Motani-Garg 容量界可达通信方案的解码延时过长问题，提出了一种新型的基于前向解码的三节点中继信道通信方案. 在该方案中，假定在第 b 个分组中的发送消息为 $w=(w_1,w_2)$，则中继需要在第 b 个分组结束时对消息 w_1 进行解码并生成一个新的消息 z，而接收机需要在第 $b+1$ 个分组结束时对消息 w_1 进行解码，然后在第 $b+2$ 个分组结束时对消息 z 和 w_2 进行解码. 分析结果表明，该通信方案也可以达到 Chong-Motani-Garg 容量界，并且该方案的解码延时只有 2 个分组长度，大大短于基于后向解码的通信方案的解码延时. 因此，三节点中继信道 Chong-Motani-Garg 容量界可以通过基于解码延时短的前向解码的通信方案达到.

关键词：可达速率；容量；后向解码；前向解码；混合策略；中继信道

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