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Multi-phase signal setting and capacity of signalized intersection

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Abstract: A capacity model of multi-phase signalized intersections is derived by a stopping-line method. It is simplified with two normal situations: one situation involves one straight lane and one left-turn lane; the other situation involves two straight lanes and one left-turn lane. The results show that the capacity is mainly relative to signal cycle length, phase length, intersection layout and following time. With regard to the vehicles arrival rates, the optimal model is derived based on each phase's remaining time balance, and it is solved by Lagrange multipliers. Therefore, the calculation models of the optimal signal cycle length and phase lengths are derived and simplified. Compared to the existing models, the proposed model is more convenient and practical. Finally, a practical intersection is chosen and its signal cycles and phase lengths are calculated by the proposed model.

Key words: traffic engineering; intersection; multi-phase signal timing; Lagrange multipliers; optimization

With the sustained and rapid development of the national economy, urban automobile possession, traffic volume and traffic demand are increasing drastically. Traffic congestion of different degrees appears universally in many metropolitans. As the joint of road networks, intersections converge traffic flows from different directions. Due to the red light time loss and mixed flows, intersection capacity is far lower than road capacity. As a result, the intersection becomes a bottleneck for excessive traffic flows from the roads and the sectors of high accident occurrences in urban road networks. In order to assure the traffic security of intersections and make full use of intersection capacity, it is an important measure to operate scientific management and control at intersections.

At present, the intersection traffic control in the cities of China mostly applies a signal control method, either a two-phase signal control method or a multi-phase signal control method. The multi-phase signal control method is a unified name of the control method in which signal phases are more than two. It separates traffic flow in time, decreases the traffic conflict spots at intersections, and improves traffic order and security when vehicles and pedestrians pass intersections. Zhou et al. [1] compared the multi-phase signal control method with the two-phase signal control method with regard to traffic conflict, capacity, service levels and so on. They held that the former is one of the effective ways to improve intersection security and service levels. Xu et al. [2] took a typical four-leg signalized intersection as an example. They calculated the intersection capacity by three common computing methods: the crashing point method, the stop line method, and the American method, and they compared the result with the measured capacity. The results indicate that the results computed by the crashing point method most closely approximates the measured results, while the other two results computed by the stop line method and the America method are both a bit greater. So the crashing point method is considered to be the most suitable method to compute the capacity under traffic conditions of mixed-traffic-flow characteristics in China.

Previously, the British TRLL(Traffic Research Laboratory of London) method is applied to design intersection signal timing[3-4], which is based on the Webster delay model. It determines the optimum cycle length by making delay least, and fixes the green interval according to the key traffic flow during each phase. Now, the method proposed by Shanghai Project Constructing Criterion is frequently used for signal phase setting[5-6]. Yang et al. [7] set up two object optimal models to calculate the signal cycle lengths by minimizing the delay and queuing vehicles as the objective functions and the saturation degree as the restrictive condition. Zhang et al. [8] set up the delay-capacity ally optimal model by taking the division between delay and capacity as the objective function.

1 Capacity of Multi-Phase Signalized Intersection

Considering a cross signalized intersection, there are four different inlet roads i, $i=1, 2, 3, 4$. It is assumed that every inlet road has special left-turn and straight signals. For a signalized intersection, the opposite inlet roads often apply the same signal phase; that is, the signal phase of inlet road 1 and inlet road 3 are the same. And the signal phases of inlet road 2 and inlet road 4 are the same. To simplify calculations, inlet road 1 and inlet road 2 are taken as the representatives. Let the total signal cycle length be recorded as L, the green time of inlet road i from direction j as l_{ij}($i=1, 2; j=1$ is straight, $j=2$ is left turn) and the yellow time as c_{ij}. When the vehicle flow of inlet road i from direction j passes the intersection stopping line, let the time spent by the first vehicle be recorded as t_{cij} and the time spent by the following vehicle as t_{fij}. Then the following formulae can be obtained:

$$\sum_{i,j=1}^{2}(l_{ij}+c_{ij}) = L \tag{1}$$

Let $c=\sum_{i,j=1}^{2} c_{ij}$, then $\sum_{i,j=1}^{2} l_{ij}+c=L$.

The yellow interval c_{ij} is between 2 s and 4 s. Parameter c is the sum of the yellow time in one cycle. It is a constant related to the intersection geometry, which mainly depends on the intersection size and the lane setting method.

If the number of the lanes of inlet road i from direction j is recorded as m_{ij}, then the number of the vehicles of the in-

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let road from the direction in one signal cycle can be calculated by

$$s_{ij} = m_{ij}\left(\frac{l_{ij} - t_{cij}}{t_{fij}} + 1\right) = m_{ij}\left(\frac{l_{ij}}{t_{fij}} - \frac{t_{cij} - t_{fij}}{t_{fij}}\right)$$

The total number of vehicles that pass the intersection in one cycle can be calculated by

$$S = \sum_{i,j=1}^{4} s_{ij} = 2\sum_{i,j=1}^{2} s_{ij} = 2\sum_{i,j=1}^{2} m_{ij}\left(\frac{l_{ij} - t_{cij}}{t_{fij}} + 1\right) = 2\sum_{i,j=1}^{2} m_{ij}\left(\frac{l_{ij}}{t_{fij}} - \frac{t_{cij} - t_{fij}}{t_{fij}}\right)$$

Then the number of vehicles that can pass the intersection in unit time, i. e., the capacity of the intersection, can be calculated by

$$N = \frac{S}{L} = 2\sum_{i,j=1}^{2} \frac{m_{ij}}{L}\left(\frac{l_{ij}}{t_{fij}} - \frac{t_{cij} - t_{fij}}{t_{fij}}\right) \qquad (2)$$

2 Distribution of Headway at Intersections

When the signal light changes from red to green, the first vehicle in the queue requires some reaction time and acceleration time. Therefore, the headway of the first vehicle is the longest and those of the following vehicles decrease one by one. But those behind the fourth vehicles are basically the same. The headways of vehicles at different positions in the queue are approximately shown in Fig. 1.

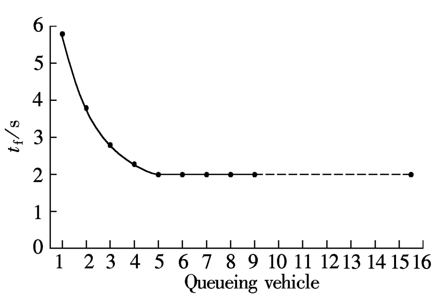

Fig. 1 Gap distribution of the following car at different positions

Based on the measured results, the average headway of the first vehicle is about 5 s, the average headway of the vehicles behind the fourth is about 2 s, and the excessive headway of the front vehicles is about 6 s. To simplify calculations, the excessive headway of the front vehicles is all recorded as the headway of the first vehicle. Therefore, when the queuing vehicles pass the intersection after the signal light changes from red to green, the headway of the first vehicle is about 8 s and the headway of the following vehicle is about 2 s. In addition, there is little difference among the headways of straight, left-turn and right-turn vehicles. Therefore, t_{cij} can be recorded as 8 s and t_{fij} can be recorded as 2 s in practical calculations.

3 Two Types of Simplified Situations

3.1 One straight lane and one left-turn lane at every inlet road

One of the simplified situations is discussed as follows. It is assumed that the grade and the shape of the two crossing roads of the intersection are basically the same. And there are only one straight lane and one left-turn lane at every inlet road. Supposing that the following car headway t_{fi1} ($i = 1, 2, 3, 4$) of the straight vehicle flow at every inlet road of the intersection is the same which is recorded as t_{f1}; and t_{fi2} ($i = 1, 2, 3, 4$) of the left-turn is also the same, which is recorded as t_{f2}, Eq. (2) can be simplified as

$$N = 2\sum_{i,j=1}^{2} \frac{1}{L}\left(\frac{l_{ij}}{t_{fij}} - \frac{t_{cij} - t_{fij}}{t_{fij}}\right) = \frac{2}{L}\left(\sum_{j=1}^{2} \frac{1}{t_{fj}}\sum_{i=1}^{2} l_{ij} - \sum_{i,j=1}^{2} \frac{t_{cij} - t_{fij}}{t_{fij}}\right) \qquad (3)$$

Let $\alpha_{ij} = \frac{t_{cij} - t_{fij}}{t_{fij}}$, then $\alpha = \sum_{i,j=1}^{2} \alpha_{ij} = \sum_{i,j=1}^{2} \frac{t_{cij} - t_{fij}}{t_{fij}}$.

Parameter α is determined by geometrical features of the intersection and vehicle performance. Then the intersection capacity is

$$N = \frac{2}{L}\left(\sum_{j=1}^{2} \frac{1}{t_{fj}}\sum_{i=1}^{2} l_{ij} - \alpha\right) \qquad (4)$$

According to Eq. (1), if the following car headway of the straight lane vehicle is approximate to that of the left-turn lane vehicle (that is, $t_{f2} = t_{f2} = t_2$), then Eq. (4) is simplified as

$$N = \frac{2}{L}\left(\frac{1}{t_f}\sum_{i,j=1}^{2} l_{ij} - \alpha\right) = \frac{2}{L}\left(\frac{L - c}{t_f} - \alpha\right) = 2\left(\frac{1}{t_f} - \frac{1}{L}\left(\frac{c}{t_f} + \alpha\right)\right) \qquad (5)$$

Eq. (5) indicates that the capacity of the signalized intersection is related to the signal cycle length and it is the reciprocal of cycle length L. That is, the capacity is larger for the longer cycle length L. But when the signal cycle length reaches a certain degree, a further increase of the signal cycle length does not affect the capacity much, a capacity N is limited to $2/t_f$ when L is limited to ∞.

If the yellow interval c_{ij} is taken as 2 s, then c is 8 s. And when t_{c1}, t_{c2}, t_{f1} and t_{f2} are taken as 8, 7, 2, 2.5 s, respectively, Eq. (5) can be simplified as

$$N = \left(1 - \frac{30}{L}\right)(\text{pcu/s}) \qquad (6)$$

Accordingly, the relationship between the intersection capacity N and the cycle length L is shown in Fig. 2.

It can be seen from Fig. 2 that the intersection capacity increases rapidly with the increase in L when the cycle length

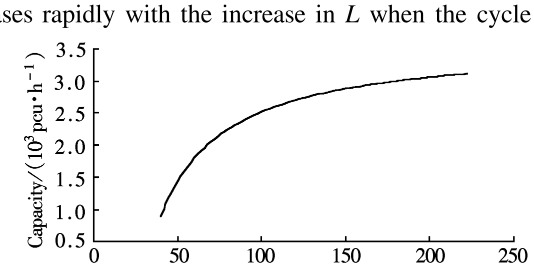

Fig. 2 Relationship between intersection capacity and phase length

L is less than 100 s. But when $L > 160$ s, the intersection capacity increases slowly with the increase in L. For example, when L increases from 80 s to 100 s, the intersection capacity increases by 270 pcu/h; but when L increases from 160 s to 180 s, the intersection capacity only increases by 75 pcu/h. In general, when vehicles pass the intersection, the delay increases with the increase in the cycle length. Therefore, the cycle length should most properly be designed between 100 s and 140 s, not longer than 160 s.

In the above discussion of the intersection capacity, the case of the right-turn vehicles is not taken into account. If a special right-turn lane is designed at an intersection, the intersection capacity will only have to add the capacity of the special right-turn lane. This is due to the fact that the right-turn signal phase can be designed along with other phases and it does not take up of the entire signal period. If the right-turn lane and the straight lane share the same inlet road, the total capacity of the intersection decreases a little due to the interaction of the straight vehicles and the right-turn ones.

3.2 Two straight lanes and one left-turn lane at every inlet road

For many common intersections in the urban roads, the proportion of straight vehicle flow is greater than those of left-turn and right-turn vehicle flows. Thereby, at many intersections, the number of straight lanes is more than those of right-turn and left-turn lanes. One of the usual situations is that the number of straight lanes is 2, and the number of right-turn and left-turn lanes are both 1. The capacity in the intersection under this condition is discussed as follows.

If m_{i1} is taken as 2, m_{i2} is taken as 1, t_{fij} is taken as 2 s, t_{cij} is taken as 8 s in Eq. (2), and the yellow interval c_{ij} is taken as 2 s on the restrictive condition of Eq. (1), then the following formula can be obtained:

$$N = 1 + \frac{l_{11} + l_{21} - 42}{L} (\text{pcu/s}) \tag{7}$$

The changes in the capacities of the above two intersections are analyzed. The difference between Eq. (7) and Eq. (6) is

$$\Delta N = \frac{l_{11} + l_{21} - 12}{L} (\text{pcu/s}) \tag{8}$$

The number of straight lanes increases from 1 to 2 after the inlet roads are broadened. Under this condition, the variance of the intersection capacity is noted as ΔN. Dividing Eq. (8) by Eq. (6), the growth rate of the capacity is given as

$$\frac{\Delta N}{N} = \frac{l_{11} + l_{21} - 12}{2(L - 30)} \tag{9}$$

When the number of the straight lane vehicles is more than that of the left-turn lane vehicles and the straight phase is longer than that of the left-turn phase, the capacity of the intersection can increase about 50% with the number of straight lanes increasing from 1 to 2.

4 Model in the Case of Vehicle Arrival Rate

The factor of vehicle delay needs to be considered while designing the signal phase of an intersection in practice. Generally, when the signal period is longer, the capacity is greater. But, meanwhile, the caused vehicle delay is longer. When the signal period is shorter, the vehicle delay is shorter; but the capacity of the intersection is smaller. Therefore, in order to make vehicle delay the shortest, while designing the signal phase, we only discuss the case that the intersection capacity is larger than the vehicle arrival rate.

The situation of the non-saturated traffic flow, that is, the case that the arrival traffic flow is smaller than the intersection capacity, is discussed as follows. Because the same signal phase is often applied in the opposite inlet roads, while designing the signal phases the main traffic demand needs to be met when the traffic flow from a certain direction is great. The average arrival rate of inlet road i from direction j is supposed to be q_{ij}. To make the arrival traffic flow pass the intersection in the signal period, the design of the signal phase length must satisfy the following condition:

$$m_{ij}\left(\frac{l_{ij}}{t_{fij}} - \frac{t_{cij} - t_{fij}}{t_{fij}}\right) > q_{ij} L \qquad i, j = 1, 2 \tag{10}$$

That is,

$$l_{ij} > t_{cij} + t_{fij}\left(\frac{q_{ij} L}{m_{ij}} - 1\right) \tag{11}$$

After dividing formula (10) by m_{ij}, the divided results are added based on i and j. Then, divide the total results by the cycle length L. Finally, the following formula can be obtained:

$$N = 2\sum_{i,j=1}^{2} \frac{1}{L}\left(\frac{l_{ij}}{t_{fij}} - \frac{t_{cij} - t_{fij}}{t_{fij}}\right) > 2\sum_{i,j=1}^{2} \frac{q_{ij}}{m_{ij}} \tag{12}$$

Formula (12) indicates that the capacity of the intersection is greater than the vehicle arrival rate.

In the case of the non-saturated flow rate, the aim of signal phase design must make the remaining time as equal as possible after the vehicles of all the inlet roads from all the directions have passed the intersection. That is to say, the following values should be as equal as possible:

$$\left(\frac{l_{ij}}{t_{fij}} - \frac{t_{cij} - t_{fij}}{t_{fij}}\right) - \frac{q_{ij}}{m_{ij}} L \qquad i, j = 1, 2 \tag{13}$$

The mathematical description in the issue is

$$\min \sum_{i,j=1}^{2} \left(\left(\frac{l_{ij}}{t_{fij}} - \frac{t_{cij} - t_{fij}}{t_{fij}}\right) - \frac{q_{ij}}{m_{ij}} L \right) \tag{14}$$

$$\text{s.t.} \sum_{i,j=1}^{2} l_{ij} = L - c$$

Using Lagrangian multiplication factors, the results of (14) are given as

$$l_{ij} = \frac{t_{fij}^2}{\sum_{i,j=1}^{2} t_{fij}^2}\left(L - c - \sum_{i,j=1}^{2}\left((t_{cij} - t_{fij}) + \frac{q_{ij} t_{fij}}{m_{ij}} L\right)\right) +$$

$$(t_{cij} - t_{fij}) + \frac{q_{ij} t_{fij}}{m_{ij}} L \tag{15}$$

Eq. (15) is just used to design phases of all directions and can be simplified a bit in practice. After the traffic flow is stable, the difference in the following times between the left-turn flow and the straight one is not great. Therefore, the following formula can be given:

$$\frac{t_{fij}^2}{\sum_{i,j=1}^{2} t_{fij}^2} \approx \frac{1}{4}$$

Then Eq. (15) can be simplified as

$$l_{ij} = \frac{1}{4}(L - c) + \left((t_{cij} - t_{fij}) - \frac{1}{4}\sum_{i,j=1}^{2}(t_{cij} - t_{fij}) \right) + t_f L\left(\frac{q_{ij}}{m_{ij}} - \frac{1}{4}\sum_{i,j=1}^{2} \frac{q_{ij}}{m_{ij}} \right) \tag{16}$$

When the differences among $(t_{cij} - t_{fij})$ $(i, j = 1, 2)$ are not great, the second part of Eq. (16) can be eliminated. Then Eq. (16) can be simplified as

$$l_{ij} = \frac{1}{4}(L - c) + t_f L\left(\frac{q_{ij}}{m_{ij}} - \frac{1}{4}\sum_{i,j=1}^{2} \frac{q_{ij}}{m_{ij}} \right) \tag{17}$$

Eq. (17) indicates that for the cross intersection designed with the special left-turn signal, the signal phase length l_{ij} is determined by the parameters such as the cycle length L, the yellow interval c, the following car headway t_f and the main traffic volume q_{ij}/m_{ij}. Hereinto, the cycle length L is determined by Eq. (12). Assuming that all the values of t_{fij} are the same and recorded as t_f, according to Eqs. (1), (3) and (12), the following formula can be given as

$$\frac{1}{L}\left(\frac{L - c}{t_f} - \alpha \right) \geqslant \sum_{i,j=1}^{2} \frac{q_{ij}}{m_{ij}}$$

Subsequently, the cycle length should meet the following condition:

$$L \geqslant \frac{c + \alpha t_f}{1 - t_f \sum_{i,j=1}^{2} \frac{q_{ij}}{m_{ij}}} \tag{18}$$

Thus, the shortest cycle length L^* should be

$$L^* = \frac{c + \alpha t_f}{1 - t_f \sum_{i,j=1}^{2} \frac{q_{ij}}{m_{ij}}} \tag{19}$$

And it can be used to calculate the cycle length.

5 Practical Example

The intersection shape of Dashikou in Zhenjiang is shown in Fig. 3. The southern inlet road has two left-turn roadways and one straight roadway, and the other three inlet roads all have one left-turn roadway and one straight roadway. The traffic flows of the inlet roads (not including the right-turn traffic flows) are listed in Tab. 1.

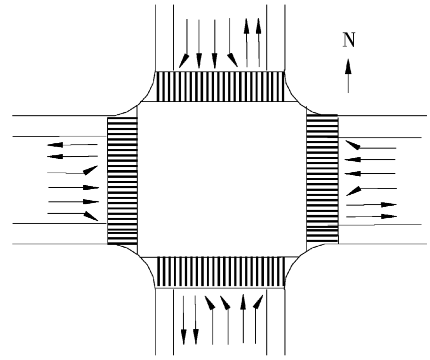

Fig. 3 Intersection shape of Dashikou in Zhenjiang

Tab. 1 Traffic flows of inlet roads pcu/h

Inlet road	Straight roadway	Left-turn roadway
East	544	181
West	786	228
South	418	626
North	428	258

When the yellow interval, the time loss of the first vehicle and the following car headway are set as 2, 6 and 2 s, respectively, the total time loss of the four phases is 32 s by the proposed method. According to Eq. (19), the cycle length is calculated as 128 s. According to Eq. (17), the length of each phase is calculated as follows: with regard to the east and west inlet roads, the straight phase is 34 s and the left-turn phase is 22 s; with regard to the south and north inlet roads, the straight phase is 36 s and the left-turn phase is 28 s.

6 Conclusions

1) The capacity of the signalized intersection increases with the increase of the signal cycle length. If the cycle length is shorter, the capacity decreases rapidly; if the cycle length is longer, the capacity increases slowly and the delay increases rapidly. As a result, the cycle length should be properly designed between 100 s and 140 s, and no more than 160 s.

2) If the right-turn traffic flow is not taken into account, the intersection capacity increases by about 50% when inlet roads are broadened from one straight lane and one left-turn lane to two straight lanes and one left-turn lane. Therefore, broadening inlet roads is one of the effective ways to improve intersection capacity.

3) In the design of the signal phases at intersections, cycle length and phase length are related to the number of inlet roads, following car headway, yellow intervals, starting time loss and so on.

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交叉口多相位信号设置及通行能力研究

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摘要:采用停车线法建立了多相位信号交叉口通行能力模型,并对直行车道和左转车道均为一条以及直行车道为2条而左转车道为一条这2种常见状态下建立的模型进行了简化,结果表明信号交叉口通行能力主要和信号周期长度、相位配置、交叉口几何设计和车辆跟车时距等因素有关.考虑车辆到达率时的信号周期长度和相位配时设置问题,建立了基于各相位剩余时间均衡的优化模型,并采用拉格朗日乘子法对优化模型进行求解,从而确定出信号周期最佳时长以及各相位时长的计算模型,并对模型进行了简化.所建立周期和相位时长的计算模型较现有的模型更简便实用.最后,选取了一个实际的交叉口,用所建立的模型对其周期和相位时长进行了设计.

关键词:交通工程;交叉口;多相位信号配时;拉格朗日乘子法;优化

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