Journal of Southeast University (English Edition) Vol. 25, No. 2, pp. 167 – 170 June 2009 ISSN 1003—7985

Downlink BER performance analysis of distributed antenna systems over shadowed Rayleigh fading channels

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Abstract: Due to the complexity of the composite fading channel, a new simplified channel model is proposed to analyze the bit error ratio (BER) performance of the distributed antenna system (DAS). First, instead of the gamma-log-normal distribution, the log-normal distribution is applied to describe the output signal to noise ratio (SNR) after maximal ratio combining (MRC) at the receiver. Then, assuming that the channel state information (CSI) is available to the transmitter, by employing the Gauss-Hermite integral, an approximate analytical expression of the BER is derived for the downlink of the DAS with antenna selective transmission and MRC. Finally, the results of a Monte Carlo simulation show that the analytical results match the simulation results. Therefore, it can be concluded that the proposed approximate channel model is effective and accurate, and the derived analytical expression can be used to evaluate the real system performance.

Key words: distributed antenna system; bit error ratio; shadowed Rayleigh fading; log-normal; gamma-log-normal

In recent years, the distributed antenna system (DAS) has been a hot topic and it is considered as a promising technology for future wireless communications[1-4]. Compared with the traditional centralized antenna system (CAS) where the antennas are co-located, the DAS comprises some separately-placed antennas and each antenna is also called an access point (AP). Furthermore, through coaxial cables or fiber optics, all the APs are connected to the main processing unit (MPU). Generally, the signaling, switching and mobility management are performed at the MPU.

Up to date, many works on the DAS have been done. The signal to interference was investigated in Ref. [5]. Ref. [6] compared the DAS with the CAS in terms of consumption power and capacity. Furthermore, the outage probability of the DAS was discussed in Refs. [7 – 8]. In addition, the ergodic capacity of the cellular DAS was discussed in Refs. [9 – 11]. From the previous works, it can be concluded that the DAS is an effective technology to enhance link reliability, increase signal quality and enhance system capacity. However, to the best of our knowledge, the bit error ratio (BER) of the DAS has not been analyzed in the available literature. Therefore, this paper tries to provide insight into the downlink BER performance of the DAS over shadowed Rayleigh fading channels with antenna selective transmission and maximal ratio combining (MRC).

Received 2008-11-18.

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Foundation items: The National High Technology Research and Development Program of China (863 Program) (No. 2007AA01Z207, 2007AA01Z268), Program for New Century Excellent Talents in University, Research Fund of National Mobile Communications Research Laboratory of Southeast University (No. 2008A06).

Citation: Chen Huamin, Chen Ming. Downlink BER performance analysis of distributed antenna systems over shadowed Rayleigh fading channels[J]. Journal of Southeast University (English Edition), 2009, 25(2): 167 – 170.

1 System Model

Fig. 1 shows the downlink of a typical DAS with N APs and one MPU. For easy description, the i-th AP is denoted as AP_i. Without loss of generality, the mobile station (MS) is equipped with L antennas. Furthermore, the antenna selective transmission is employed to maximize the output SNR after MRC at the MS. Then, if AP_i is selected for transmission, the received signals at the MS can be expressed as

$$\boldsymbol{Y}_i = \{y_i^{(1)}, \dots, y_i^{(L)}\}^{\mathrm{T}} = \sqrt{E_s}\underbrace{\{h_i^{(1)}, \dots, h_i^{(L)}\}^{\mathrm{T}}}_{\boldsymbol{H}_i} x + \boldsymbol{Z} \in \mathbf{C}^{L\times 1} \tag{1}$$

where the j-th entry of $\boldsymbol{Y}_i$, $y_i^{(j)}$ $(j = 1, 2, \dots, L)$ is the received signal at the j-th antenna of the MS, E_s is the transmit signal power, $h_i^{(j)}$ in the channel vector $\boldsymbol{H}_i$ denotes the composite channel fading between AP_i and the j-th antenna of the MS, x is the transmitted signal from AP_i with unitary power, and $\boldsymbol{Z}$ is the circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $E\{\boldsymbol{Z}\boldsymbol{Z}^{\dagger}\} = N_0\boldsymbol{I}_L$. Note that $\boldsymbol{I}_L$ is an $L\times L$ identity matrix.

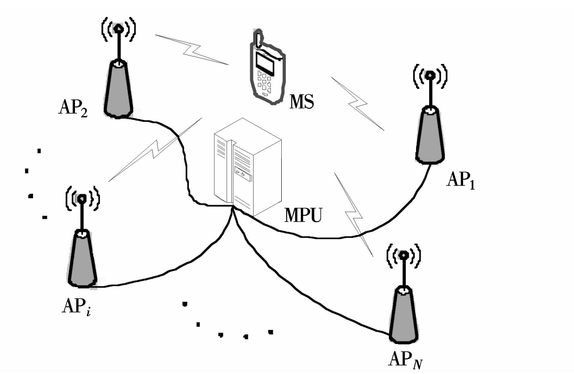

Fig. 1 An example of DAS structure

2 Distribution of the Output SNR after MRC

Due to the structural features of the DAS, $h_i^{(j)}$ can be modeled as

$$h_i^{(j)} = g_i^{(j)} \sqrt{P_i S_i} \tag{2}$$

where $g_i^{(j)}$ represents the small-scale fast fading between AP_i and the j-th receive antenna of the MS, and P_i and S_i denote the path loss and the shadowing effect between AP_i and the MS. From a practical standpoint, the same path loss term P_i and the shadowing effect term S_i are shared by $g_i^{(1)}, \dots, g_i^{(L)}$. Furthermore, $g_i^{(j)}$, P_i and S_i are independent of each other.

The envelope of the fast fading is assumed to undergo a Rayleigh fading, which means the squared envelope $|g_i^{(j)}|^2$ follows an exponential distribution. The path loss term P_i in Eq. (2) can be modeled as [12]

$$P_i = \left(\frac{d_0}{d_i}\right)^{\beta_i} \tag{3}$$

where β_i is the path loss exponent, d_0 is the reference distance and d_i represents the distance from the AP_i to the MS. The shadowing effect term S_i in Eq. (2) is log-normally distributed and the mean of $10\lg S_i$ is assumed to be zero (in dB) for easy analysis. As a result, the probability density function (PDF) of S_i can be given as[12]

$$f_{S_i}(s) = \frac{\xi}{\sqrt{2\pi}\sigma_i s}\exp\left(-\frac{(10\lg s)^2}{2\sigma_i^2}\right) \qquad s>0 \tag{4}$$

where σ_i (in dB) is the standard deviation of $10\lg S_i$ and $\xi = \frac{10}{\ln 10}$.

From Eqs. (1) and (2), the output SNR after MRC can be obtained as

$$\gamma_i = \sum_{j=1}^{L}\gamma_i^{(j)} = \Omega_i\sum_{j=1}^{L}|g_i^{(j)}|^2 \tag{5}$$

where $\gamma_i^{(j)}$ denotes the instantaneous SNR at the j-th antenna, and $\Omega_i = \frac{E_s P_i S_i}{N_0}$. Determined by S_i, Ω_i is also distributed log-normally and its PDF can be achieved as[13]

$$f_{\Omega_i}(\omega) = \frac{\xi}{\sqrt{2\pi}\sigma_i\omega}\exp\left(-\frac{(10\lg\omega-\mu_i)^2}{2\sigma_i^2}\right) \qquad \omega>0 \tag{6}$$

where $\mu_i = 10\lg\left(\frac{E_s P_i}{N_0}\right)$ is the mean of $10\lg\Omega_i$. Under the assumption that $g_i^{(1)}, \ldots, g_i^{(L)}$ are independent of each other, the conditional moment generating function (MGF) of γ_i given Ω_i can be expressed as

$$M_{\gamma_i|\Omega_i}(p\mid\omega) = \prod_{j=1}^{L}M_{\gamma_i^{(j)}|\Omega_i}(p\mid\omega) \tag{7}$$

where $M_{\gamma_i^{(j)}|\Omega_i}(p\mid\omega)$ is the conditional MGF of $\gamma_i^{(j)}$, and can be derived as[14]

$$M_{\gamma_i^{(j)}|\Omega_i}(p\mid\omega) = \int_0^{+\infty} f_{\gamma_i^{(j)}|\Omega_i}(t\mid\omega)\exp(pt)\,\mathrm{d}t = \int_0^{+\infty}\frac{1}{\omega}\exp\left(-\frac{t}{\omega}\right)\exp(pt)\,\mathrm{d}t = (1-p\omega)^{-1} \tag{8}$$

where $f_{\gamma_i^{(j)}|\Omega_i}(t\mid\omega)$ is the conditional PDF of $\gamma_i^{(j)}$. Then, by the inverse Laplace transformation of $M_{\gamma_i|\Omega_i}(-p\mid\omega)$, the PDF of γ_i conditioned on Ω_i can be obtained as

$$f_{\gamma_i|\Omega_i}(t\mid\omega) = \mathscr{L}^{-1}\{M_{\gamma_i|\Omega_i}(-p\mid\omega)\} = \mathscr{L}^{-1}\{(1+p\omega)^{-L}\} = \frac{t^{L-1}}{\Gamma(L)\omega^L}\exp\left(-\frac{t}{\omega}\right) \tag{9}$$

where $\mathscr{L}^{-1}\{\cdot\}$ denotes the inverse Laplace transformation. Finally, the PDF of γ_i can be derived from Eqs. (6) and (8) as

$$f_{\gamma_i}(t) = \int_0^{+\infty}\frac{L^L t^{L-1}}{\Gamma(L)\tau^L}\exp\left(-\frac{Lt}{\tau}\right)\frac{\xi}{\sqrt{2\pi}\sigma_i\tau}\cdot \exp\left[-\frac{(10\lg\tau-\mu_i-10\lg L)^2}{2\sigma_i^2}\right]\mathrm{d}\tau \tag{10}$$

where $\tau = L\omega$. Obviously, it can be found that γ_i follows a gamma-log-normal distribution.

However, it is not straightforward to evaluate Eq. (10). Here, a log-normal distribution is used as a substitute for the gamma-log-normal in Eq. (10), and the PDF of γ_i can be approximated as[15]

$$f_{\gamma_i}(t) \cong \frac{\xi}{\sqrt{2\pi}\hat{\sigma}_i t}\exp\left(-\frac{(10\lg t-\hat{\mu}_i)^2}{2\hat{\sigma}_i^2}\right) \qquad t\geqslant 0 \tag{11}$$

where $\hat{\mu}_i$ is the mean of the approximate distribution and can be expressed as

$$\hat{\mu}_i = \mu_i + 10\lg L + \xi[\psi(L) - \ln(L)] \tag{12}$$

where $\psi(\cdot)$ is the Euler psi function[16]. Furthermore, $\hat{\sigma}_i^2$ in Eq. (10) is the variance of the approximate distribution and can be achieved as

$$\hat{\sigma}_i^2 = \sigma_i^2 + \xi^2\zeta(2, L) \tag{13}$$

where $\zeta(\cdot,\cdot)$ is Reimann's zeta function[16]. The PDF curves of these two distributions are shown in Fig. 2. The curves indicate that Eq. (11) can approximate Eq. (10) very well.

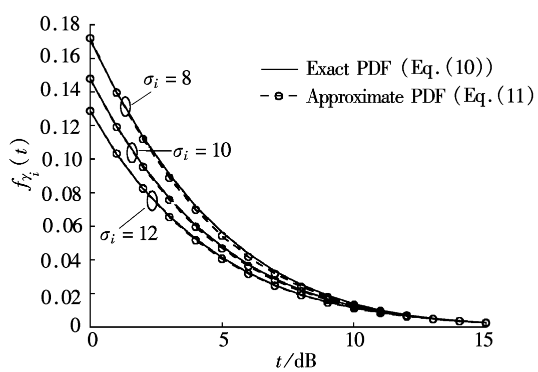

Fig. 2 PDF curves of the gamma-log-normal and log-normal distribution

3 BER Analysis with Antenna Selective Transmission

Suppose that a selective transmission scheme is applied and only one AP is selected for transmission to maximize the output SNR at the receiver, provided that the channel state information between AP_i and the MS is known to the transmitter. Accordingly, the transmission scheme can be expressed by the following rule as

$$\gamma = \max\{\gamma_1, \ldots, \gamma_N\} \tag{14}$$

Because of the large space among APs, it is reasonable to assume $\gamma_1, \ldots, \gamma_N$ are independent of each other. Therefore, the cumulative distribution function (CDF) of γ can be obtained as[13]

$$
\begin{aligned}
F_\gamma(r) &= \prod_{i=1}^{N} F_{\gamma_i}(r) = \\
&\prod_{i=1}^{N}\left[\int_0^r \frac{\xi}{\sqrt{2\pi}\hat{\sigma}_i t}\exp\left(-\frac{(10\lg t-\hat{\mu}_i)^2}{2\hat{\sigma}_i^2}\right)\mathrm{d}r\right] = \\
&\prod_{i=1}^{N}\left[1-\frac{1}{2}\mathrm{erfc}\left(\frac{10\lg r-\hat{\mu}_i}{\sqrt{2}\hat{\sigma}_i}\right)\right]
\end{aligned} \tag{15}
$$

where $F_{\gamma_i}(r)$ is the CDF of γ_i and $\mathrm{erfc}(\cdot)$ is the complementary error function[17]. Differentiating Eq. (15) with respect to r yields the PDF of γ as

$$
f_\gamma(r) = \sum_{i=1}^{N}\left[f_{\gamma_i}(r)\prod_{k=1,k\neq i}^{N}F_{\gamma_k}(r)\right] \tag{16}
$$

Consequently, the average BER can be expressed as[12]

$$
\begin{aligned}
P_\mathrm{b} &= \int_0^{+\infty}\alpha Q(\sqrt{\lambda r})f_\gamma(r)\,\mathrm{d}r = \\
&\sum_{i=1}^{N}\int_0^{+\infty}\alpha Q(\sqrt{\lambda r})\left[\prod_{k=1,k\neq i}^{N}F_{\gamma_k}(r)\right]f_{\gamma_i}(r)\,\mathrm{d}r
\end{aligned} \tag{17}
$$

where $Q(\cdot)$ is the Gaussian function, α and λ depend on the modulation type. Obviously, there is no closed form for the above expression. However, the Gauss-Hermite integral is employed and Eq. (17) can be approximated as[17]

$$
\begin{aligned}
P_\mathrm{b} \cong \frac{1}{\sqrt{\pi}}\sum_{i=1}^{N}\sum_{n=1}^{N_p}\Bigg\{&\alpha H_n Q\left(\sqrt{\lambda 10^{(\sqrt{2}\hat{\sigma}_i t_n+\hat{\mu}_i)/20}}\right)\cdot \\
&\prod_{k=1,k\neq i}^{N}\left[1-\frac{1}{2}\mathrm{erfc}\left(\frac{10\lg r-\hat{\mu}_k}{\sqrt{2}\hat{\sigma}_k}\right)\right]\Bigg\}
\end{aligned} \tag{18}
$$

where t_n and H_n are the base points and weight factors of the N_p-order Hermite polynomial, respectively.

4 Simulation Results

The analytical results and the simulation results are presented in this section. Fig. 3 depicts the simulation scenery, where four APs are located at the four corners of a square area and the MS is randomly distributed. Unless specified otherwise, the main simulation parameters are listed in Tab. 1.

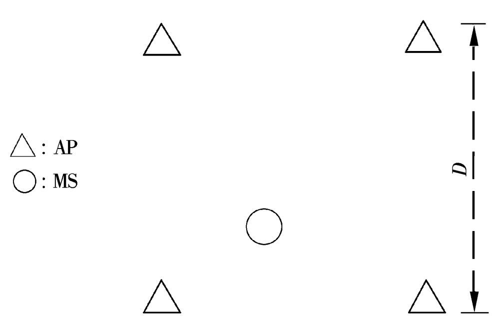

Fig. 3 Simulation scenery

Tab. 1 Main simulation parameters

Parameter	Value	Parameter	Value
N	4	β_i	4
L	1	d_0/m	10
α	1	σ_i/dB	8
λ	2	D/m	500

Fig. 4 and Fig. 5 show the BER performance of the DAS vs. the transmit SNR(E_s/N_0). It can be easily noted that the BER performance is improved with the increase of E_s/N_0. Fig. 4 plots the BER performance as a function of the receive antenna number. Obviously, an improvement in the BER can be obtained by increasing the number of receive antennas. Furthermore, Fig. 5 reveals the relationship between the BER and the path loss exponent. It can be known that the BER value is reduced as the path loss exponent decreases. Moreover, it should be emphasized that good agreement is observed between the simulation results and the analytical values.

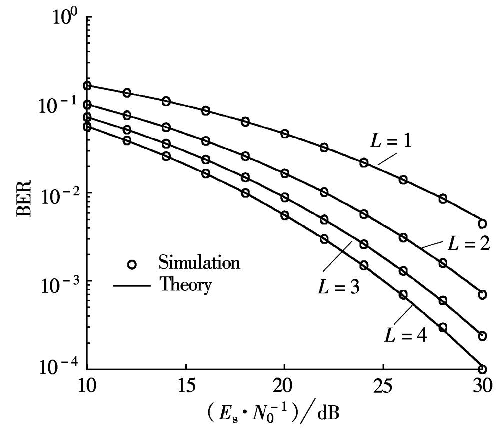

Fig. 4 BER vs. transmit SNR (E_s/N_0) with different numbers of receive antennas

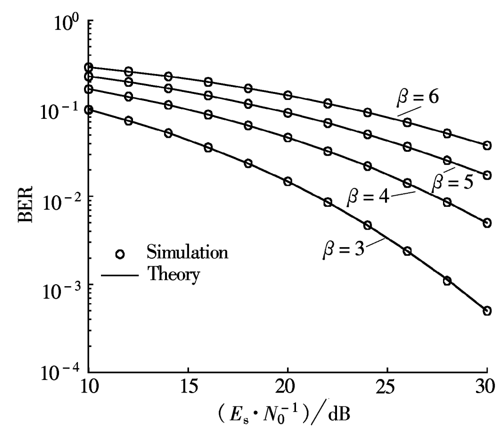

Fig. 5 BER vs. transmit SNR (E_s/N_0) with different path loss exponents when $\beta_i=\beta$

5 Conclusion

Under the antenna selective transmission strategy at the transmitter and MRC at the receiver, the BER performance of the DAS is studied over shadowed Rayleigh fading channels in this paper. Without loss of generality, the output SNR after MRC is modeled by a log-normal distribution instead of the gamma-log-normal distribution. An approximate analytical expression of BER is derived for the downlink DAS by applying the Gauss-Hermite integral. It can be concluded from the simulation results that the derived analytical expression can be applied to precisely evaluate the BER performance of the DAS.

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阴影 Rayleigh 衰落信道条件下分布式天线系统的下行 BER 性能分析

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摘要：针对复合衰落信道的复杂性，提出了一种简化的信道模型来研究分布式天线系统的误比特率性能. 首先，基于阴影 Rayleigh 衰落信道，在接收端进行最大比合并，并用对数正态分布替代复杂的伽玛-对数正态分布，描述所得信噪比的随机特性. 其次，假设发送端已知信道状态信息，推导了分布式天线系统基于天线选择发送的下行误比特率表达式，表达式采用了 Gauss-Hermite 近似以便于理论计算. 最后，仿真结果显示理论值和仿真值能够很好地吻合. 因此，可以认为所提出的简化模型是有效的和精确的，同时也表明所推导的解析表达式能够用于精确评估系统的实际性能.

关键词：分布式天线系统；误比特率；阴影 Rayleigh 衰落；对数正态；伽玛-对数正态

中图分类号：TN929.5