Journal of Southeast University (English Edition) Vol. 25, No. 2, pp. 189 – 192 June 2009 ISSN 1003—7985

Life prediction for vacuum fluorescent display using maximum likelihood estimation

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Abstract: In order to obtain the life information of the vacuum fluorescent display (VFD) in a short time, a model of constant stress accelerated life tests (CSALT) is established with its filament temperature increased, and four constant stress tests are conducted. The Weibull function is applied to describe the life distribution of the VFD, and the maximum likelihood estimation (MLE) and its iterative flow chart are used to calculate the shape parameters and the scale parameters. Furthermore, the accelerated life equation is determined by the least square method, the Kolmogorov-Smirnov test is performed to verify whether the VFD life meets the Weibull distribution or not, and self-developed software is employed to predict the average life and the reliable life. Statistical data analysis results demonstrate that the test plans are feasible and versatile, that the VFD life follows the Weibull distribution, and that the VFD accelerated model satisfies the linear Arrhenius equation. The proposed method and the estimated life information of the VFD can provide some significant guideline to its manufacturers and customers.

Key words: vacuum fluorescent display; accelerated life test; constant stress; Weibull distribution; maximum likelihood estimation

The vacuum fluorescent display (VFD) is a kind of flat panel display, which consists of a cathode, a grid and an anode with phosphor as shown in Fig. 1. It is widely used in many fields, such as electronic products, instruments and automobiles. Rapid changes in technology, and shorter periods in VFD development, make it even more useful and reliable in the field of flat panel display. However, it is known that the more reliable the device, the more difficult it is to measure its reliability[1]. For example, VFD life can be up to ten thousands hours. However, the VFD will have to be renewed before conventional life tests are terminated under normal working conditions. This paper presents an approach to performing an accelerated life test (ALT) for the VFD, which enables rapid and accurate estimation of VFD life.

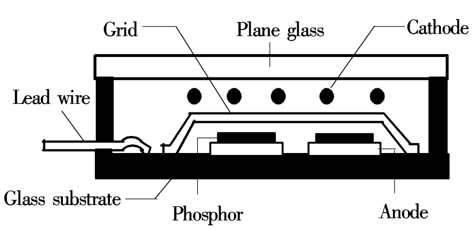

Fig. 1 VFD structure

The ALT is commonly used to obtain information on the life distribution of products or materials[2]. The principle is to test units under stress levels greater than the usual or operational conditions so as to induce early failures. It can save test time as well as test expense and manpower. Many researchers have paid much more attention to the ALT. Bosc et al. [3] presented a new test method for micro-machined chemical sensors and evaluated the sensor reliability in the range of the requirements of applications using gas sensors. Brunold et al. [4] developed a procedure for an accelerated life test of solar absorber surfaces within the framework of the working group MSTC (Materials in Solar Thermal Collectors) of the IEA-SHCP (International Energy Agency-Solar Heating and Cooling Programme). Rosenbaum et al. [5] compared several popular accelerated test methods for projecting SiO_2 lifetime distributions or failure rates. Gouno[6] studied the step-stress accelerated life test and presented a practical method to analyze temperature step-stress accelerated life test data. Tang et al. [7] considered optimal test plans involving life distributions with failure-free life, i. e., where there is an unknown threshold parameter below which no failure will occur. Xie and Pecht[8] presented a probabilistic-approach-based reliability prediction model of semiconductor light emitting devices. To the best of our knowledge, so far there have been no research reports on VFD accelerated life test except our work, in which the VFD life is estimated by the constant-step ALT based on the least square method (LSM) [9].

The ALT is commonly divided into constant stress test, step stress test and progressive stress test. For the constant stress accelerated life test (CSALT) plan, the samples are divided into several groups, each of which is tested at a constant stress level until the specified time (also called censored time) or the specified number of failures (also called censored number). The CSALTs have some advantages, e. g. simple test methods, ripe theory and precise test data. In addition, the maximum likelihood estimation (MLE) is more rapid and accurate than other numerical analysis methods. In view of the above advantages, four CSALTs are performed in this paper. After the VFD life is described by the Weibull distribution function, the shape parameters and the scale parameters are estimated based on the MLE and the accelerated life line is plotted. Furthermore, the VFD failure patterns and the life distributions are revealed and the life prediction software is self-developed. Accurate and rapid estimation of VFD life can serve as the basis for follow-up research on the design and manufacture of the VFD.

Received 2008-10-25.

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Foundation items: Undergraduate Education Highland Construction Project of Shanghai, the Key Course Construction of Shanghai Education Committee (No. 20075302), the Key Technology R&D Program of Shanghai Municipality (No. 08160510600).

Citation: Zhang Jianping, Wang Ruitao. Life prediction for vacuum fluorescent display using maximum likelihood estimation[J]. Journal of Southeast University (English Edition), 2009, 25(2): 189 – 192.

1 Test Plan

1.1 Basic assumptions

In order to define a criterion for VFD lifetime, the following assumptions are made[9]:

1) Under any constant level of stress, the life distribution of the VFD conforms to the following two-parameter Weibull distribution:

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^m\right] \qquad t > 0 \tag{1}$$

where m and η are positive, and they are the shape parameter and the scale parameter, respectively.

2) The VFD accelerated model satisfies the linear Arrhenius equation. Namely, the relationship between stress and life is defined as

$$\ln\eta = \alpha + \frac{\beta}{T} \tag{2}$$

where α and β are unknown parameters, and T is the filament absolute temperature (K).

3) The VFD failure mechanism is unchangeable within the stress range from normal stress T_0 to accelerated ones T_i ($i = 1, 2, 3, 4$). That is, the shape parameter m will not be changed, which means

$$m_0 = m_1 = m_2 = m_3 = m_4 \tag{3}$$

1.2 The method of accelerated life test

The VFD life is mainly affected by its cathode temperature, so its filament temperature (monitored by voltage) is selected as the accelerated stress.

For the VFD cathode, the temperature cannot exceed 850 ℃. If the cathode temperature is too high, new failure mechanisms will soon occur. On the other hand, if the cathode temperature is set too low, the test period will last too long. Therefore, four different temperatures (T_1 = 1 023.00 K, T_2 = 1 055.56 K, T_3 = 1 087.85 K, T_4 = 1 123.33 K) are chosen as accelerated stress levels. The total number of the samples under each stress level n_i is 20. In addition, it is necessary to explain that the normal working stress T_0 of this type of VFD is 923.95 K.

The failure time of each test sample is automatically saved by recording equipment when the VFD luminance decreases to below 210 cd/m^2 (green phosphor powder). Furthermore, the complete life test is carried out; that is to say, all the test samples are up to failure criterion under accelerated testing conditions.

2 Maximum Likelihood Estimation of Weibull Parameters

According to Eq. (1), the probability density function (PDF) of the Weibull function can be written as

$$f(t) = \frac{m}{\eta}\left(\frac{t}{\eta}\right)^{m-1}\exp\left[-\left(\frac{t}{\eta}\right)^m\right] \tag{4}$$

Thus, the corresponding likelihood function is

$$L(m, \eta) = \prod_{j=1}^{n_i}\frac{m}{\eta}\left(\frac{t_j}{\eta}\right)^{m-1}\exp\left[-\left(\frac{t_j}{\eta}\right)^m\right] = \left(\frac{m}{\eta}\right)^{n_i}\prod_{j=1}^{n_i}\left(\frac{t_j}{\eta}\right)^{m-1}\exp\left[-\sum_{j=1}^{n_i}\left(\frac{t_j}{\eta}\right)^m\right] \tag{5}$$

where t_j ($j = 1, 2, \ldots, n_i$) is the failure time at each accelerated stress T_i ($i = 1, 2, 3, 4$). Then, the corresponding log-likelihood function is

$$\ln L(m, \eta) = n_i\ln\left(\frac{m}{\eta}\right) + \sum_{j=1}^{n_i}\ln\left(\frac{t_j}{\eta}\right)^{m-1} - \sum_{j=1}^{n_i}\left(\frac{t_j}{\eta}\right)^m \tag{6}$$

After derivation with respect to m and η, Eq. (6) is transformed as

$$\frac{\partial \ln L}{\partial m} = \frac{n_i}{m} + \sum_{j=1}^{n_i}\ln\frac{t_j}{\eta} - \sum_{j=1}^{n_i}\left(\frac{t_j}{\eta}\right)^m\ln\frac{t_j}{\eta} \tag{7}$$

$$\frac{\partial \ln L}{\partial \eta} = -\frac{n_i}{\eta} - \frac{n_i(m-1)}{\eta} + \frac{m}{\eta}\sum_{j=1}^{n_i}\left(\frac{t_j}{\eta}\right)^m \tag{8}$$

Let the right-hand side be zero in Eqs. (7) and (8), that is, $\frac{\partial \ln L}{\partial m} = 0$ and $\frac{\partial \ln L}{\partial \eta} = 0$, one can obtain

$$\frac{1}{m} = \frac{\sum_{j=1}^{n_i} t_j^m \ln t_j}{\sum_{j=1}^{n_i} t_j^m} - \frac{1}{n_i}\sum_{j=1}^{n_i}\ln t_j \tag{9}$$

$$\eta = \sqrt[m]{\frac{1}{n}\sum_{j=1}^{n_i} t_j^m} \tag{10}$$

Eq. (9) has a unique positive solution[10], so a simple iterative algorithm is given as

$$m_{k+1} = [h(m_k) - \tilde{w}]^{-1} \qquad m_0 = 2.0 \tag{11}$$

where m_k and m_{k+1} represent the k and $k+1$ times approximation, respectively; $h(m)$ and $\tilde{w}$ are respectively shown as

$$h(m) = \sum_{j=1}^{n_i} t_j^m \ln t_j \Big/ \sum_{j=1}^{n_i} t_j^m, \quad \tilde{w} = \frac{1}{n_i}\sum_{j=1}^{n_i}\ln t_j \tag{12}$$

It is easy to prove that $\mathrm{d}h(m)/\mathrm{d}m > 0$, that is, $h(m)$ is a monotonically increasing function. Furthermore, a method of accelerating convergence is used to iterate in this paper, which is shown as follows[11]:

$$m_{k+1} = \frac{m_k + [h(m_k) - \tilde{w}]^{-1}}{2} \qquad m_0 = 2.0 \tag{13}$$

Using the method mentioned above, the estimated value of the shape parameter m is calculated from Eq. (13). Substituting the estimated value m into Eq. (10), the estimated value of the scale parameter η can be obtained. The flow chart of this iteration procedure can be seen in Fig. 2.

3 Statistical Analysis of Test Data

3.1 Test data

According to the test plan mentioned above, four constant stress ALTs are performed. The failure time data of four constant stress tests are listed in Tab. 1.

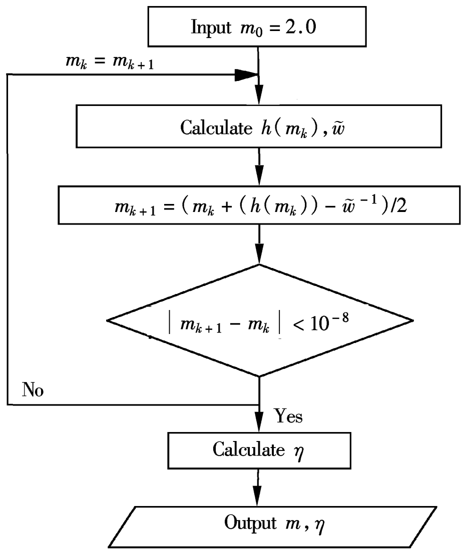

Fig. 2 Flow chart of iteration procedure

Tab. 1 Sample failure time of CSALTs at T_1, T_2, T_3, T_4

Failure time	T_1	T_2	T_3	T_4
t_1	1 509. 00	555. 33	213. 33	74. 80
t_2	1 553. 00	562. 67	220. 67	79. 20
t_3	1 591. 50	570. 50	231. 67	83. 60
t_4	1 597. 00	581. 00	235. 33	93. 08
t_5	1 602. 50	586. 50	239. 00	94. 77
t_6	1 641. 00	596. 40	242. 67	96. 46
t_7	1 657. 50	600. 80	246. 33	104. 92
t_8	1 663. 00	605. 20	253. 14	106. 62
t_9	1 668. 50	609. 60	256. 29	108. 31
t_{10}	1 685. 00	621. 33	259. 43	111. 89
t_{11}	1 729. 00	628. 67	262. 57	114. 40
t_{12}	1 747. 33	638. 75	265. 71	118. 80
t_{13}	1 754. 67	641. 50	268. 86	120. 10
t_{14}	1 773. 00	644. 25	277. 50	123. 20
t_{15}	1 789. 50	647. 00	283. 00	127. 60
t_{16}	1 795. 00	649. 75	288. 50	148. 85
t_{17}	1 800. 50	652. 50	299. 50	151. 41
t_{18}	1 839. 00	655. 25	305. 00	151. 93
t_{19}	1 883. 00	669. 00	310. 50	171. 94
t_{20}	2 037. 00	713. 00	327. 00	176. 56

3.2 Parameters calculation

The test data processing based on the MLE is achieved by the self-developed software, which can be used instead of the complex calculation of Weibull parameters. The software of life prediction for the VFD has some advantages, such as high speed and precision in computing and expedient application in engineering.

Based on the MLE mentioned in section 2, the iteration number k_i, the shape parameter m_i and the scale parameter η_i under each stress are shown in Tab. 2. From the data of the iteration number, it is obvious that the method of accelerating convergence is useful and effective.

Tab. 2 Iteration number k_i, shape parameter m_i and scale parameter η_i at T_i

Stress level	T_1	T_2	T_3	T_4
k_i	10	11	13	11
m_i	13. 26	16. 23	9. 22	4. 38
η_i	1 774. 85	639. 96	278. 04	129. 27

3.3 Accelerated life equation

The data of $1/T_i$ and $\ln\eta_i (i = 1, 2, 3, 4)$ under four constant stresses as well as the fitting curve are plotted in Fig. 3 with the aid of the self-developed software. The parameters of the fitting curve, that is, the unknown parameters of Eq. (2) can be obtained by LSM as follows: $\alpha = -21.89$, $\beta = 29\,985.48$. So the accelerated life equation can be written as

$$\ln\eta = -21.89 + 29\,985.48/T \tag{14}$$

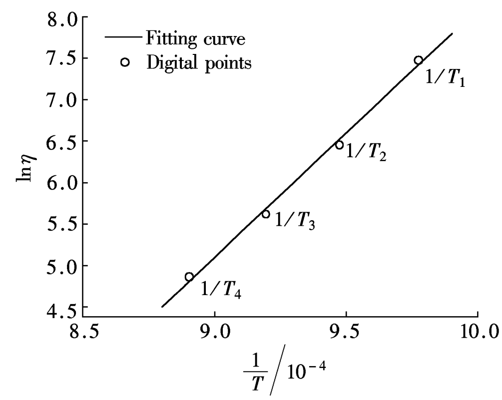

Fig. 3 Curve of life characteristic pattern

The accelerated life characteristic pattern for each stress level is found in Fig. 3. The determination coefficient of the fitting curve is 0. 996 8, which exhibits that the VFD accelerated model satisfies the linear Arrhenius equation very well.

3.4 K-S test

In order to verify whether the VFD life follows the Weibull distribution or not, the Kolmogorov-Smirnov (K-S) theory[12] is employed in this paper. A higher significance level ($\alpha = 0.2$) is selected though the number of test samples is smaller ($n_i = 20$). The results of the K-S test at four accelerated stresses calculated by the self-developed software are all equal to 0, and the values of statistic KSSTAT are $\text{KSSTAT}_1 = 0.148\,4$, $\text{KSSTAT}_2 = 0.130\,6$, $\text{KSSTAT}_3 = 0.130\,1$, $\text{KSSTAT}_4 = 0.144\,8$, respectively, which are all far less than the critical value CV (CV = 0. 231 6). So it is verified that the K-S tests at each $T_i (i = 1, 2, 3, 4)$ pass.

4 Life Estimation and Comparison

Under the condition of the Weibull distribution, the average life μ and the reliable life t_R for the VFD at T_0 can be respectively expressed as

$$\mu = \eta_0 \Gamma\left(1 + \frac{1}{m}\right), \quad t_R = \eta_0 \left(\ln \frac{1}{R}\right)^{\frac{1}{m}} \tag{15}$$

where $\Gamma(\cdot)$ is the Gamma function; R is the given reliability

($R = 0.9$ in most instances); η_0, m are the scale parameters and the shape parameters of life estimation at T_0, respectively.

According to the third basic assumption, m can be the weighted average as

$$m = \sum_{i=1}^{4} n_i m_i \bigg/ \sum_{i=1}^{4} n_i \tag{16}$$

Substituting $T_0 = 923.95$ K into Eq. (14), the scale parameter η_0 under normal working stress T_0 can be obtained as follows: $\eta_0 = 38\ 860.52$ h. From the statistical data m_i ($i = 1, 2, 3, 4$) in Tab. 2 and Eq. (16), the shape parameter m ($m_0 = m$) is 10.77. Combining Eq. (15), the average life μ and the reliable life t_R at T_0 can be respectively calculated as follows: $\mu = 37\ 083.6$ h, $t_R = 31\ 533.2$ h.

In comparison with the results of Ref. [9], the errors of the average life and the reliable life (shown in Tab. 3) are 16.56% and 16.49%, respectively. Because the data from CSALTs are more accurate than the ones from the step stress ALT, the results in this study are relatively precise and the information of VFD life can be well represented.

Tab. 3 Comparison of the average life and the reliable life

Method	Average life/h	Relative error/%	Reliable life/h	Relative error/%
Ref. [9]	44 446		37 762	
This paper	37 083.6	16.56	31 533.2	16.49

5 Conclusions

According to the above statistical analysis of four CSALTs for VFD by MLE, the conclusions are drawn as follows:

1) The proposed test plan is feasible and versatile; the VFD life follows the Weibull distribution, and the results support the Arrhenius model that the life of the VFD has an exponential relationship with the reciprocal of the operating temperature.

2) The self-developed life software makes it rapid and effective to perform the complex analysis on ALT data.

3) This study means to pave the way for follow-up research of design and manufacture of the VFD, as well as being of guiding significance for the manufacturers and users of the VFD.

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基于极大似然法的真空荧光显示器寿命预测研究

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摘要:为了在短时间内获得真空荧光显示器(VFD)的寿命信息,通过加大其灯丝温度建立了加速寿命试验模型,开展了4组恒定应力加速寿命试验. 采用威布尔函数描述VFD寿命分布,利用极大似然法(MLE)及其迭代流程图估计出形状参数和尺度参数. 通过最小二乘法确定了VFD加速寿命方程,对VFD寿命是否符合威布尔分布进行了Kolmogorov-Smirnov检验,并利用自行开发的寿命预测软件计算出平均寿命和可靠寿命. 数据统计分析结果表明,试验设计方案正确可行,VFD的寿命服从威布尔分布,VFD加速模型符合线性阿伦尼斯方程. 所提出的试验方法和估计出的VFD寿命对生产厂商和用户有很强的指导意义.

关键词:真空荧光显示器;加速寿命试验;恒定应力;威布尔分布;极大似然法

中图分类号:TN141;TN102