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Improved delay-dependent stability criteria for stochastic systems with time-varying interval delay

Yu Jianjiang[1,2] Zhang Kanjian[1] Fei Shumin[1]

([1]School of Automation, Southeast University, Nanjing 210096, China)

([2]School of Information Science and Technology, Yancheng Teachers University, Yancheng 224002, China)

Abstract: The problem of the stability for a class of stochastic systems with time-varying interval delay and the norm-bounded uncertainty is investigated. Utilizing the information of both the lower and the upper bounds of the interval time-varying delay, a novel Lyapunov-Krasovskii functional is constructed. The delay-dependent sufficient criteria are derived in terms of linear matrix inequalities (LMIs), which can be easily checked by the LMI in the Matlab toolbox. Based on the Jensen integral inequality, neither model transformations nor bounding techniques for cross terms is employed, so the derived criteria are less conservative than the existing results. Meanwhile, the computational complexity of the obtained stability conditions is reduced because no redundant matrix is introduced. A numerical example is given to show the effectiveness and the benefits of the proposed method.

Key words: delay-dependent stability; stochastic system; interval delay; linear matrix inequalities(LMIs)

Time delays are often the main sources of instability and poor performance of a control system[1–2]. So, the robust stability analysis of dynamic systems with delays has attracted considerable attention over the past decade. In general, stability results can be classified into two types: delay-dependent stability criteria and delay-independent stability criteria. It is well known that delay-dependent stability conditions are generally less conservative than delay-independent conditions, especially when the size of the delay is small. Therefore, more and more attention has been focused on the derivation of delay-dependent stability criteria and many effective approaches have been provided to reduce the conservatism[3–10]. Furthermore, interval time-varying delay is a special type of time delay in practical engineering systems, and has been investigated in Refs. [11 – 15], in which the lower bound is not restricted to be 0.

On the other hand, stochastic systems with time delays have come to play an important role in many branches of science and industry. Many efforts have been devoted to extending the results of deterministic systems to stochastic systems[16–17]. The problems of robust stability, stabilization, H_∞ control and filtering for stochastic time-delay systems have received increasing attention[18–24].

In this paper, the robust stability of the uncertain stochastic system with time-varying interval delays is investigated. By virtue of the information of both the lower and the upper bounds, a new Lyapunov-Krasovskii functional is proposed to drive some new delay-dependent stability criteria. Based on the Jensen integral inequality, the new delay-dependent criteria are established in terms of the LMI. Model transformations and bounding techniques for cross terms are not employed. The advantage of the approach is illustrated by a numerical example.

1 Problem Formulation and Preliminaries

Consider the following stochastic delayed system

$$\mathrm{d}\boldsymbol{x}(t)=[\boldsymbol{A}(t)\boldsymbol{x}(t)+\boldsymbol{A}_1(t)\boldsymbol{x}(t-\tau(t))]\mathrm{d}t+\boldsymbol{g}(t,\boldsymbol{x}(t),\boldsymbol{x}(t-\tau(t)))\mathrm{d}\omega(t) \qquad \boldsymbol{x}(t)=\boldsymbol{\phi}(t);\ t\in[-\tau_M,0] \tag{1}$$

where $\boldsymbol{x}(t)=[x_1(t),x_2(t),\dots,x_n(t)]^{\mathrm{T}}\in\mathbf{R}^n$ are the static variables, and $\tau(t)$ is a time-varying interval delay and satisfies $0\leqslant\tau_m\leqslant\tau(t)\leqslant\tau_M$, $\dot{\tau}(t)\leqslant\mu<\infty$. $\boldsymbol{\phi}(t)\in L^2_{F_0}([-\tau_M,0];\mathbf{R}^n)$ is the initial condition. $\omega(t)$ is an m-dimensional Brownian motion defined on $(\boldsymbol{\Omega},\boldsymbol{F},\{\boldsymbol{F}_t\}_{t\geqslant0},\boldsymbol{P})$ with a natural filtration $\{\boldsymbol{F}_t\}_{t\geqslant0}$ generated by $\{\omega(s);0\leqslant s\leqslant t\}$, and it satisfies

$$\boldsymbol{E}\{\mathrm{d}\omega(t)\}=0,\ \ \boldsymbol{E}\{\mathrm{d}\omega^2(t)\}=\mathrm{d}t \tag{2}$$

The matrices $\boldsymbol{A}(t)=\boldsymbol{A}+\Delta\boldsymbol{A}(t)$, $\boldsymbol{B}(t)=\boldsymbol{B}+\Delta\boldsymbol{B}(t)$, where $\boldsymbol{A},\boldsymbol{B}\in\mathbf{R}^{n\times n}$ are the constant matrices. $\Delta\boldsymbol{A}(t)$ and $\Delta\boldsymbol{B}(t)$ are time-varying parameter uncertainties. The uncertainties are said to be admissible if the following assumptions are satisfied.

Assumption 1

$$[\Delta\boldsymbol{A}(t)\quad \Delta\boldsymbol{A}_1(t)]=\boldsymbol{M}\boldsymbol{F}(t)[\boldsymbol{E}_1\quad \boldsymbol{E}_2] \tag{3}$$

where $\boldsymbol{M},\boldsymbol{E}_1$, and $\boldsymbol{E}_2$ are the given matrices. The uncertain matrix $\boldsymbol{F}(t)$ satisfies

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Biographies: Yu Jianjiang(1975—), male, graduate; Zhang Kanjian(corresponding author), male, doctor, professor, kjzhang@ seu. edu. cn.

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$$\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t) \leqslant \boldsymbol{I} \qquad \forall t \in \mathbf{R} \tag{4}$$

Assumption 2 $\boldsymbol{g}(t) = \boldsymbol{g}(t, \boldsymbol{x}(t), \boldsymbol{x}(t-\tau(t))) \in \mathbf{R}^{n\times m}$ is locally Lipschitz continuous and satisfies the linear growth condition

$$\mathrm{tr}[\boldsymbol{g}^{\mathrm{T}}(t)\boldsymbol{g}(t)] \leqslant \|\boldsymbol{G}_1\boldsymbol{x}(t)\|^2 + \|\boldsymbol{G}_2\boldsymbol{x}(t-\tau(t))\|^2 \tag{5}$$

where $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ are constant matrices.

Definition 1 For system (1), the trivial solution is said to be mean-square asymptotically stable if

$$\lim_{t\to\infty}\boldsymbol{E}\|\boldsymbol{x}(t)\|^2 = 0 \tag{6}$$

The purpose of this paper is to derive the delay-dependent criteria for the mean-square asymptotical stability of system (1). To obtain our main results, we need the following lemmas:

Lemma 1 (Schur complement) Given constant matrices $\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2, \boldsymbol{\Omega}_3$, where $\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}_1^{\mathrm{T}}$ and $\boldsymbol{\Omega}_2 > \boldsymbol{0}$, then $\boldsymbol{\Omega}_1 + \boldsymbol{\Omega}_3^{\mathrm{T}}\boldsymbol{\Omega}_2^{-1}\boldsymbol{\Omega}_3 < \boldsymbol{0}$, if and only if $\begin{bmatrix} \boldsymbol{\Omega}_1 & \boldsymbol{\Omega}_3^{\mathrm{T}} \\ \boldsymbol{\Omega}_3 & -\boldsymbol{\Omega}_2 \end{bmatrix} < \boldsymbol{0}$, or $\begin{bmatrix} -\boldsymbol{\Omega}_2 & \boldsymbol{\Omega}_3 \\ \boldsymbol{\Omega}_3^{\mathrm{T}} & \boldsymbol{\Omega}_1 \end{bmatrix} < \boldsymbol{0}$.

Lemma 2 Let $\boldsymbol{\Phi}$ be a given symmetric matrix, and $\boldsymbol{M}$ and $\boldsymbol{G}$ be matrices with appropriate dimensions. Then, for any $\boldsymbol{F}(t)$ satisfying $\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t) \leqslant \boldsymbol{I}$, we have the following inequality

$$\boldsymbol{\Phi} + \boldsymbol{M}\boldsymbol{F}(t)\boldsymbol{G} + \boldsymbol{G}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{M}^{\mathrm{T}} < \boldsymbol{0} \tag{7}$$

holds if and only if there exists a scalar $\varepsilon > 0$, such that

$$\boldsymbol{\Phi} + \varepsilon^{-1}\boldsymbol{M}\boldsymbol{M}^{\mathrm{T}} + \varepsilon\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G} < \boldsymbol{0} \tag{8}$$

Lemma 3 (Jensen inequality) For any constant matrix $\boldsymbol{M} \in \mathbf{R}^{n\times n}$, $\boldsymbol{M} = \boldsymbol{M}^{\mathrm{T}} > 0$, scalars r_1 and r_2 satisfying $r_1 < r_2$, and a vector function $\boldsymbol{\omega}$: $[r_1, r_2] \to \mathbf{R}^n$ such that the integrations concerned are well defined, then

$$\Big(\int_{r_1}^{r_2}\boldsymbol{\omega}(s)\,\mathrm{d}s\Big)^{\mathrm{T}}\boldsymbol{M}\Big(\int_{r_1}^{r_2}\boldsymbol{\omega}(s)\,\mathrm{d}s\Big) \leqslant (r_2 - r_1)\int_{r_1}^{r_2}\boldsymbol{\omega}^{\mathrm{T}}(s)\boldsymbol{M}\boldsymbol{\omega}(s)\,\mathrm{d}s \tag{9}$$

2 Main Results

In this section, a new Lyapunov-Krasovskii functional is constructed and the following improved stability criterion is obtained.

Theorem 1 For given scalars $0 \leqslant \tau_m \leqslant \tau_M \leqslant 0$ and μ, the uncertain stochastic system (1) is robustly asymptotically stable in mean square if there exist scalar $\varepsilon > 0$, positive matrices $\boldsymbol{P} > \boldsymbol{0}$, $\boldsymbol{Q} > 0$, $\boldsymbol{R}_1 > \boldsymbol{0}$, $\boldsymbol{R}_2 > \boldsymbol{0}$, $\boldsymbol{R}_3 > \boldsymbol{0}$, $\begin{bmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} \\ \boldsymbol{Q}_{21} & \boldsymbol{Q}_{22} \end{bmatrix} > \boldsymbol{0}$, $\begin{bmatrix} \boldsymbol{Z}_{11} & \boldsymbol{Z}_{12} \\ \boldsymbol{Z}_{21} & \boldsymbol{Z}_{22} \end{bmatrix} > \boldsymbol{0}$, $n\times n$ matrix $\boldsymbol{S}$, such that the following LMI holds:

$$\boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{P}\boldsymbol{A}_1 & \boldsymbol{\Lambda}_{13} & \boldsymbol{0} & \boldsymbol{A}^{\mathrm{T}}\boldsymbol{S} & \boldsymbol{\Lambda}_{16} & \boldsymbol{0} & \boldsymbol{P}\boldsymbol{M} & \varepsilon\boldsymbol{E}_1^{\mathrm{T}} \\ & \boldsymbol{\Lambda}_{22} & \boldsymbol{0} & \boldsymbol{R}_2 & \boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{S} & \boldsymbol{0} & \boldsymbol{R}_2 & \boldsymbol{0} & \varepsilon\boldsymbol{E}_2^{\mathrm{T}} \\ & & \boldsymbol{\Lambda}_{33} & -\boldsymbol{Q}_{12} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ & & & \boldsymbol{\Lambda}_{44} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ & & & & \boldsymbol{\Lambda}_{55} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M} & \boldsymbol{0} \\ & & & & & \boldsymbol{\Lambda}_{66} & -\boldsymbol{Z}_{12} & \boldsymbol{0} & \boldsymbol{0} \\ & & & & & & \boldsymbol{\Lambda}_{77} & \boldsymbol{0} & \boldsymbol{0} \\ & & & & & & & -\varepsilon\boldsymbol{I} & \boldsymbol{0} \\ & & & & & & & & -\varepsilon\boldsymbol{I} \end{bmatrix} < 0 \tag{10}$$

where

$$\begin{gathered} \boldsymbol{\Lambda}_{11} = \boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{Q} + \boldsymbol{Q}_{11} - \boldsymbol{R}_1 - \boldsymbol{R}_3 + \boldsymbol{Z}_{11} + \boldsymbol{G}_1^{\mathrm{T}}\boldsymbol{P}\boldsymbol{G}_1, \ \boldsymbol{\Lambda}_{13} = \boldsymbol{Q}_{12} + \boldsymbol{R}_1, \ \boldsymbol{\Lambda}_{16} = \boldsymbol{R}_3 + \boldsymbol{Z}_{12} \\ \boldsymbol{\Lambda}_{22} = -2\boldsymbol{R}_2 - (1-\mu)\boldsymbol{Q} + \boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{P}\boldsymbol{G}_2, \ \boldsymbol{\Lambda}_{33} = \boldsymbol{Q}_{22} - \boldsymbol{Q}_{11} - \boldsymbol{R}_1, \ \boldsymbol{\Lambda}_{44} = -\boldsymbol{Q}_{22} - \boldsymbol{R}_2 \\ \boldsymbol{\Lambda}_{55} = -\boldsymbol{S} - \boldsymbol{S}^{\mathrm{T}} + \frac{\tau_M^2}{4}\boldsymbol{R}_1 + \delta^2\boldsymbol{R}_2 + \frac{\tau_m^2}{4}\boldsymbol{R}_3, \ \boldsymbol{\Lambda}_{66} = -\boldsymbol{R}_3 + \boldsymbol{Z}_{22} - \boldsymbol{Z}_{11}, \ \boldsymbol{\Lambda}_{77} = -\boldsymbol{R}_2 - \boldsymbol{Z}_{22}, \ \delta = \tau_{\mathrm{M}} - \tau_{\mathrm{m}} \end{gathered} \tag{11}$$

Proof Define a new vector $\boldsymbol{y}(t) \in \mathbf{R}^n$, such that

$$\boldsymbol{y}(t)\,\mathrm{d}t = \mathrm{d}\boldsymbol{x}(t), \ \boldsymbol{y}(t) = \boldsymbol{\varphi}(t) \qquad t \in [-\tau_M, 0] \tag{12}$$

From (12), we have

$$\int_{t-\tau(t)}^{t} \boldsymbol{y}(s)\,\mathrm{d}s = \boldsymbol{x}(t) - \boldsymbol{x}(t-\tau(t)) \tag{13}$$

Choose a Lyapunov-Krasovskii functional as

$$\boldsymbol{V}(\boldsymbol{x}(t),t) = \boldsymbol{V}_1(\boldsymbol{x}(t),t) + \boldsymbol{V}_2(\boldsymbol{x}(t),t) + \boldsymbol{V}_3(\boldsymbol{x}(t),t) + \boldsymbol{V}_4(\boldsymbol{x}(t),t) \tag{14}$$

where

$$\boldsymbol{V}_1(\boldsymbol{x}(t),t) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{x}(t) + \int_{t-\tau(t)}^{t} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}\boldsymbol{x}(s)\,\mathrm{d}s$$

$$\boldsymbol{V}_2(\boldsymbol{x}(t),t) = \frac{\tau_M}{2}\int_{-\frac{\tau_M}{2}}^{0}\int_{t+s}^{t} \boldsymbol{y}^{\mathrm{T}}(\theta)\boldsymbol{R}_1\boldsymbol{y}(\theta)\,\mathrm{d}\theta\mathrm{d}s + \delta\int_{-\tau_M}^{-\tau_m}\int_{t+s}^{t} \boldsymbol{y}^{\mathrm{T}}(\theta)\boldsymbol{R}_2\boldsymbol{y}(\theta)\,\mathrm{d}\theta\mathrm{d}s + \frac{\tau_m}{2}\int_{-\frac{\tau_m}{2}}^{0}\int_{t+s}^{t} \boldsymbol{y}^{\mathrm{T}}(\theta)\boldsymbol{R}_3\boldsymbol{y}(\theta)\,\mathrm{d}\theta\mathrm{d}s$$

$$\boldsymbol{V}_3(\boldsymbol{x}(t),t) = \int_{t-\frac{\tau_M}{2}}^{t} \begin{bmatrix} \boldsymbol{x}(s) \\ \boldsymbol{x}\left(s-\frac{\tau_M}{2}\right) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} \\ \boldsymbol{Q}_{21} & \boldsymbol{Q}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(s) \\ \boldsymbol{x}\left(s-\frac{\tau_M}{2}\right) \end{bmatrix} \mathrm{d}s$$

$$\boldsymbol{V}_4(\boldsymbol{x}(t),t) = \int_{t-\frac{\tau_m}{2}}^{t} \begin{bmatrix} \boldsymbol{x}(s) \\ \boldsymbol{x}\left(s-\frac{\tau_m}{2}\right) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{Z}_{11} & \boldsymbol{Z}_{12} \\ \boldsymbol{Z}_{21} & \boldsymbol{Z}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(s) \\ \boldsymbol{x}\left(s-\frac{\tau_m}{2}\right) \end{bmatrix} \mathrm{d}s$$

with $\boldsymbol{P} > \boldsymbol{0}$, $\boldsymbol{Q} > 0$, $\boldsymbol{R}_i > \boldsymbol{0}(i=1,2,3)$, $\begin{bmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} \\ \boldsymbol{Q}_{21} & \boldsymbol{Q}_{22} \end{bmatrix} > \boldsymbol{0}$, $\begin{bmatrix} \boldsymbol{Z}_{11} & \boldsymbol{Z}_{12} \\ \boldsymbol{Z}_{21} & \boldsymbol{Z}_{22} \end{bmatrix} > \boldsymbol{0}$.

It can be derived by the Itô's differential formula that

$$\mathrm{d}\boldsymbol{V}(\boldsymbol{x}(t),t) = L\boldsymbol{V}(\boldsymbol{x}(t),t)\,\mathrm{d}t + 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{g}(t)\,\mathrm{d}\omega(t) \tag{15}$$

The infinitesimal generator $L\boldsymbol{V}(\boldsymbol{x}(t),t)$ is given by

$$L\boldsymbol{V}(\boldsymbol{x}(t),t) = L\boldsymbol{V}_1(\boldsymbol{x}(t),t) + L\boldsymbol{V}_2(\boldsymbol{x}(t),t) + L\boldsymbol{V}_3(\boldsymbol{x}(t),t) + L\boldsymbol{V}_4(\boldsymbol{x}(t),t) \tag{16}$$

and

$$\begin{aligned} L\boldsymbol{V}_1(\boldsymbol{x}(t),t) &= 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}[\boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{A}_1(t)\boldsymbol{x}(t-\tau(t))] + \boldsymbol{g}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{g}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{x}(t) - (1-\dot{\tau}(t))\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}\boldsymbol{x}(t-\tau(t)) \leqslant \\ & 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}[\boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{A}_1(t)\boldsymbol{x}(t-\tau(t))] + \boldsymbol{g}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{g}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{x}(t) - (1-\mu)\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}\boldsymbol{x}(t-\tau(t)) \end{aligned} \tag{17}$$

$$\begin{aligned} L\boldsymbol{V}_2(\boldsymbol{x}(t),t) &= \boldsymbol{y}^{\mathrm{T}}(t)\left(\frac{\tau_M^2}{4}\boldsymbol{R}_1 + \delta^2\boldsymbol{R}_2 + \frac{\tau_m^2}{4}\boldsymbol{R}_3\right)\boldsymbol{y}(t) - \frac{\tau_M}{2}\int_{t-\frac{\tau_M}{2}}^{t} \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{R}_1\boldsymbol{y}(s)\,\mathrm{d}s - \delta\int_{t-\frac{\tau_M}{2}}^{t-\frac{\tau_m}{2}} \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{R}_2\boldsymbol{y}(s)\,\mathrm{d}s - \\ & \frac{\tau_m}{2}\int_{t-\frac{\tau_m}{2}}^{t} \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{R}_3\boldsymbol{y}(s)\,\mathrm{d}s \end{aligned} \tag{18}$$

According to lemma 2, we have

$$-\frac{\tau_M}{2}\int_{t-\frac{\tau_M}{2}}^{t} \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{R}_1\boldsymbol{y}(s)\,\mathrm{d}s \leqslant \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}\left(t-\frac{\tau_M}{2}\right) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\boldsymbol{R}_1 & \boldsymbol{R}_1 \\ \boldsymbol{R}_1 & -\boldsymbol{R}_1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}\left(t-\frac{\tau_M}{2}\right) \end{bmatrix} \tag{19}$$

$$\begin{aligned} -\delta\int_{t-\tau_M}^{t-\tau_m} \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{R}_2\boldsymbol{y}(s)\,\mathrm{d}s &= -\delta\int_{t-\tau(t)}^{t-\tau_m} \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{R}_2\boldsymbol{y}(s)\,\mathrm{d}s - \delta\int_{t-\tau_M}^{t-\tau(t)} \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{R}_2\boldsymbol{y}(s)\,\mathrm{d}s \leqslant \begin{bmatrix} \boldsymbol{x}(t-\tau_m) \\ \boldsymbol{x}(t-\tau(t)) \end{bmatrix}^{\mathrm{T}} \cdot \\ & \begin{bmatrix} -\boldsymbol{R}_2 & \boldsymbol{R}_2 \\ \boldsymbol{R}_2 & -\boldsymbol{R}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t-\tau_m) \\ \boldsymbol{x}(t-\tau(t)) \end{bmatrix} + \begin{bmatrix} \boldsymbol{x}(t-\tau(t)) \\ \boldsymbol{x}(t-\tau_M^{\mathrm{T}}) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\boldsymbol{R}_2 & \boldsymbol{R}_2 \\ \boldsymbol{R}_2 & -\boldsymbol{R}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t-\tau(t)) \\ \boldsymbol{x}(t-\tau_M) \end{bmatrix} \end{aligned} \tag{20}$$

and

$$-\frac{\tau_m}{2}\int_{t-\frac{\tau_m}{2}}^{t} \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{R}_3\boldsymbol{y}(s)\,\mathrm{d}s \leqslant \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}\left(t-\frac{\tau_m}{2}\right) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\boldsymbol{R}_3 & \boldsymbol{R}_3 \\ \boldsymbol{R}_3 & -\boldsymbol{R}_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}\left(t-\frac{\tau_m}{2}\right) \end{bmatrix} \tag{21}$$

It is also easy to obtain

$$L\boldsymbol{V}_3(\boldsymbol{x}(t),t) = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}\left(t-\frac{\tau_M}{2}\right) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} \\ \boldsymbol{Q}_{21} & \boldsymbol{Q}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}\left(t-\frac{\tau_M}{2}\right) \end{bmatrix} - \begin{bmatrix} x\left(t-\frac{\tau_M}{2}\right) \\ \boldsymbol{x}(t-\tau_M) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} \\ \boldsymbol{Q}_{21} & \boldsymbol{Q}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}\left(t-\frac{\tau_M}{2}\right) \\ \boldsymbol{x}(t-\tau_M) \end{bmatrix} \tag{22}$$

$$LV_4(\boldsymbol{x}(t),t) = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}\left(t-\frac{\tau_m}{2}\right) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{Z}_{11} & \boldsymbol{Z}_{12} \\ \boldsymbol{Z}_{21} & \boldsymbol{Z}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}\left(t-\frac{\tau_m}{2}\right) \end{bmatrix} - \begin{bmatrix} \boldsymbol{x}\left(t-\frac{\tau_m}{2}\right) \\ \boldsymbol{x}(t-\tau_m) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{Z}_{11} & \boldsymbol{Z}_{12} \\ \boldsymbol{Z}_{21} & \boldsymbol{Z}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}\left(t-\frac{\tau_m}{2}\right) \\ \boldsymbol{x}(t-\tau_m) \end{bmatrix} \tag{23}$$

On the other hand,

$$\mathrm{tr}\{\boldsymbol{g}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{g}(t)\} \leqslant \mathrm{tr}\{\boldsymbol{\xi}_1^{\mathrm{T}}(t)\begin{bmatrix} \boldsymbol{G}_1^{\mathrm{T}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{G}_2^{\mathrm{T}} \end{bmatrix}\boldsymbol{P}\begin{bmatrix} \boldsymbol{G}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{G}_2 \end{bmatrix}\boldsymbol{\xi}_1(t)\} \leqslant \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{G}_1^{\mathrm{T}}\boldsymbol{P}\boldsymbol{G}_1\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{P}\boldsymbol{G}_2\boldsymbol{x}(t-\tau(t)) \tag{24}$$

where $\boldsymbol{\xi}_1^{\mathrm{T}}(t) = (\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}^{\mathrm{T}}(t-\tau(t)))^{\mathrm{T}}$.

From Eq. (12), for any matrix $\boldsymbol{S} \in \mathbf{R}^{n\times n}$, we have

$$2\boldsymbol{y}^{\mathrm{T}}(t)\boldsymbol{S}^{\mathrm{T}}\{[\boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{A}_1(t)\boldsymbol{x}(t-\tau(t)) - \boldsymbol{y}(t)]\mathrm{d}t + \boldsymbol{g}(t)\mathrm{d}\omega(t)\} = 0 \tag{25}$$

According to Eqs. (16) to (25), we have

$$\mathrm{d}V(\boldsymbol{x}(t),t) = L\bar{V}(\boldsymbol{x}(t),t)\mathrm{d}t + 2[\boldsymbol{P}\boldsymbol{x}(t) + \boldsymbol{S}\boldsymbol{y}(t)]^{\mathrm{T}}\boldsymbol{g}(t)\mathrm{d}\omega(t) \tag{26}$$

where

$$L\bar{V}(\boldsymbol{x}(t),t) = LV(\boldsymbol{x}(t),t) + 2\boldsymbol{y}^{\mathrm{T}}(t)\boldsymbol{S}^{\mathrm{T}}[\boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{A}_1(t)\boldsymbol{x}(t-\tau(t)) - \boldsymbol{y}(t)]\mathrm{d}t \leqslant \boldsymbol{\xi}^{\mathrm{T}}(t)\hat{\boldsymbol{\Lambda}}\boldsymbol{\xi}(t) \tag{27}$$

with

$$\boldsymbol{\xi}^{\mathrm{T}}(t) = \left[\boldsymbol{\xi}_1^{\mathrm{T}}(t)\boldsymbol{x}^{\mathrm{T}}\left(t-\frac{\tau_M}{2}\right)\boldsymbol{x}^{\mathrm{T}}(t-\tau_M)\boldsymbol{y}^{\mathrm{T}}(t)\boldsymbol{x}^{\mathrm{T}}\left(t-\frac{\tau_m}{2}\right)\boldsymbol{x}^{\mathrm{T}}(t-\tau_m)\right]$$

$$\hat{\boldsymbol{\Lambda}} = \begin{bmatrix} \hat{\boldsymbol{\Lambda}}_{11} & \boldsymbol{P}\boldsymbol{A}_1(t) & \boldsymbol{\Lambda}_{13} & \boldsymbol{0} & \boldsymbol{A}(t)^{\mathrm{T}}\boldsymbol{S} & \boldsymbol{\Lambda}_{16} & \boldsymbol{0} \\ & \hat{\boldsymbol{\Lambda}}_{22} & \boldsymbol{0} & \boldsymbol{R}_2 & \boldsymbol{A}_1(t)^{\mathrm{T}}\boldsymbol{S} & \boldsymbol{0} & \boldsymbol{R}_2 \\ & & \boldsymbol{\Lambda}_{33} & -\boldsymbol{Q}_{12} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ & & & \boldsymbol{\Lambda}_{44} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ & & & & \boldsymbol{\Lambda}_{55} & \boldsymbol{0} & \boldsymbol{0} \\ & & & & & \boldsymbol{\Lambda}_{66} & \boldsymbol{0} \\ & & & & & & \boldsymbol{\Lambda}_{77} \end{bmatrix} \tag{28}$$

and $\hat{\boldsymbol{\Lambda}}_{11} = \boldsymbol{P}\boldsymbol{A}(t) + \boldsymbol{A}^{\mathrm{T}}(t)\boldsymbol{P} + \boldsymbol{Q} + \boldsymbol{Q}_{11} - \boldsymbol{R}_1 - \boldsymbol{R}_3 + \boldsymbol{Z}_{11} + \boldsymbol{G}_1^{\mathrm{T}}\boldsymbol{P}\boldsymbol{G}_1$, $\hat{\boldsymbol{\Lambda}}_{22} = -2\boldsymbol{R}_2 - (1-\mu)\boldsymbol{Q} + \boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{P}\boldsymbol{G}_2$.

Then, $\hat{\boldsymbol{\Lambda}}$ can be rewritten as

$$\hat{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda}_0 + \begin{bmatrix} \boldsymbol{P}\boldsymbol{M} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{F}(t)[\boldsymbol{E}_1 \quad \boldsymbol{E}_2 \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0}] + \begin{bmatrix} \boldsymbol{E}_1^{\mathrm{T}} \\ \boldsymbol{E}_2^{\mathrm{T}} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{F}(t)[\boldsymbol{M}^{\mathrm{T}}\boldsymbol{P} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{M}^{\mathrm{T}}\boldsymbol{S} \quad \boldsymbol{0} \quad \boldsymbol{0}] \tag{29}$$

where

$$\boldsymbol{\Lambda}_0 = \begin{bmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12} & \boldsymbol{Q}_{12} + \boldsymbol{R}_1 & \boldsymbol{0} & \boldsymbol{A}^{\mathrm{T}}\boldsymbol{S} & \boldsymbol{R}_3 + \boldsymbol{Z}_{12} & \boldsymbol{0} \\ & \boldsymbol{\Lambda}_{22} & \boldsymbol{0} & \boldsymbol{R}_2 & \boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{S} & \boldsymbol{0} & \boldsymbol{R}_2 \\ & & \boldsymbol{\Lambda}_{33} & -\boldsymbol{Q}_{12} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ & & & \boldsymbol{\Lambda}_{44} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ & & & & \boldsymbol{\Lambda}_{55} & \boldsymbol{0} & \boldsymbol{0} \\ & & & & & \boldsymbol{\Lambda}_{66} & \boldsymbol{0} \\ & & & & & & \boldsymbol{\Lambda}_{77} \end{bmatrix} \tag{30}$$

From lemma 1 and lemma 2, $\boldsymbol{\Lambda} < \boldsymbol{0}$ is equivalent to $\hat{\boldsymbol{\Lambda}} < \boldsymbol{0}$, which implies that system (1) is robustly asymptotically stable in the mean-square sense. This completes the proof.

Remark 1 In theorem 1, the Lyapunov functional (14) is more general, and the obtained stability is less conservative than the existing ones. Meanwhile, only one slack matrix $\boldsymbol{S}$ is involved, and the computational complexity is reduced. So, our stability criteria are more efficient and less conservative.

Remark 2 When μ is unknown, by setting $\boldsymbol{Q}=\boldsymbol{0}$ in (14), we can obtain a delay-dependent and rate-independent mean-square asymptotically stable criterion of system (1) from theorem 1.

Remark 3 When handling the term $\mathrm{tr}\{\boldsymbol{g}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{g}(t)\}$, some previous works, such as Ref. [20], assume that $\boldsymbol{P}<\lambda\boldsymbol{I}$, where λ is the maximum eigenvalue of $\boldsymbol{P}$. This conservative restriction is removed in our method.

Next, we consider system (1) with the routine delay case described by $0\leqslant\tau(t)\leqslant\tau_M$.

Corollary 1 For given scalars $0\leqslant\tau_M\leqslant 0$ and μ, the uncertain stochastic system(1) is robustly asymptotically stable in mean square if there exist scalar $\varepsilon>0$, positive matrices $\boldsymbol{P}>\boldsymbol{0}$, $\boldsymbol{Q}>0$, $\boldsymbol{R}_1>\boldsymbol{0}$, $\boldsymbol{R}_2>\boldsymbol{0}$, $\begin{bmatrix}\boldsymbol{Q}_{11} & \boldsymbol{Q}_{12}\\ \boldsymbol{Q}_{21} & \boldsymbol{Q}_{22}\end{bmatrix}>\boldsymbol{0}$, $n\times n$ matrix $\boldsymbol{S}$, such that the following LMI holds:

$$\boldsymbol{\Xi}=\begin{bmatrix}\boldsymbol{\Xi}_{11} & \boldsymbol{\Xi}_{12} & \boldsymbol{Q}_{12}+\boldsymbol{R}_1 & \boldsymbol{0} & \boldsymbol{A}^{\mathrm{T}}\boldsymbol{S} & \boldsymbol{PM} & \varepsilon\boldsymbol{E}_1^{\mathrm{T}}\\ & \boldsymbol{\Xi}_{22} & \boldsymbol{0} & \boldsymbol{R}_2 & \boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{S} & \boldsymbol{0} & \varepsilon\boldsymbol{E}_2^{\mathrm{T}}\\ & & \boldsymbol{\Xi}_{33} & -\boldsymbol{Q}_{12} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ & & & -\boldsymbol{Q}_{22}-\boldsymbol{R}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ & & & & \boldsymbol{\Xi}_{55} & \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M} & \boldsymbol{0}\\ & & & & & -\varepsilon\boldsymbol{I} & \boldsymbol{0}\\ & & & & & & -\varepsilon\boldsymbol{I}\end{bmatrix}<0 \tag{31}$$

where $\boldsymbol{\Xi}_{11}=\boldsymbol{PA}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{Q}+\boldsymbol{Q}_{11}-\boldsymbol{R}_1-\boldsymbol{R}_2+\boldsymbol{G}_1^{\mathrm{T}}\boldsymbol{PG}_1$, $\boldsymbol{\Xi}_{12}=\boldsymbol{PA}_1+\boldsymbol{R}_2$, $\boldsymbol{\Xi}_{22}=-2\boldsymbol{R}_2-(1-\mu)\boldsymbol{Q}+\boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{PG}_2$, $\boldsymbol{\Xi}_{33}=\boldsymbol{Q}_{22}-\boldsymbol{Q}_{11}-\boldsymbol{R}_1$, $\boldsymbol{\Xi}_{55}=-\boldsymbol{S}-\boldsymbol{S}^{\mathrm{T}}+\frac{\tau_M^2}{4}\boldsymbol{R}_1+\delta^2\boldsymbol{R}_2$.

Proof Choose the Lyapunov-Krasovskii functional as

$$\begin{aligned}V(\boldsymbol{x}(t),t)={}&\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Px}(t)+\int_{t-\tau(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Qx}(s)\,\mathrm{d}s+\frac{\tau_M}{2}\int_{-\frac{\tau_M}{2}}^{0}\int_{t+s}^{t}\boldsymbol{y}^{\mathrm{T}}(\theta)\boldsymbol{R}_1\boldsymbol{y}(\theta)\,\mathrm{d}\theta\mathrm{d}s+\tau_M\int_{-\frac{\tau_M}{2}}^{0}\int_{t+s}^{t}\boldsymbol{y}^{\mathrm{T}}(\theta)\boldsymbol{R}_2\boldsymbol{y}(\theta)\,\mathrm{d}\theta\mathrm{d}s+{}\\&\int_{t-\frac{\tau_M}{2}}^{t}\begin{bmatrix}\boldsymbol{x}(s)\\ \boldsymbol{x}\left(s-\frac{\tau_M}{2}\right)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{Q}_{11} & \boldsymbol{Q}_{12}\\ \boldsymbol{Q}_{21} & \boldsymbol{Q}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(s)\\ \boldsymbol{x}\left(s-\frac{\tau_M}{2}\right)\end{bmatrix}\end{aligned} \tag{32}$$

The proof of corollary 1 is similar to that of theorem 1, and it is omitted here.

3 Numerical Example

In this section, we provide a numerical example to show the effectiveness of our results.

Example 1 Consider the delayed uncertain stochastic system (1) with

$$\boldsymbol{A}=\begin{bmatrix}-2 & 0\\ 1 & -1\end{bmatrix},\ \boldsymbol{A}_1=\begin{bmatrix}-1 & 0\\ -0.5 & -1\end{bmatrix},\ \boldsymbol{M}=\boldsymbol{I}$$

$$\boldsymbol{E}_1=\boldsymbol{E}_2=0.1\boldsymbol{I},\ \boldsymbol{G}_1=\boldsymbol{G}_2=\sqrt{0.1}\boldsymbol{I} \tag{33}$$

First, we consider the constant time delay case. Tab. 1 lists the maximal allowable delays given by corollary 1 and the existing methods. It is obvious that corollary 1 is much less conservative than the methods in the literature. Then, for the time-varying interval delay, the maximal allowable delays given by our results are illustrated in Tab. 2.

From Tab. 1 and Tab. 2, we can see that our results are less conservative and more general than those in the literature.

Tab. 1 Comparison of maximal allowable delays by different methods ($\mu=0$)

Methods	τ_M
Ref. [20]	1.181 2
Ref. [19]	1.364 0
Ref. [21]	1.527 0
Ref. [23]	1.56
Ref. [24]	2.898 7
Corollary 1	2.933 0

Tab. 2 The maximal allowable delays by different τ_m and μ

τ_m	μ				
	0	0.1	0.5	0.9	Unknown
0	2.933 0	2.486 7	1.508 3	1.040 8	1.010 3
0.1	2.934 4	2.488 0	1.510 7	1.058 5	1.037 8
0.3	2.939 9	2.494 1	1.521 0	1.118 7	1.118 2

4 Conclusion

This paper presents improved results to test the robust stability of stochastic delayed systems with admissible uncertainty and time-varying delays in a range. The results are obtained by constructing a new class of Lyapunov-Krasovskii functional. Neither model transformation nor cross-term bounding techniques are used in this paper. Therefore, the presented criteria are less conservative than the existing ones, and have been demonstrated by a numerical example.

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区间时滞随机系统的时滞相关稳定性改进判据

于建江[1,2]　张侃健[1]　费树岷[1]

([1] 东南大学自动化学院,南京 210096)
([2] 盐城师范学院信息科学与技术学院,盐城 224002)

摘要:讨论了一类具有区间时变时滞的不确定随机系统的稳定性问题.利用区间时滞的上、下界信息,构造了一个新颖的 Lyapunov-Krasovskii 泛函.以线性矩阵不等式(LMIs)形式给出了时滞相关稳定性的充分判据,利用 Matlab 工具箱可以很容易对这些判据进行检验.推导过程基于 Jensen 积分不等式方法,避免了系统模型变换和交叉项有界等易于产生保守性的方法的使用,故得出判据的保守性小于文献中已有的结果.由于在获得的稳定性条件中没有引入多余的矩阵变量,因此所得判据的计算复杂度明显降低.最后,用一个数值例子说明了该方法的有效性和具有的优势.

关键词:时滞相关稳定性;随机系统;区间时滞;线性矩阵不等式

中图分类号:TP271$^+$.61