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Robust admissibility analysis of uncertain switched singular systems: a switched Lyapunov function approach

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Abstract: The robust admissibility analysis of a class of uncertain discrete-time switched linear singular(SLS) systems for arbitrary switching laws is addressed. The parameter uncertainty is assumed to be norm-bounded. First, by using the switched Lyapunov function approach, some new sufficient conditions ensuring the nominal discrete-time SLS system to be regular, casual and asymptotically stable for arbitrary switching laws are derived in terms of linear matrix inequalities. Then, the robust admissibility condition for the uncertain discrete-time SLS systems is presented. The obtained results can be viewed as an extension of previous works on the switched Lyapunov function approach from the regular switched linear systems to the switched linear singular cases. Numerical examples show the reduced conservatism and effectiveness of the proposed conditions.

Key words: switched singular systems; robust admissibility; switched Lyapunov function; parameter uncertainty

Recently, switched systems have received increasing attention in the control field. Switched systems are a class of hybrid systems consisting of a family of continuous- (or discrete-) time subsystems and a switching rule that specifies the switching among them[1–2]. A survey of basic problems in stability and design of switched systems has been proposed recently in Ref. [3]. As pointed out in Ref. [3], one of the interesting problems in switched systems is to find non(or less)-conservative conditions to guarantee the stability of the systems for arbitrary switching laws. A powerful tool regarding this issue is the multiple Lyapunov functions (MLF) approach[4–5]. The switched Lyapunov function (SLF), which attracts the poly-quadratic stability idea for a polytopic time varying uncertain system to solve a class of discrete-time switched control problems, essentially belongs to the MLF approach, and can be considered as a tradeoff between those conservative methodologies (using a single common Lyapunov function) and the ones less conservative but numerically harder to be checked[5].

Within the general class of switched systems, switched linear singular(SLS) systems form an important subclass—they are suitable models for many natural and man-made phenomena, for example, dynamic economic systems, electrical networks and robotics[6–8]. Therefore, it is important and, in fact, necessary to study SLS systems. Since the regularity, impulse elimination, state consistence and stability should be considered simultaneously[9], the analysis and synthesis of SLS systems are more complicated than those of regular systems. Recently, some basic research results on SLS systems have been given in Refs. [10 – 17]. However, all the above-mentioned works are based on a common Lyapunov function (or common Lyapunov-like inequalities) method, which tends to give more conservative conditions.

In this paper, we investigate the robust admissibility analysis of the discrete-time SLS systems for arbitrary switching laws and norm-bounded uncertainties. By using the switched Lyapunov function approach, some new sufficient conditions ensuring the discrete SLS system to be admissible for arbitrary switching laws are derived in terms of LMIs. The results extend the previous works on the switched Lyapunov function approach from regular switched linear systems to switched singular cases. Two examples are given to show the reduced conservatism and effectiveness of the proposed conditions.

Throughout this paper, $\mathbf{C}$ denotes the set of all complex numbers. $\mathbf{R}$ is the set of real numbers, and $\mathbf{R}^{n\times n}$ is the set of $n\times n$ dimensional real matrices. $\boldsymbol{M}>\mathbf{0}(\boldsymbol{M}<\mathbf{0})$ means that $\boldsymbol{M}$ is positive definite(negative definite). $\boldsymbol{M}\geqslant\mathbf{0}(\boldsymbol{M}\leqslant\mathbf{0})$ means that $\boldsymbol{M}$ is positive semi-definite (negative semi-definite).

1 Problem Formulation and Preliminaries

Consider the following uncertain discrete-time SLS system

$$\widetilde{\boldsymbol{E}}_{\sigma}\tilde{\boldsymbol{x}}(k+1)=(\widetilde{\boldsymbol{A}}_{\sigma}+\Delta\widetilde{\boldsymbol{A}}_{\sigma})\tilde{\boldsymbol{x}}(k) \tag{1}$$

where $\tilde{\boldsymbol{x}}(k)\in\mathbf{R}^n$ is the state. The right continuous function σ: $\{0, 1, \dots\}\to\mathscr{l}=\{1, 2, \dots, l\}$ is the switching law. Moreover, $\sigma=i$ implies that the i-th subsystem is activated. $\widetilde{\boldsymbol{E}}_i\in\mathbf{R}^{n\times n}$ and $0<\operatorname{rank}\widetilde{\boldsymbol{E}}_i\leqslant n$. $\widetilde{\boldsymbol{A}}_i(i\in\mathscr{l})$ are constant matrices of approximate dimensions. $\Delta\widetilde{\boldsymbol{A}}_i(i\in\mathscr{l})$ are the norm-bounded parameter uncertain matrices of the form

$$\Delta\widetilde{\boldsymbol{A}}_i=\widetilde{\boldsymbol{G}}_i\boldsymbol{\Gamma}_i(\rho)\widetilde{\boldsymbol{F}}_i \qquad \forall\, i\in\mathscr{l} \tag{2}$$

where $\widetilde{\boldsymbol{G}}_i$, $\widetilde{\boldsymbol{F}}_i(i\in\mathscr{l})$ are the known real constant matrices with appropriate dimensions and $\boldsymbol{\Gamma}_i(\rho)(i\in\mathscr{l})$ are the uncertainty matrices satisfies

$$\boldsymbol{\Gamma}_i^{\mathrm{T}}(\rho)\boldsymbol{\Gamma}_i(\rho)\leqslant\boldsymbol{I} \qquad \forall\,\rho\in\Sigma \tag{3}$$

where Σ is a compact set in $\mathbf{R}$.

As in Refs. [15, 17], the following assumption is made:

Assumption 1 There exist l invertible matrices $\boldsymbol{M}_1, \boldsymbol{M}_2$,

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…, M_l and an invertible matrix N such that $M_i\widetilde{E}_iN = \mathrm{diag}\{I_r, 0\} := E$, $\forall i \in \mathscr{L}$

Under assumption 1 with the transformation $x = N^{-1}\tilde{x} = [x_1^{\mathrm{T}} \quad x_2^{\mathrm{T}}]^{\mathrm{T}}$, system (1) is restricted system equivalent (r. s. e.) to

$$Ex(k+1) = (A_\sigma + \Delta A_\sigma)x(k) \tag{4}$$

where E is defined in assumption 1, $A_i = M_i\widetilde{A}_iN := \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}$ with $A_{i11} \in \mathbf{R}^{r\times r}$ and $A_{i22} \in \mathbf{R}^{(n-r)\times(n-r)}$, and $\Delta A_i = G_i\Gamma_i(\rho)F_i$ with $G_i = M_i\widetilde{G}_i$ and $F_i = \widetilde{F}_iN$, $\forall i \in \mathscr{L}$.

Definition 1[12, 15] Consider the SLS system

$$E_\sigma x(k+1) = A_\sigma x(k) \tag{5}$$

System (5) is said to be regular if there exists a constant scalar $s_i \in \mathbf{C}$ such that $\det(s_iE_i - A_i) \neq 0$, $\forall i \in \mathscr{L}$; system (5) is said to be causal if it is regular and for all $s_i \in \mathbf{C}$, $\deg(\det(s_iE_i - A_i)) = \mathrm{rank}\, E_i$, $\forall i \in \mathscr{L}$; system (5) is said to be asymptotically stable if $\lim\limits_{k\to\infty} x(k) = 0$; system (5) is said to be admissible if it is regular, casual and asymptotically stable for arbitrary switching laws.

The robust admissibility analysis problem to be addressed in this paper is to develop conditions guaranteeing that the uncertain discrete SLS system (1) is admissible for arbitrary switching laws and parameter uncertainties in formulae (2) and (3). To this end, the following lemmas are needed.

Lemma 1[18] The system $Ex(k+1) = Ax(k)$ is admissible if and only if there exists a symmetric matrix P such that $E^{\mathrm{T}}PE \geqslant 0$ and $A^{\mathrm{T}}PA - E^{\mathrm{T}}PE < 0$.

Lemma 2[5] The switched system described as

$$x(k+1) = A_\sigma x(k)$$

where σ is a switching rule which takes its values in the finite set $\mathscr{L} = \{1, 2, \ldots, l\}$, and is asymptotically stable for arbitrary switching laws if there exist l positive-definite matrices $P_1, P_2, \ldots, P_l$ such that

$$P_i - A_i^{\mathrm{T}}P_jA_i > 0 \qquad \forall (i,j) \in \mathscr{L}\times\mathscr{L}$$

2 Main Results

In this section, we present some LMI-based sufficient conditions under which the switched singular system (1) is robustly admissible for arbitrary switching laws. First, by using matrix inequalities, we present the following theorem.

Theorem 1 Under assumption 1, the SLS system (1) with $\Gamma_i(\rho) = 0 (i \in \mathscr{L})$ is admissible for arbitrary switching laws if there exist l symmetric matrices $P_1, P_2, \ldots, P_l (P_i \in \mathbf{R}^{n\times n}, i \in \mathscr{L})$ such that the following two sets of inequalities hold

$$\widetilde{E}_iP_i\widetilde{E}_i^{\mathrm{T}} \geqslant 0 \qquad \forall i \in \mathscr{L} \tag{6}$$

$$\widetilde{A}_iP_j\widetilde{A}_i^{\mathrm{T}} - \widetilde{E}_iP_i\widetilde{E}_i^{\mathrm{T}} < 0 \qquad \forall (i,j) \in \mathscr{L}\times\mathscr{L} \tag{7}$$

Proof Under assumption 1, by remark 2, system (1) with $\Gamma_i(\rho) = 0$ is r. s. e. to

$$\left.\begin{aligned} x_1(k+1) &= A_{\sigma11}x_1(k) + A_{\sigma12}x_2(k) \\ A_{\sigma21}x_1(k) &+ A_{\sigma22}x_2(k) = 0 \end{aligned}\right\} \tag{8}$$

Therefore, we only need to prove that (8) is admissible for arbitrary switching laws. If (6) and (7) hold, then from Ref. [17], we know that subsystems $(\widetilde{E}_i, \widetilde{A}_i)$, $\forall i \in \mathscr{L}$, are regular and causal. So by Ref. [9], A_{i22}, $\forall i \in \mathscr{L}$, is nonsingular. Furthermore, by definition 1, system (1) with $\Gamma_i(\rho) = 0$ is regular and causal.

We now prove the asymptotic stability of system (1) with $\Gamma_i(\rho) = 0$. Let $N^{-1}P_iN^{-\mathrm{T}} = \begin{bmatrix} P_{i11} & P_{i12} \\ P_{i12}^{\mathrm{T}} & P_{i22} \end{bmatrix}$. Then by (6), we can obtain $P_{i11} \geqslant 0$. Substituting $M_i\widetilde{E}_iN$, $N^{-1}P_iN^{-\mathrm{T}}$ and $M_i\widetilde{A}_iN$ into (7) gives

$$\begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}\begin{bmatrix} P_{j11} & P_{j12} \\ P_{j12}^{\mathrm{T}} & P_{j22} \end{bmatrix}\begin{bmatrix} A_{i11}^{\mathrm{T}} & A_{i21}^{\mathrm{T}} \\ A_{i12}^{\mathrm{T}} & A_{i22}^{\mathrm{T}} \end{bmatrix} - \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} P_{i11} & P_{i12} \\ P_{i12}^{\mathrm{T}} & P_{i22} \end{bmatrix}\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} < 0 \quad \forall (i,j) \in \mathscr{L}\times\mathscr{L} \tag{9}$$

Pre- and post-multiplying the left-hand-side matrix of (9) by $\begin{bmatrix} I & -A_{i12}A_{i22}^{-1} \\ 0 & I \end{bmatrix}$ and its transpose, we obtain

$$\begin{bmatrix} (A_{i11} - A_{i12}A_{i22}^{-1}A_{i21})P_{j11}(A_{i11} - A_{i12}A_{i22}^{-1}A_{i21})^{\mathrm{T}} - P_{i11} & * \\ * & * \end{bmatrix} < 0 \quad \forall (i,j) \in \mathscr{L}\times\mathscr{L} \tag{10}$$

where $*$ represents a matrix not used in the following discussion. Inequality (10) implies

$$(A_{i11} - A_{i12}A_{i22}^{-1}A_{i21})P_{j11}(A_{i11} - A_{i12}A_{i22}^{-1}A_{i21})^{\mathrm{T}} - P_{i11} < 0 \qquad \forall (i,j) \in \mathscr{L}\times\mathscr{L} \tag{11}$$

Using inequality (11) and noticing $P_{i11} \geqslant 0$, we obtain

$$P_{i11} > 0 \qquad \forall i \in \mathscr{L} \tag{12}$$

Furthermore, by (8) and the non-singularity of A_{i22}, we have

$$x_1(k+1) = (A_{i11} - A_{i12}A_{i22}^{-1}A_{i21})x_1(k) \qquad \forall i \in \mathscr{L} \tag{13}$$

Then by lemma 2, the sub-state $x_1(k)$ is asymptotically convergent to zero, and so is $x_2(k)$ by $x_2(k) = -A_{i22}^{-1}A_{i21}x_1(k)$. Therefore, system (8) is asymptotically stable for arbitrary switching laws, and so is the system (1) with $\Gamma_i(\rho) = 0$.

Remark 1 When $\widetilde{E}_i = I$ and $\Gamma_i(\rho) = 0$, system (1) reduces to a regular switched system and theorem 1 coincides with the stability conditions in theorem 2[5]. Therefore, theorem 1 can be regarded as a generalization of the reported results for regular switched systems to singular switched systems.

Remark 2 The condition in theorem 1 is less conservative than that in theorem 1[15] since the common Lyapunov-like inequalities are replaced by a set of switched Lyapunov-like inequalities, which is similar to the regular switched linear systems case.

By introducing the matrix $\Phi \in \mathbf{R}^{n\times(n-r)}$ satisfying rank Φ

$= n - r$ and $\widetilde{\boldsymbol{E}}_i \boldsymbol{\Phi} = \boldsymbol{0} (i \in \mathscr{L})$, we can obtain the following corollary which transforms the matrix inequality conditions to strict LMIs conditions.

Corollary 1 Under assumption 1, if there exist positive-definite matrices $\boldsymbol{X}_i \in \mathbf{R}^{n \times n} (i \in \mathscr{L})$ and symmetric matrices $\boldsymbol{Y}_i \in \mathbf{R}^{(n-r) \times (n-r)} (i \in \mathscr{L})$, such that the following one set of inequalities

$$\widetilde{\boldsymbol{A}}_i (\boldsymbol{X}_j + \boldsymbol{\Phi} \boldsymbol{Y}_j \boldsymbol{\Phi}^{\mathrm{T}}) \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} - \widetilde{\boldsymbol{E}}_i \boldsymbol{X}_i \widetilde{\boldsymbol{E}}_i^{\mathrm{T}} < \boldsymbol{0} \qquad \forall (i, j) \in \mathscr{L} \times \mathscr{L} \tag{14}$$

or the following one set of strict-LMIs

$$\begin{bmatrix} \widetilde{\boldsymbol{A}}_i \boldsymbol{\Phi} \boldsymbol{Y}_j \boldsymbol{\Phi}^{\mathrm{T}} \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} - \widetilde{\boldsymbol{E}}_i \boldsymbol{X}_i \widetilde{\boldsymbol{E}}_i^{\mathrm{T}} & \widetilde{\boldsymbol{A}}_i \boldsymbol{X}_j \\ \boldsymbol{X}_j \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} & -\boldsymbol{X}_j \end{bmatrix} < \boldsymbol{0} \qquad \forall (i, j) \in \mathscr{L} \times \mathscr{L} \tag{15}$$

hold, then the SLS system (1) with $\boldsymbol{\Gamma}_i(\rho) = \boldsymbol{0} (i \in \mathscr{L})$ is admissible for arbitrary switching laws.

Proof (sufficiency) Suppose that there exist l positive-definite matrices $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_l$, l matrices $\boldsymbol{Y}_1, \boldsymbol{Y}_2, \ldots, \boldsymbol{Y}_l$ and matrix $\boldsymbol{\Phi}$ such that (14) holds. By setting $\boldsymbol{P}_i = \boldsymbol{X}_i + \boldsymbol{\Phi} \boldsymbol{Y}_i \boldsymbol{\Phi}^{\mathrm{T}}$ $(i \in \mathscr{L})$, we can easily obtain (6) and (7) with the help of (14).

(Necessities) Suppose that l symmetric matrices $\boldsymbol{P}_1, \boldsymbol{P}_2, \ldots, \boldsymbol{P}_l$ are the solutions of (6) and (7). Choose symmetric matrices $\boldsymbol{X}_{i22} (i \in \mathscr{L})$ such that

$$\begin{bmatrix} \boldsymbol{P}_{i11} & \boldsymbol{P}_{i12} \\ \boldsymbol{P}_{i12}^{\mathrm{T}} & \boldsymbol{X}_{i22} \end{bmatrix} > \boldsymbol{0} \qquad \forall i \in \mathscr{L} \tag{16}$$

This is feasible because from the proof procedure of theorem 1 we can have $\boldsymbol{P}_{i11} > \boldsymbol{0}$ and by the Schur complement formula it suffices to choose $\boldsymbol{X}_{i22} > \boldsymbol{P}_{i12}^{\mathrm{T}} \boldsymbol{P}_{i11}^{-1} \boldsymbol{P}_{i12}$. Let $\boldsymbol{X}_i = \boldsymbol{N} \begin{bmatrix} \boldsymbol{P}_{i11} & \boldsymbol{P}_{i12} \\ \boldsymbol{P}_{i12}^{\mathrm{T}} & \boldsymbol{X}_{i22} \end{bmatrix} \boldsymbol{N}^{\mathrm{T}}$ and $\boldsymbol{Y}_i = \boldsymbol{P}_{i22} - \boldsymbol{X}_{i22} (i \in \mathscr{L})$. Then, by (16) and the nonsingularity of $\boldsymbol{N}$, we have $\boldsymbol{X}_i > \boldsymbol{0}$. Let $\boldsymbol{\Phi} = \boldsymbol{N} [0 \quad \boldsymbol{I}_{n-r}]^{\mathrm{T}}$ and by (16), we obtain $\boldsymbol{P}_i = \boldsymbol{X}_i + \boldsymbol{\Phi} \boldsymbol{Y}_i \boldsymbol{\Phi}^{\mathrm{T}} (i \in \mathscr{L})$. Therefore, (14) follows from (7). Moreover, by using the Schur complement formula, (15) is equivalent to (14).

Denote

$$\bar{\boldsymbol{P}}(\boldsymbol{X}_i, \boldsymbol{Y}_i) = \boldsymbol{X}_i + \boldsymbol{\Phi} \boldsymbol{Y}_i \boldsymbol{\Phi}^{\mathrm{T}} \qquad \forall i \in \mathscr{L} \tag{17}$$

By a similar proof procedure to those of theorem 1 and corollary 1, we obtain the following lemma.

Lemma 3 Under assumption 1, if there exist positive-definite matrices $\boldsymbol{X}_i \in \mathbf{R}^{n \times n} (i \in \mathscr{L})$, symmetric matrices $\boldsymbol{Y}_i \in \mathbf{R}^{(n-r) \times (n-r)} (i \in \mathscr{L})$, such that

$$(\widetilde{\boldsymbol{A}}_i + \Delta \widetilde{\boldsymbol{A}}_i) \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) (\widetilde{\boldsymbol{A}}_i + \Delta \widetilde{\boldsymbol{A}}_i)^{\mathrm{T}} - \widetilde{\boldsymbol{E}}_i \boldsymbol{X}_i \widetilde{\boldsymbol{E}}_i^{\mathrm{T}} < \boldsymbol{0} \qquad \forall (i, j) \in \mathscr{L} \times \mathscr{L} \tag{18}$$

then the uncertain SLS system (1) is robustly admissible for arbitrary switching laws.

We now present strict LMIs conditions under which the uncertain SLS system (1) is robustly admissible.

Theorem 2 Under assumption 1, if and only if there exist positive-definite matrices $\boldsymbol{X}_i \in \mathbf{R}^{n \times n} (i \in \mathscr{L})$, symmetric matrices $\boldsymbol{Y}_i \in \mathbf{R}^{(n-r) \times (n-r)} (i \in \mathscr{L})$, positive scalars $\varepsilon_{ij}, \gamma_{ij} (i \in \mathscr{L}, j \in \mathscr{L})$, such that the following strict LMIs hold

$$\begin{bmatrix} \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} - \widetilde{\boldsymbol{E}}_i \boldsymbol{X}_i \widetilde{\boldsymbol{E}}_i^{\mathrm{T}} + \gamma_{ij} \widetilde{\boldsymbol{G}}_i \widetilde{\boldsymbol{G}}_i^{\mathrm{T}} & \widetilde{\boldsymbol{A}}_i \bar{P}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{F}}_i^{\mathrm{T}} \\ (\widetilde{\boldsymbol{A}}_i \bar{P}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{F}}_i^{\mathrm{T}})^{\mathrm{T}} & -(\gamma_{ij} \boldsymbol{I} - \boldsymbol{Q}_{ij}) \end{bmatrix} < \boldsymbol{0} \qquad \forall (i, j) \in \mathscr{L} \times \mathscr{L} \tag{19}$$

where $\boldsymbol{Q}_{ij} = \varepsilon_{ij} \boldsymbol{I} + \widetilde{\boldsymbol{F}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{F}}_i^{\mathrm{T}}$, then the uncertain SLS system (1) is robustly admissible for arbitrary switching laws.

Proof From (19), it follows that $\gamma_{ij} \boldsymbol{I} - \boldsymbol{Q}_{ij} > \boldsymbol{0}$. By (2), (3) and the definition of $\boldsymbol{Q}_{ij}$, we obtain

$$\begin{aligned}
&(\widetilde{\boldsymbol{A}}_i + \Delta \widetilde{\boldsymbol{A}}_i) \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) (\widetilde{\boldsymbol{A}}_i + \Delta \widetilde{\boldsymbol{A}}_i)^{\mathrm{T}} - \widetilde{\boldsymbol{E}}_i \boldsymbol{X}_i \widetilde{\boldsymbol{E}}_i^{\mathrm{T}} = \\
&\quad \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} + \Delta \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} + \\
&\quad \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \Delta \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} + \Delta \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \Delta \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} - \widetilde{\boldsymbol{E}}_i \boldsymbol{X}_i \widetilde{\boldsymbol{E}}_i^{\mathrm{T}} \leqslant \\
&\quad \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} + \widetilde{\boldsymbol{G}}_i \boldsymbol{\Gamma}_i(\rho) \widetilde{\boldsymbol{F}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} + \\
&\quad \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) (\widetilde{\boldsymbol{G}}_i \boldsymbol{\Gamma}_i(\rho) \widetilde{\boldsymbol{F}}_i)^{\mathrm{T}} + \widetilde{\boldsymbol{G}}_i \boldsymbol{\Gamma}_i(\rho) \boldsymbol{Q}_{ij} (\widetilde{\boldsymbol{G}}_i \boldsymbol{\Gamma}_i(\rho))^{\mathrm{T}} - \\
&\quad \widetilde{\boldsymbol{E}}_i \boldsymbol{X}_i \widetilde{\boldsymbol{E}}_i^{\mathrm{T}} = -(\widetilde{\boldsymbol{G}}_i \boldsymbol{\Gamma}_i(\rho) - \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{F}}_i^{\mathrm{T}} (\gamma_{ij} \boldsymbol{I} - \boldsymbol{Q}_{ij})^{-1}) \cdot \\
&\quad (\gamma_{ij} \boldsymbol{I} - \boldsymbol{Q}_{ij}) (\widetilde{\boldsymbol{G}}_i \boldsymbol{\Gamma}_i(\rho) - \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{F}}_i^{\mathrm{T}} (\gamma_{ij} \boldsymbol{I} - \boldsymbol{Q}_{ij})^{-1})^{\mathrm{T}} + \\
&\quad \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} - \widetilde{\boldsymbol{E}}_i \boldsymbol{X}_i \widetilde{\boldsymbol{E}}_i^{\mathrm{T}} + \gamma_{ij} \widetilde{\boldsymbol{G}}_i \boldsymbol{\Gamma}_i(\rho) \boldsymbol{\Gamma}_i^{\mathrm{T}}(\rho) \widetilde{\boldsymbol{G}}_i^{\mathrm{T}} + \\
&\quad \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{F}}_i^{\mathrm{T}} (\gamma_{ij} \boldsymbol{I} - \boldsymbol{Q}_{ij})^{-1} \widetilde{\boldsymbol{F}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} \leqslant \\
&\quad \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} - \widetilde{\boldsymbol{E}}_i \boldsymbol{X}_i \widetilde{\boldsymbol{E}}_i^{\mathrm{T}} + \gamma_{ij} \widetilde{\boldsymbol{G}}_i \boldsymbol{\Gamma}_i(\rho) \boldsymbol{\Gamma}_i^{\mathrm{T}}(\rho) \widetilde{\boldsymbol{G}}_i^{\mathrm{T}} + \\
&\quad \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{F}}_i^{\mathrm{T}} (\gamma_{ij} \boldsymbol{I} - \boldsymbol{Q}_{ij})^{-1} \widetilde{\boldsymbol{F}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} = \\
&\quad \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{A}}_i^{\mathrm{T}} - \widetilde{\boldsymbol{E}}_i \boldsymbol{X}_i \widetilde{\boldsymbol{E}}_i^{\mathrm{T}} + \gamma_{ij} \widetilde{\boldsymbol{G}}_i \widetilde{\boldsymbol{G}}_i^{\mathrm{T}} + \\
&\quad \widetilde{\boldsymbol{A}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{F}}_i^{\mathrm{T}} (\gamma_{ij} \boldsymbol{I} - \boldsymbol{Q}_{ij})^{-1} \widetilde{\boldsymbol{F}}_i \bar{\boldsymbol{P}}(\boldsymbol{X}_j, \boldsymbol{Y}_j) \widetilde{\boldsymbol{A}}_i^{\mathrm{T}}
\end{aligned}$$

By (19) and the Schur complement formula, (18) holds. Then, by lemma 3, we can conclude that the SLS system (1) is robustly admissible for arbitrary switching laws.

3 Numerical Examples

Example 1 Consider system (1) with $\sigma \in \mathscr{L} = \{1, 2\}$ and

$$\widetilde{\boldsymbol{E}}_1 = \widetilde{\boldsymbol{E}}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{\boldsymbol{A}}_1 = \begin{bmatrix} 0.9 & 0 & 1 \\ 0 & 0.5 & -0.1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\widetilde{\boldsymbol{A}}_2 = \begin{bmatrix} -0.4 & 0 & -0.5 \\ 0.1 & -0.5 & -1 \\ -0.3 & -0.1 & 0.5 \end{bmatrix}$$

We set $\boldsymbol{\Phi} = [0 \quad 0 \quad 1]^{\mathrm{T}}$. Using the Matlab LMI control toolbox, the LMIs in corollary 2 in Ref. [15] are not feasible for this example. Therefore, we cannot conclude whether this system is admissible for arbitrary switching laws or not by corollary 2 in Ref. [15]. However, we can conclude that this system is admissible for arbitrary switching laws by solving the LMIs in (17), where a set of feasible solutions is obtained as follows:

$$\boldsymbol{X}_1 = \begin{bmatrix} 82.0859 & 19.3564 & 30.9463 \\ 19.3564 & 52.3091 & 39.4432 \\ 30.9463 & 39.4432 & 71.7134 \end{bmatrix}$$

$$\boldsymbol{X}_2 = \begin{bmatrix} 50.2535 & 34.3759 & 23.6681 \\ 34.3759 & 79.8489 & 48.1885 \\ 23.6681 & 48.1885 & 69.3152 \end{bmatrix}$$

$$\boldsymbol{Y}_1 = -197.0242, \quad \boldsymbol{Y}_2 = -133.8238$$

This means, for this example, corollary 1 in this paper is less conservative than corollary 2 in Ref. [15].

We set the initial value as $[1 \quad 2 \quad 1]^{\mathrm{T}}$ and let subsystem 1 be activated when $k \leqslant 5$ and subsystem 2 be activated when $k > 5$. Fig. 1 shows that all the state trajectories converge to the origin quickly.

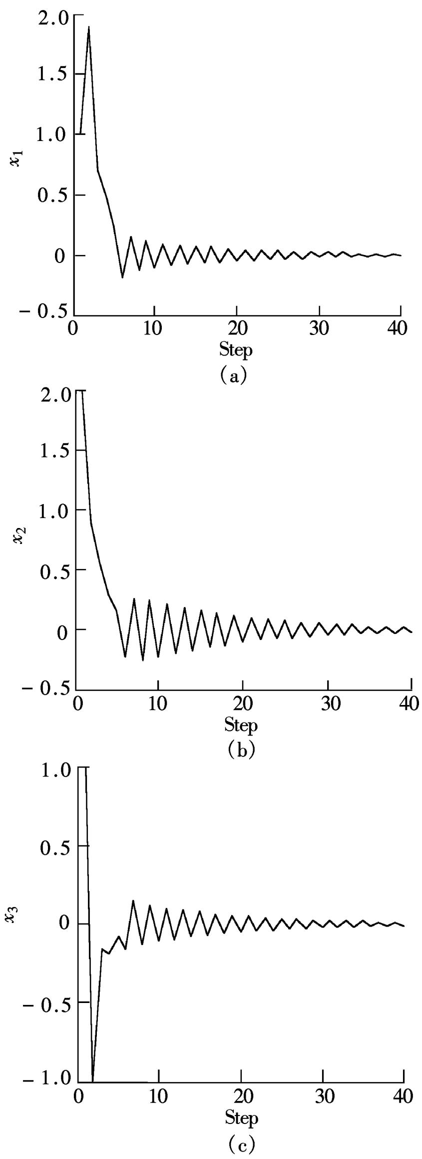

Fig. 1 State x_1, x_2 and x_3 (when switching between two systems). (a) x_1; (b) x_2; (c) x_3

Example 2 Consider system (1) with $\sigma \in \mathscr{I} = \{1, 2\}$ and

$$\widetilde{\boldsymbol{E}}_1 = \widetilde{\boldsymbol{E}}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{\boldsymbol{A}}_1 = \begin{bmatrix} 1.4472 & 0.7700 & 1.8714 \\ 0.7300 & 1.2500 & 0.6800 \\ 1.5200 & 0.8500 & 1.1900 \end{bmatrix}$$

$$\widetilde{\boldsymbol{A}}_2 = \begin{bmatrix} -0.7159 & 1.6320 & -0.1582 \\ 0.3560 & -0.4320 & 0.5640 \\ 1.1000 & 0.5720 & 1.0160 \end{bmatrix}$$

$$\widetilde{\boldsymbol{G}}_1 = [0.1 \quad 0.2 \quad 0.1]^{\mathrm{T}}, \quad \widetilde{\boldsymbol{G}}_2 = [0.2 \quad 0 \quad 0.13]^{\mathrm{T}}$$

$$\widetilde{\boldsymbol{F}}_1 = [0.2 \quad 0.1 \quad 0.2], \quad \widetilde{\boldsymbol{F}}_2 = [0 \quad 0.12 \quad 0.12]$$

Setting $\boldsymbol{\Phi} = [0 \quad 0 \quad 1]^{\mathrm{T}}$ and solving the feasibility problem of LMIs (19) gives the following solution:

$$\boldsymbol{X}_1 = \begin{bmatrix} 361.7218 & 10.5121 & -463.7764 \\ 10.5121 & 51.1258 & -22.0846 \\ -463.7764 & -22.0846 & 780.2397 \end{bmatrix}$$

$$\boldsymbol{X}_2 = \begin{bmatrix} 337.3973 & -14.0327 & -413.8702 \\ -14.0327 & 58.1922 & -22.0797 \\ -413.8702 & -22.0797 & 719.2177 \end{bmatrix}$$

$$\boldsymbol{Y}_1 = -325.4296, \quad \boldsymbol{Y}_2 = -278.2854$$

$$\varepsilon_{11} = 94.0426, \quad \gamma_{11} = 254.6283$$

$$\varepsilon_{12} = 22.8041, \quad \gamma_{12} = 93.2373$$

$$\varepsilon_{21} = 29.6115, \quad \gamma_{21} = 167.3273$$

$$\varepsilon_{22} = 13.9098, \quad \gamma_{22} = 93.3242$$

The LMIs in theorem 2 in Ref. [15] are not feasible for this example. It is clear that, for this example, theorem 2 in this paper is less conservative than theorem 2 in Ref. [15].

4 Conclusion

In this paper, the robust admissibility of uncertain discrete-time SLS systems for arbitrary switching laws and norm-bounded uncertainties is investigated. The main contribution of this paper is that the switched Lyapunov function approach is extended from regular switched systems to singular switched cases. Examples have been provided to show the reduced conservation and effectiveness of the proposed conditions. Possible research topic in the future study is to extend the results to the cases of uncertain SLS systems whose matrices $\widetilde{\boldsymbol{E}}_\sigma$ do not share a common rank.

References

[1] Morse A S. *Control using logic-based switching*[M]. London: Springer-Verlag, 1997.

[2] Ling H, Michel A, Ye H. Stability analysis of switched systems[C]//*Proceedings of the 35th IEEE Conference on Decision and Control*. Kobe, Japan, 1996: 1208 − 1212.

[3] Liberzon D, Morse A S. Basic problems in stability and design of switched system[J]. *IEEE Control Systems Magazine*, 1999, **19**(5): 59 − 70.

[4] Branicky M S. Multiple Lyapunov functions and other analysis tools for switched and hybrid systems[J]. *IEEE Transactions on Automatic Control*, 1998, **43**(4): 475 − 482.

[5] Daafouz J, Riedinger P, Iung C. Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach [J]. *IEEE Transactions on Automatic Control*, 2002, **47**(11): 1883 − 1887.

[6] Begola C, Carmen C, Elena S. Positive N-periodic descriptor control systems[J]. *System and Control Letters*, 2004, **53**(5): 407 − 414.

[7] Massarini A, Reggiani U, Kazimierczuk M K. Analysis of networks with ideal switches by state equations[J]. *IEEE Transactions on Circuits and Systems Ⅰ: Fundamental Theory and Applications*, 1997, **44**(8): 692 − 697.

[8] Mills J K, Goldenberg A A. Force and position control of ma-

nipulators during constrained motion tasks[J]. *IEEE Transactions on Robotics and Automation*, 1989, **5**(1): 30 − 46.
[9] Dai L. *Singular control systems*[M]. Berlin: Springer-Verlag, 1989.
[10] Meng B, Zhang J F. Admissible switched feedback control design of singular systems [C]//*Proceedings of the* 23*rd Chinese Control Conference*. Wuxi, China, 2004: 1615 − 1619.
[11] Yin Y J, Liu Y Z, Zhao J. Stability of a class of switched linear singular systems[J]. *Control and Decision*, 2006, **21**(1): 24 − 27. (in Chinese)
[12] Xie G M, Wang L. Stability and stabilization of switched descriptor systems under arbitrary switching[C]//*Proceedings of the* 2004 *IEEE International Conference on Systems*, *Man and Cybernetics*. Hague, Netherlands, 2004: 779 − 783.
[13] Zhai G S, Kou R, Imae J, et al. Stability analysis and design for switched descriptor systems [C]//*Proceedings of the* 2006 *IEEE International Symposium on Intelligent Control*. Munich, Germany, 2006: 482 − 487.
[14] Zhai G S, Xu X P, Imae J, et al. Qualitative analysis of switched discrete-time descriptor systems[C]//*Proceedings of the* 22*nd IEEE International Symposium on Intelligent Control, Part of IEEE Multi-Conference on Systems and Control*. Singapore, 2007: 196 − 201.
[15] Meng B, Zhang J F. Output feedback based admissible control of switched linear singular systems[J]. *Acta Automatica Sinica*, 2006, **32**(2): 181 − 185. (in Chinese)
[16] Boukas E. On robust stability of singular systems with random abrupt changes[J]. *Nonlinear Analysis*, 2005, **63**(3): 301 − 310.
[17] Lin C, Wang Q, Lee T. Stability and stabilization of a class of fuzzy time-delay descriptor systems[J]. *IEEE Transactions on Fuzzy Systems*, 2006, **14**(4): 542 − 551.
[18] Xu S Y, Lam J. Robust stability and stabilization of discrete singular systems: an equivalent characterization[J]. *IEEE Transactions on Automatic Control*, 2004, **49**(4): 568 − 574.

不确定切换奇异系统的鲁棒容许性分析：切换 Lyapunov 函数方法

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摘要：研究了一类不确定离散切换线性奇异(SLS)系统任意切换律下的鲁棒容许性问题. 假设系统参数不确定性满足范数有界条件. 首先，采用切换 Lyapunov 函数方法，给出了一些新的保证名义离散 SLS 系统任意切换律下正则、无脉冲以及渐进稳定的充分条件，且条件表示为线性矩阵不等式形式. 基于获得的条件，进一步给出了保证不确定离散 SLS 系统任意切换律下鲁棒容许的条件. 将正常切换系统的切换 Lyapunov 函数方法推广到奇异切换系统. 数值例子说明了该方法保守性的降低及可行性.

关键词：切换奇异系统；鲁棒容许性；Lyapunov 函数；参数不确定
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