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Analysis of substrate eddy effects and distribution effects in silicon-based inductor model

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Abstract: The concepts of substrate eddy influence factor and distribution-effects-occurring frequency are presented. The effects of substrate resistivity and inductor spiral length on the substrate eddy and distribution effects are captured. The substrate eddy influence factors of an inductor (6 turn, 3 060 μm in length) fabricated on low (1 Ω·cm) and high resistivity (1 000 Ω·cm) silicon substrates are 0.3 and 0.04, and the distribution-effects-occurring frequencies are 1.8 GHz and 14.5 GHz, respectively. The measurement results show that the equivalent circuit model of the inductor on low resistivity silicon must take into consideration substrate eddy effects and distribution effects. However, the circuit model of the inductor on high resistivity silicon cannot take into account the substrate eddy effects and the distribution effects at the frequencies of interest. Its simple model shows agreement with the measurements, and the contrast is within 7%.

Key words: planar spiral inductors; substrate eddy effects; distribution effects; equivalent circuit model

In recent years, the low cost of Si IC fabrication and the potential for integration with baseband circuits make Si be a better choice in RF IC[1-2]. An inductor is an important passive component for turning, matching and filtering. Nowadays, the need for fast and accurate inductor models has become apparent. Unfortunately, various parasitic effects and numerous loss mechanisms in RF inductors make model building difficult [3-5]. In order to establish a model of distribution effects and substrate eddy effects, a model with a complex structure and many components is necessary.

In fact, because of the difference in inductor layouts and substrate resistivity, the forms of the models are not the same. In this paper, the substrate eddy effects and the distribution effects are investigated by theoretical analysis and an electromagnetic simulator (HFSS). The concepts of substrate eddy influence factors and distribution-effects-occurring frequencies are presented, and they can be used for inductor model selection. The accuracy of the model selection method is proved by experiment.

1 Substrate Eddy Effects and Distribution Effects

1.1 Substrate eddy effects

The substrate loss is a major loss of an inductor at very high frequencies[6]. Fig. 1 shows two types of substrate loss. The electric field penetrates into the substrate leading to the displacement current, which is responsible for the electric (capacitive) substrate loss. By Lenz's law, while the time-varying RF current flows into the inductor, an opposite image current (substrate eddy current) is induced in the substrate to resist a change in the field.

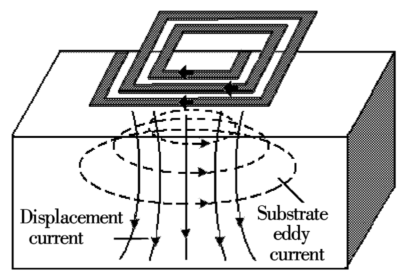

Fig. 1 The illustration of substrate loss

A popular model for an inductor is shown in Fig. 2(a). The ladder circuit (L_0, L_1, R_0 and R_1) is used to model the skin and proximity effects[7-8]. C_{is} represents the oxide capacitance. R_{si} and C_{si} are the substrate resistance and capacitance to ground. C_s models the ports' capacitive coupling of the inductor. C_{is}-R_{si}-C_{si} represents the displacement current

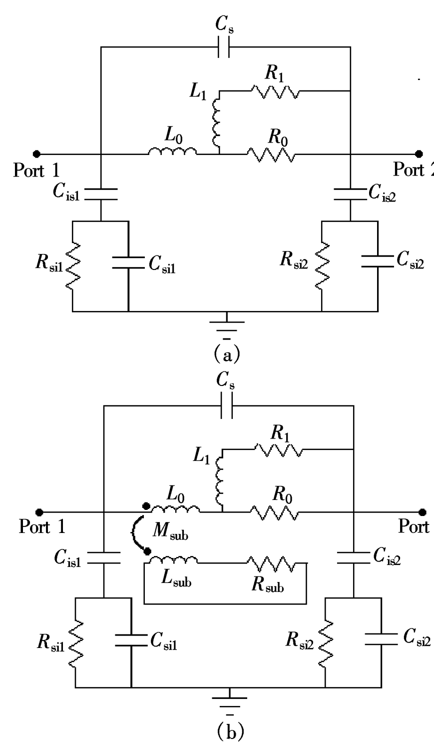

Fig. 2 The equivalent circuit of inductors. (a) Without substrate eddy effects; (b) With substrate eddy effects

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loss in the substrate. The substrate eddy effects are modeled by a transformer as indicated in Fig. 2(b). M_{sub} is the mutual inductance between the inductor and the substrate eddy current. R_{sub} and L_{sub} are the resistance and inductance associated with eddy current flow. According to the circuit relationship of the inductor coil (L_s and R_s) and coupled transformer loop (L_{sub}, R_{sub} and M_{sub}), we can obtain

$$V = I_1(R_s + j\omega L_s) + j\omega M_{sub} I_2 \tag{1}$$

$$0 = j\omega M_{sub} I_1 + I_2(R_{sub} + j\omega L_{sub}) \tag{2}$$

where R_s and L_s are the resistance and the inductance of the ladder structure. The actual impedance of the inductor coil considering substrate eddy effects is derived from Eq. (1) and Eq. (2) as

$$Z_{in} = \frac{V}{I_1} = \left(R_s + \frac{R_{sub}\omega^2 M_{sub}^2}{R_{sub}^2 + \omega^2 L_{sub}^2}\right) + j\omega\left(L_s - \frac{L_{sub}\omega^2 M_{sub}^2}{R_{sub}^2 + \omega^2 L_{sub}^2}\right) \tag{3}$$

As a result, the effective series inductance L_{eff} and resistance R_{eff} in the model shown in Fig. 2(b) are demonstrated by

$$L_{eff} = L_s - L_{sub}\left(\frac{\omega^2 M_{sub}^2}{R_{sub}^2 + \omega^2 L_{sub}^2}\right) \tag{4}$$

$$R_{eff} = R_s + R_{sub}\left(\frac{\omega^2 M_{sub}^2}{R_{sub}^2 + \omega^2 L_{sub}^2}\right) \tag{5}$$

Eq. (4) and Eq. (5) can be approximated as

$$L_{eff} \approx L_s \tag{6}$$

$$R_{eff} \approx R_s + \frac{\omega^2 M_{sub}^2}{R_{sub}} \tag{7}$$

From Eq. (6) and Eq. (7), the primary effects of the substrate eddy effects lead to an increase in effective series resistance, and the decrease of the inductance is slight.

1.2 Distribution effects

Some deviations are observed in the measurement and simulation of inductors. As shown in Fig. 3(a), the HFSS simulation shows that R_{eff} is equal to the DC resistance at first, and then rises with frequencies, but after R_{eff} reaches a maximum value, it begins to drop until reaching a negative value. This phenomenon is due to the lumped nature of capacitive substrate coupling that overrides some other effects, and can only be explained and modeled by the distributed topology model. A 2-Π distributed model (see Fig. 4) is proposed in Ref. [4]. The effective resistance of the distributed model can be obtained by[4]

$$R_{eff} = R_0 + R_1 - \frac{\omega^2 L_0 L_2}{R_{si}} \tag{8}$$

From Eq. (8), R_{eff} can reach a negative value at high enough frequencies.

As shown in Fig. 3(b), the single-Π models of the inductor only reflect the increase in R_{eff} with frequencies, and the distributed model can reflect the decrease with higher frequencies.

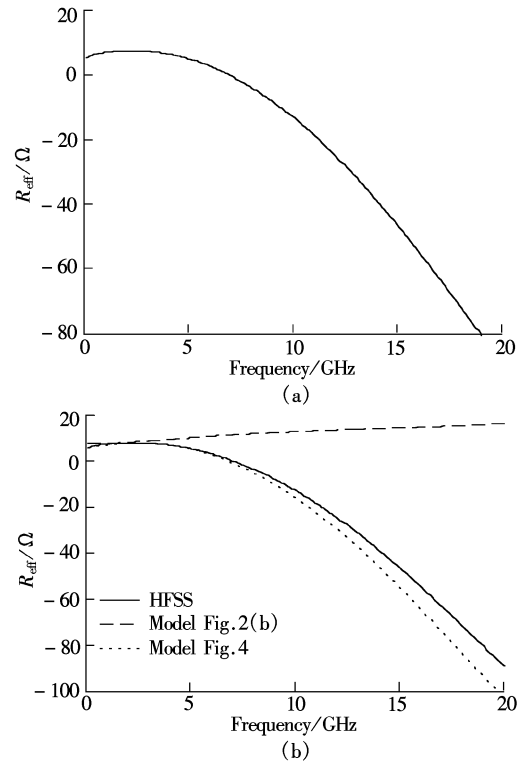

Fig. 3 Influence of distribution effects. (a) Simulated R_{eff} with HFSS; (b) Comparison of simulated R_{eff} with two models

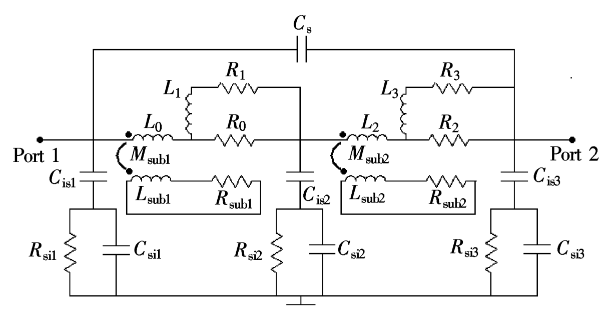

Fig. 4 The equivalent circuit of inductors with substrate eddy effects and distribution effects

2 Distribution-Effects-Occurring Frequency and Substrate Eddy Influence Factor

The inductor model with substrate eddy and distribution effects (see Fig. 4) is somewhat complicated and has a large number of elements. In fact, because of the difference between inductor layouts and substrate resistivities, inductor models are not the same. The concepts of the substrate eddy influence factor and the distribution-effects-occurring frequency are presented in this section, and they can be used for inductor model selection.

2.1 Distribution-effects-occurring frequency

The distribution-effects-occurring frequency f_c is defined as the frequency at which R_{eff} reaches the maximum and will start to decrease rapidly. More than 200 square spiral inductors with different layouts and substrate resistivities are simulated by HFSS to obtain various f_c. The simulation shows that f_c is closely related to the inductor length and substrate resistivity. The substrate thickness and the type and thickness of the dielectric layer impact f_c so slightly that they can be ignored. Fig. 5 shows f_c as a function of substrate resistivities and inductor lengths. The expression of f_c is obtained by data-fitting techniques

$$f_c = \beta l_{total}^{\alpha_1} \rho_{si}^{\alpha_2} \tag{9}$$

where f_c is the distribution-effects-occurring frequency (GHz), l_{total} is the total length of the inductor (μm), ρ_{si} is the resistivity of substrate (Ω·cm). The coefficients β, α_1 and α_2 are layout dependent. For the square spiral inductor, they are 1 284.7, −0.86 and 0.35, respectively.

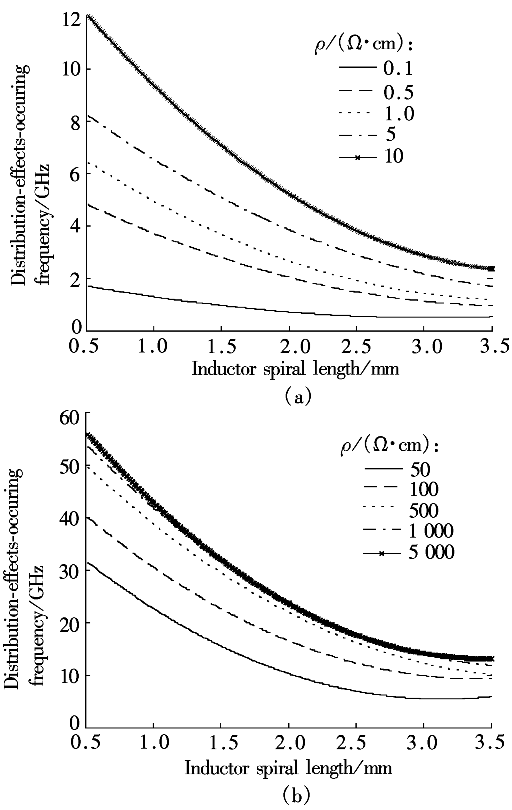

Fig. 5 Distribution-effects-occurring frequency f_c vs. ρ_{si} and l_{total}. (a) On low resistivity silicon; (b) On high resistivity silicon

According to Fig. 5 and Eq. (9), for a large area inductor on low resistivity substrate, f_c is low, and the distribution effects must be considered. Contrarily, f_c is high for a small area inductor on high resistivity substrate, and the distribution effects can be omitted at certain frequencies.

2.2 Substrate eddy influence factor

As discussed above, the primary effect of substrate eddy effects is an increase in R_{eff}, while the distribution effects override other effects very soon, which makes the substrate eddy effects incalculable. It is certain that the substrate eddy effects have the greatest impact on R_{eff} at f_c. The ratio of the increased resistance caused by substrate eddy effects R_{eddy} to R_{eff} is defined as the substrate eddy influence factor θ_{eddy} at f_c,

$$\theta_{eddy} = \frac{R_{eddy}}{R_{eff}} \tag{10}$$

From Eq. (7), Eq. (10) can be simplified as

$$\theta_{eddy} = \left. \frac{1}{1 + \frac{R_s R_{sub}}{\omega^2 M_{sub}^2}} \right|_{\omega = 2\pi f_c} \tag{11}$$

If $R_s(f_c)$, $R_{sub}(f_c)$ and $M_{sub}(f_c)$ are obtained, θ_{eddy} can be calculated. $R_s(f_c)$ is described in Refs. [7−9]. R_{sub} and M_{sub} are dependent on frequency, substrate characters and the parameters of the inductor. R_{sub} can be expressed as

$$R_{sub} = \frac{l \rho_{si}}{\alpha w t_{eff}} \tag{12}$$

where l and w represent the length and the width of the inductor. The area that the eddy current occupies is larger than the metal line area, which is represented by α factor[3], and in this paper, α is 1.42. The effective thickness of the substrate t_{eff} is given by

$$t_{eff} = \delta_{sub} \left[1 - \exp\left(-\frac{t_{sub}}{\delta_{sub}} \right) \right] \tag{13}$$

where t_{sub} is the thickness of the substrate, and δ_{sub} is the skin depth of the substrate,

$$\delta_{sub} = \sqrt{\frac{2\rho_{si}}{\omega \mu_{si} \mu_0}} \tag{14}$$

By the transformer loop formed by L_0, L_1, R_0, R_1, L_{sub}, R_{sub} and M_{sub}, we can obtain

$$\frac{L_s}{L_{sub}} = \frac{-jk\omega M_{sub} R_{sub} + k\omega^2 M_{sub} L_{sub}}{R_{sub}^2 + (\omega L_{sub})^2} \approx \frac{k\omega^2 M_{sub} L_{sub}}{R_{sub}^2 + (\omega L_{sub})^2} \tag{15}$$

where k is the coupling coefficient between the inductor and the substrate eddy current,

$$k = \left(\frac{r^2}{r^2 + h^2} \right)^{3/2} \tag{16}$$

where r is the average diameter of the spiral, h is the geometric mean distance between the inductor and the substrate eddy current, which is approximately $0.5t_{eff}$. M_{sub} is given as

$$M_{sub} = k\sqrt{L_{sub} L_s} \tag{17}$$

In Eq. (11), R_{sub} is far larger than R_s and $j\omega M_{sub}$, and it is the most important influencing factor of θ_{eddy}. As expressed in Eq. (12), R_{sub}, R_s and M_{sub} are all dependent on inductor layouts, so the influence of inductor layouts on θ_{eddy} is eliminated. Based on the discussion and expression, a large series of square inductors with different layouts and substrate resistivities are calculated. Fig. 6 shows the relationship between θ_{eddy} and substrate resistivities.

As shown in Fig. 6, if substrate resistivity is low, θ_{eddy} is large, and substrate eddy effects must be considered in models. For high resistivity substrate, θ_{eddy} is very small. Substrate eddy effects can be omitted in models.

3 Experiment Verifications

θ_{eddy} and f_c are presented in section 2, and they can be used

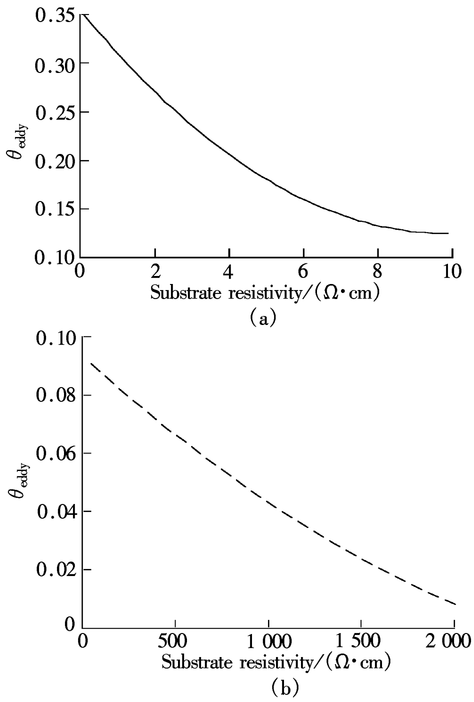

Fig. 6 Substrate eddy influence factor θ_{eddy} vs. ρ_{si}. (a) On low resistivity silicon; (b) On high resistivity silicon

for model selection. In order to verify the accuracy of the methods, inductors are fabricated on two different silicon wafers, with resistivities of 1 Ω·cm and 1 000 Ω·cm. The fabrication process uses two-metal (Al). Fig. 7 shows the SEM photographs of inductors. Two-port S-parameters are measured using an HP8719ES network analyzer and Cascade Microtech GSG probes. The pad parasitic influence is deembedded using an open pad structure. The effective inductance L_{11} and quality factor Q can be extracted by[10-11]

$$L_{11} = \frac{\mathrm{Im}(-1/Y_{11})}{2\pi f} \tag{18}$$

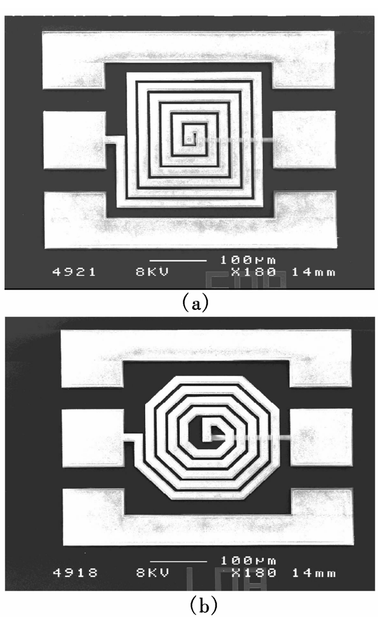

Fig. 7 SEM photographs of planar spiral inductors. (a) Square structure; (b) Octagon structure

$$Q = -\frac{\mathrm{Im}(Y_{11})}{\mathrm{Re}(Y_{11})} \tag{19}$$

Take a 4.8 nH inductor for example. Its width is 12 μm, space is 6 μm, length is 3 060 μm and the metal thickness is 2 μm. If the inductor is fabricated on low resistivity silicon, its f_c is 1.8 GHz, and θ_{eddy} is 0.3, which means that the resistance caused by the substrate eddy effects accounts for nearly 30% in total resistance at 1.8 GHz. So, the model of the inductor should include both distribution effects and substrate eddy effects due to low f_c and high θ_{eddy}.

The single-Π and the distributed 2-Π model of the inductor on low resistivity silicon are shown in Fig. 2(b) and Fig. 4. The model parameters can be calculated or extracted by the methods in Refs. [7-9]. Fig. 8 shows the comparison of measured and simulated S-parameters, L_{11}, and Q of the in-

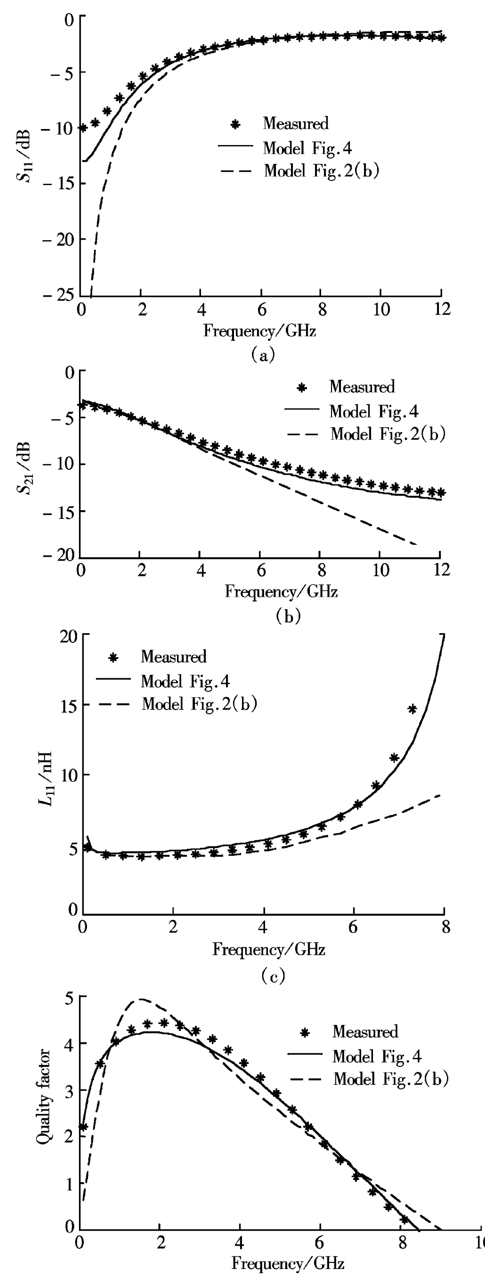

Fig. 8 Comparison of measured and simulated parameters on low resistivity silicon. (a) S_{11}; (b) S_{21}; (c) L_{11}; (d) Q

ductor. It can be seen that the simulation results of distributed model match the measured data well, but the errors of the single-Π model are large.

If the inductor is fabricated on high resistivity silicon, its f_c is 14.5 GHz, and θ_{eddy} is 0.04, which means that the resistance caused by substrate eddy effects accounts for only 4% mostly in total resistance. So, due to high f_c and low θ_{eddy}, the models of the inductor do not include both distribution effects and substrate eddy effects below 14.5 GHz.

The simple single-Π and distributed 2-Π models of the inductor on high resistivity silicon are shown in Fig. 2(a) and Fig. 4. Fig. 9 shows the comparison of measured and simulated results. It can be seen that the contrast between the simple single-Π model and the measurements is both within 7%. The simple single-Π model and the distributed 2-Π model are almost equivalent for inductors on high resistivity silicon.

As discussed above, the model selection method using f_c and θ_{eddy} is scalable and convenient. In a word, the simple single-Π model is valid for small inductors on high resistivity silicon and the distributed 2-Π model is valid for large inductors on low resistivity silicon.

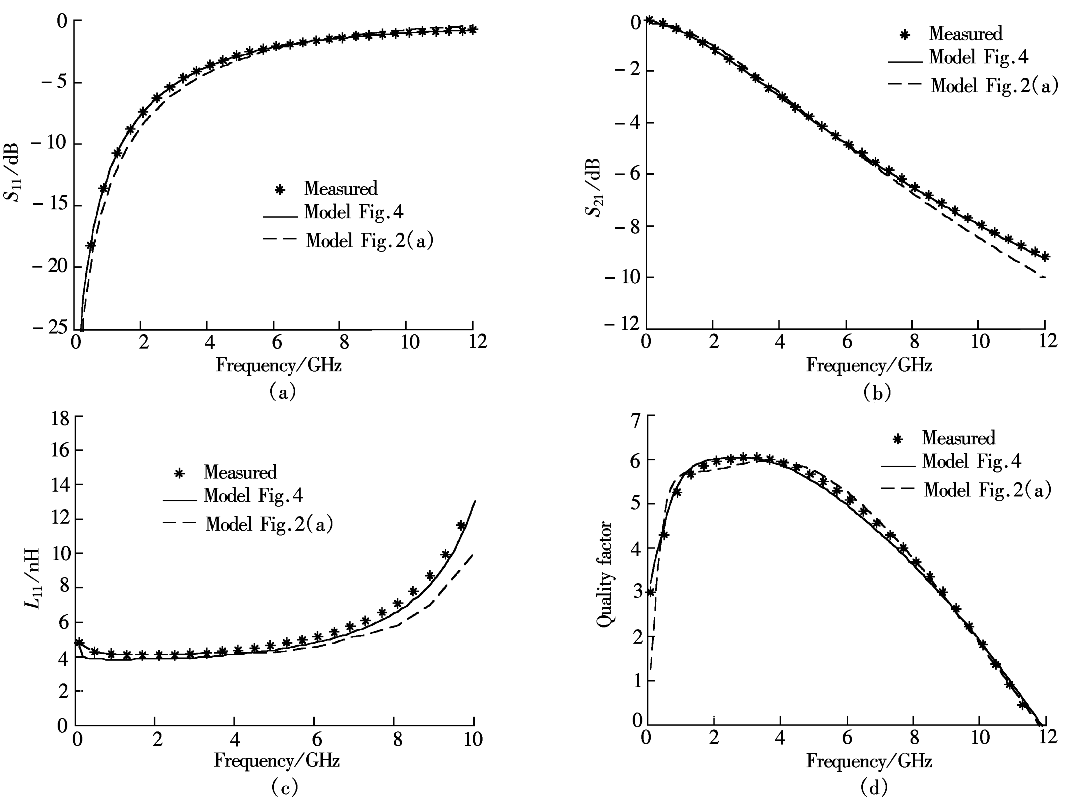

Fig. 9 Comparison of measured and simulated parameters on high resistivity silicon. (a) S_{11}; (b) S_{21}; (c) L_{11}; (d) Q

4 Conclusion

In this paper, the substrate eddy effects and the distribution effects are investigated by theoretical analysis and electromagnetic simulators. The concepts of θ_{eddy} and f_c are presented, and they can be used for model selection. The accuracy of the method is proved by experiment. The work in this paper is practical and useful for RF inductor design.

References

[1] Lakdawala Hasnain, Zhu Xu, Luo Hao, et al. Micromachined high-Q inductors in a 0.18 μm copper interconnect low-k dielectric CMOS process [J]. *IEEE Journal of Solid-State Circuits*, 2002, **37**(3): 394 - 403.

[2] Burghartz J N, Edelstein D C, Jenkins K A, et al. Monolithic spiral inductors fabricated using a VLSI Cu-damascene interconnect technology and low-loss substrates [C]//*Technical Digest: International Electron Devices Meeting*. San Francisco, CA, USA, 1996: 99 - 102.

[3] Huo X, Chan Philip C H, Chen Kevin J, et al. A physical model for on-chip spiral inductors with accurate substrate modeling [J]. *IEEE Transactions on Electron Devices*, 2006, **53**(12): 2942 - 2949.

[4] Gao Wei, Yu Zhiping. Scalable compact circuit model and synthesis for RF CMOS spiral inductors [J]. *IEEE Transactions on Microwave Theory and Technique*, 2006, **54**(3): 1055 - 1064.

[5] Rotella Francis M, Blaschke Volker, Howard David. A broad-band scalable lumped-element inductor model using analytic expressions to incorporate skin effect, substrate loss, and proximity effect [C]//*Technical Digest: International Electron Devices Meeting*. San Francisco, CA, USA, 2002: 471 - 474.

[6] Kuhn William B, Yanduru Naveen K. Spiral inductor substrate loss modeling in silicon RFICs [J]. *Microwave Journal*, 1999, **42**(3): 66 - 81.

[7] Yue C Patrick, Wong S Simon. Physical modeling of spiral inductors on silicon [J]. *IEEE Transactions on Electron Devices*, 2000, **47**(3): 560 - 568.

[8] Kim S, Neikirk D P. Compact equivalent circuit model for the skin effect [C]//*IEEE MTT-S International Microwave Symposium Digest*. San Francisco, CA, USA, 1996, **3**: 1815 - 1818.

[9] Cao Yu, Groves Robert A, Huang Xuejue, et al. Frequency-independent equivalent-circuit model for on-chip spiral in-

ductors [J]. *IEEE Journal of Solid-State Circuits*, 2003, **38**(3):419－426.

[10] Niknejad Ali M, Meyer Robert G. Analysis, design, and optimization of spiral inductors and transformers for Si RF IC's [J]. *IEEE Journal of Solid-State Circuits*, 1998, **33**(10):1470－1481.

[11] Koutsoyannopoulos Yorgos K, Papananos Yannis. Systematic analysis and modeling of integrated inductors and transformers in RF IC design [J]. *IEEE Transactions on Circuits and Systems* Ⅱ: *Analog and Digital Signal Processing*, 2000, **47**(8):699－713.

硅基电感模型中衬底涡流效应和分布效应分析

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摘要:提出了衬底涡流影响因子和分布效应发生频率的概念,该概念能够准确地反映衬底涡流效应和分布效应与衬底电阻率、电感线圈长度等参数的关系. 一个制作在低阻硅和高阻硅衬底上的 6 圈,线圈长度为 3 060 μm 的电感,其衬底涡流影响因子分别为 0.3 和 0.04,分布效应发生频率分别为 1.5 GHz 和 14.5 GHz. 实验表明,低阻硅上该电感的等效电路模型必须包含衬底涡流效应和分布效应;而高阻硅上该电感的等效电路模型可以不包含衬底涡流效应和分布效应(测试范围为 0.1～12 GHz),其简化等效电路模型仍具有较高的精度,误差在 7% 之内.

关键词:平面电感;衬底涡流效应;分布效应;等效电路模型

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