

# Simplified stiffness-based approach for seismic performance evaluation of moment-resisting frame

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**Abstract:** Based on the concept of stiffness degradation, a damage index of the whole frame and the storey is proposed for the frame seismic performance evaluation. The index is compatible with the non-linear static analysis (e. g. the pushover analysis), and the structural damage is considered via plastic hinges. Simultaneously, a practical approach is developed to obtain the relationships between the proposed index and earthquake intensities based on the capacity spectrum method. The proposed index is then illustrated through two low-rise reinforced concrete frames, and it is also compared with some other indices. The results indicate that the proposed index is on the safe side and not sensitive to the lateral load pattern. The storey index is helpful to reflect the storey damage and to uncover the position of the weak storey. Finally, the relationship between performance levels and damage index values is also proposed through statistical analysis for the performance-based seismic evaluation.

**Key words:** damage index; seismic evaluation; capacity spectrum method; frame; lateral load pattern; pushover analysis; performance level; stiffness degradation

When a building is subjected to an earthquake, the structural and nonstructural members may experience damage. The seismic damage may be measured in terms of the expected states according to performance-based concepts: minor damage, repairable damage, severe damage and collapse. And each damage state can be quantified by damage indices. These damage indices are used as a criteria in structural design provisions as well as in the evaluation and retrofit of existing buildings<sup>[1]</sup>, such that the damage in a structure does not exceed an acceptable threshold.

After reviewing some of the important researches on structure damage models, we develop a simplified approach for seismic performance evaluation with the concept of stiffness degradation. The proposed approach is based on the pushover analysis and design response spectra.

## 1 Previous Researches and Proposed Evaluation Approach

### 1.1 Previous researches on damage indices

The analytical damage models may involve various degrees of complexity as they account for the characteristics of a structure and its seismic response. They can be broadly di-

vided into three classes as follows:

1) Empirical damage indices<sup>[2]</sup> The empirical damage models are based on statistics of observed structural damage following earthquakes. Although these damage observations may be subjective, they provide useful qualitative information on the overall seismic performance of structural systems. Empirical damage indices do not take into consideration the mechanics of materials that undergo large inelastic deformation and are not capable of rationally predicting the reserve strength of the structure.

2) Strength-based damage indices They were first proposed in 1968 by Shiga et al.<sup>[3]</sup> and later were applied by Yang et al.<sup>[4]</sup>. Strength-based damage indices are easy to calculate and do not require response analysis. However, the indices must be calibrated using a large database from tests or observations following earthquakes.

3) Response-based damage indices Response-based damage indices are based on the response parameters of elements (local) or the whole structure (global). The analysis attempts to relate damage to the capacity of the structure to undergo maximum deformation and/or cumulative damage. There are some examples of response-based damage indices, such as ductility ratio, interstorey drift, maximum permanent drift, low cycle fatigue, Park and Ang index<sup>[5]</sup>, final softening index, etc.

In 1999, Ghobarah et al.<sup>[6]</sup> developed an index using the initial slopes of two performance curves, which are obtained from the pushover analysis before and after the application of the expected ground motion, as follows:

$$D_G = 1 - \frac{k_{\text{final}}}{k_{\text{initial}}} \quad (1)$$

By the initial slopes of the base shear-storey drift relationship for the  $n$ -th storey, the stiffness damage index for each storey can also be obtained from

$$D_G^n = 1 - \frac{k_{\text{final}}^n}{k_{\text{initial}}^n} \quad (2)$$

Then, Ghobarah et al. compared the proposed damage index with three other damage indices. The results show that there is a reasonable correlation among various damage indices by completely different approaches.

### 1.2 Proposed evaluation approach with pseudo-stiffness degradation index

The Ghobarah index is compatible with the pushover analysis procedure, which is suitable for seismic evaluation of buildings. The index is based on the comparison of the stiff-

Received 2008-11-17.

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**Foundation item:** The National Basic Research Program of China (973 Program) (No. 2007CB714200).

**Citation:** Yu Qi, Meng Shaoping, Wu Jing, et al. Simplified stiffness-based approach for seismic performance evaluation of moment-resisting frame[J]. Journal of Southeast University (English Edition), 2009, 25(2): 241 – 246.

ness of structures before and after the application of an earthquake. However, there are two main drawbacks: 1) The procedure is comparatively complex for performing a non-linear dynamic analysis with the expected ground motion; 2) Each value of the Ghobarah index is obtained by a specific non-linear dynamic analysis. If a structure performance is required when experiencing earthquakes of different intensities, the earthquake records scaled to different peak ground accelerations have to be input into the model. The non-linear dynamic analysis will be repeated. But what we are concerned about is the buildings' (new or existing) seismic performance under future earthquakes, of which the characteristics (intensity, frequency and duration) are aleatoric. It is not appropriate to utilize a specific earthquake record to perform the evaluation procedure of structures. In the following part, a simplified damage index is presented and the corresponding seismic evaluation approach is developed based on the design response spectra.

### 1.2.1 Proposed simplified damage index

The basic concept of the proposed index is the same as that of the Ghobarah index. But it is easier to be implemented without non-linear dynamic analyses. The procedure is shown as follows:

- 1) Model the structure to be evaluated.
- 2) Perform the first pushover analysis on the model without damage and record the development process of the plastic hinges on all the beams and the columns until the structure forms a typical yielding mechanism. The initial slope  $k_0$  resulting from this pushover curve can be obtained.
- 3) Modify the original model in step 1). Release the rotation freedom at the end of the beams and/or the columns where the first group of hinges forms. Then carry out the pushover analysis on the new model. With the second pushover curve we can obtain a new initial stiffness  $k_1$ .
- 4) Modify the model in step 3). Release the rotation freedom of the second group of hinges at the end of the components and perform the pushover analysis on the subsequent model. The initial stiffness  $k_2$  with the new pushover curve can be recorded.
- 5) Repeat the same steps as with the development process of the plastic hinges, and we can obtain a series of degraded stiffnesses  $k_3, k_4, \dots$  until the structure collapses.
- 6) Substitute the stiffnesses calculated in steps 2) to 5) into Eq. (1). The proposed pseudo-stiffness degradation index is defined as

$$D_i = 1 - \frac{k_i}{k_0} \quad (3)$$

where  $k_i$  represents the initial stiffness of the whole frame when the  $i$ -th group of plastic hinges forms and  $k_0$  represents the initial stiffness of the whole frame without damage. The damage index for the  $j$ -th storey can also be obtained in the same way as Eq. (2),

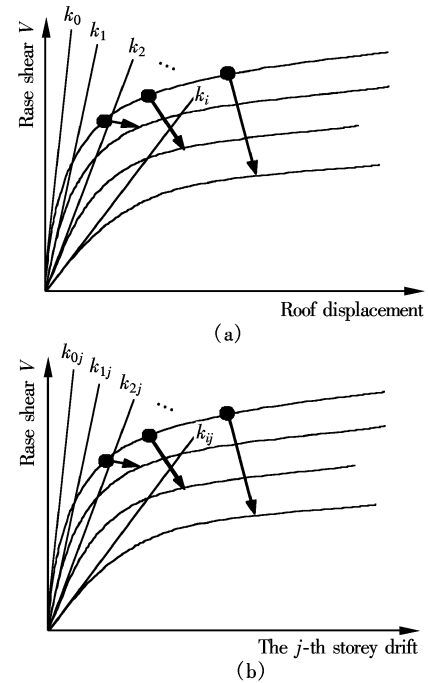
$$D_{ij} = 1 - \frac{k_{ij}}{k_{0j}} \quad (4)$$

where  $k_{ij}$  is the initial slope of the base shear-storey drift relationship of the  $j$ -th storey when the  $i$ -th of group of plastic

hinges forms and  $k_{0j}$  is the slope of the same relationship of the  $j$ -th storey without damage.

The proposed damage index is consistent with the procedure of the pushover analysis. Values of the indices range from zero to one. Zero represents no damage while one corresponds to collapse. However, in practical terms, collapse may be defined at a lower damage value due to a certain percentage loss of stiffness. The storey damage index in Eq. (4) can be used to determine the distribution of damage to various storeys.

Eq. (3) and Eq. (4) indicate that there are different damage values for different points on the first pushover curve in Fig. 1. Therefore, in order to assess the seismic performance of structures with the proposed index, the relationship between points on the first pushover curve and earthquake intensities has to be established.



**Fig. 1** Proposed damage index with pushover analysis.  
(a) Overall damage index; (b) Storey damage index

### 1.2.2 Relationship between pushover curve and peak ground acceleration(PGA)

As discussed above, it is unsuitable to utilize a specific earthquake record to conduct the evaluation procedure of structures. The design response spectrum is then proposed as a statistical result of many different earthquake records to rationally and conveniently assess the seismic performance of structures, such as the capacity spectrum method(CSM)<sup>[7-8]</sup>. In the CSM, an iterative process is required by various effective viscous dampings  $\zeta_{eq}$  to catch the performance points on the capacity spectrum. The point is consistent with the demand spectrum (transformed by the design spectrum) and then transformed to the corresponding target displacement which is used to evaluate the structure performance.

Thus, the CSM is based on finding the point on the pushover curve with expected earthquake intensity (demand spectrum) by an iterative calculation. Contrarily, an earthquake intensity (represented by PGA) can also be found with a specific point on the pushover curve but does not necessitate it-

erations. In this section, a methodology to obtain the relationship between a pushover curve and earthquake intensities (PGA) is discussed. Then the relationships between the proposed damage index  $D$  and the PGA can be established to evaluate the seismic performance of structures under earthquakes of various intensities.

1) The pushover curve (base shear  $V$  vs. roof displacement  $\Delta_r$ ) of a multi-degrees-of-freedom (MDOF) system can be converted to an acceleration-displacement response spectra (ADRS) format of equivalent single-degree-of-freedom (ESDOF) called the capacity spectrum by the following equations<sup>[7]</sup>:

$$S_a = \frac{V}{M_1^*} \quad (5)$$

$$S_d = \frac{\Delta_r}{\Gamma_1 \phi_{r,1}} \quad (6)$$

$$M_1^* = \frac{\left( \sum_{i=1}^N m_i \phi_{i,1} \right)^2}{\sum_{i=1}^N m_i \phi_{i,1}^2} \quad (7)$$

$$\Gamma_1 = \frac{\sum_{i=1}^N m_i \phi_{i,1}}{\sum_{i=1}^N m_i \phi_{i,1}^2} \quad (8)$$

where  $M_1^*$  is the modal mass coefficient for the first natural mode and  $\Gamma_1$  represents the modal participation factor.  $S_a$  and  $S_d$  are the spectral acceleration and the spectral displacement, respectively.

2) A specific design response spectrum ( $S_a$  vs.  $T$ ) can also be converted to the ADRS format ( $S_a$  vs.  $S_d$ ) called the demand spectrum by the following equation:

$$S_d = \frac{T^2}{4\pi^2} S_a \quad (9)$$

In the code for the seismic design of buildings<sup>[9]</sup>, a typical earthquake design response spectrum is presented in Fig. 2, where  $\alpha$  is the earthquake affecting coefficient:

$$\alpha = \frac{S_a}{g} = \frac{S_a}{\text{PGA}} \frac{\text{PGA}}{g} = \beta k \quad 1 \leq \beta \leq 2.25 \quad (10)$$

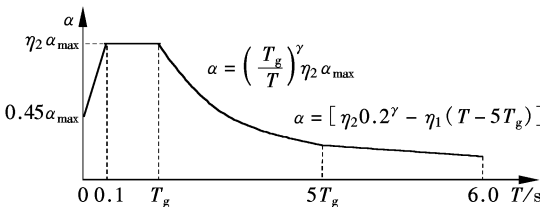


Fig. 2 Earthquake affecting coefficient curve<sup>[9]</sup>

3) For an arbitrary point ( $S_{di}$ ,  $S_{ai}$ ) on the capacity spectrum, the effective viscous damping  $\zeta_{eq}$  can be computed by a bilinear representation of the capacity spectrum as<sup>[7]</sup>

$$\zeta_{eq} = \kappa \zeta_0 + 0.05 = \kappa \frac{E_D}{E_{so}} + 0.05 \quad (11)$$

where  $\kappa$  is the damping modification factor,  $E_D$  the energy dissipated by damping, and  $E_{so}$  is the maximum strain energy. According to Fig. 2, there are a group of design spectra representing different earthquake intensities with effective viscous damping  $\zeta_{eq}$ , and one of them should intersect the capacity spectrum at the point ( $S_{di}$ ,  $S_{ai}$ ). The period  $T$  and earthquake affecting coefficient  $\alpha$  are obtained by substituting the intersection point ( $S_{di}$ ,  $S_{ai}$ ) to Eq. (9) and Eq. (10), respectively. Consequently, the maximum earthquake affecting coefficient  $\alpha_{max}$  can be concluded by Fig. 2.

4) The maximum value  $\beta_{max} = \frac{S_a}{\text{PGA}}$  is 2.25 according to Ref. [9]. Then, the PGA can be computed from Eq. (10) as

$$\text{PGA} = \frac{\alpha_{max}}{\beta_{max}} g = \frac{\alpha_{max}}{2.25} g \quad (12)$$

5) Select another point on the capacity spectrum and repeat step 3) and step 4), and we can also obtain the corresponding PGA. As a result, the relationship between the capacity spectrum and PGA is found. Setp 3) to step 5) are illustrated in Fig. 3.

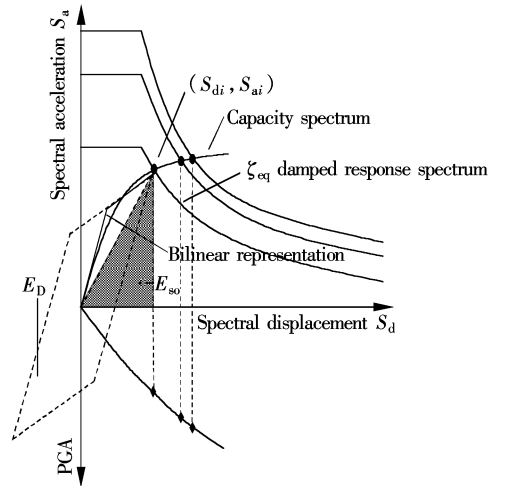


Fig. 3 Relationship between capacity spectrum and PGA

6) Finally, the relationship between the capacity spectrum and the PGA is transformed to that between the pushover curve and the PGA by Eq. (5) and Eq. (6) (see Fig. 4).

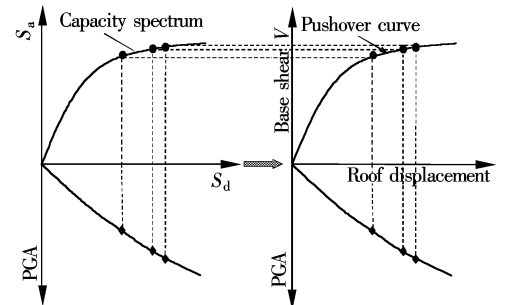


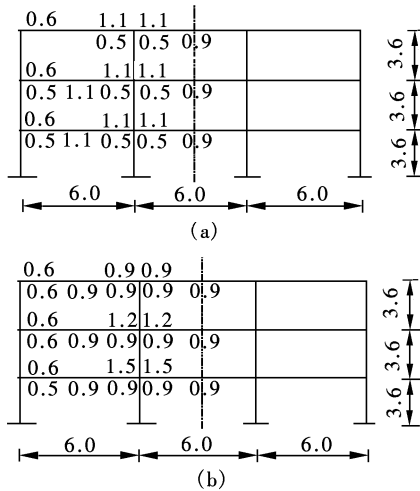
Fig. 4 Transformation from capacity spectrum to pushover curve

According to the definition of the proposed damage index, each point on the pushover curve is associated with a specific damage state  $D_i$ . Up to the present, the relationship between the proposed damage index  $D_i$  and the PGA is established.

## 2 Application of Proposed Seismic Evaluation Approach

### 2.1 Example of moment-resisting frame<sup>[6]</sup>

In order to demonstrate the proposed damage index and the evaluation approach, two typical three-storey RC frames in Ref. [6] are utilized in this study. Of the first one designed according to the 1963 ACI code, all the beams are 250 mm × 600 mm, the interior columns are 400 mm × 400 mm with a 1% reinforcement ratio, and the exterior columns are 300 mm × 300 mm with a 1.25% reinforcement ratio. Nevertheless, of the second RC frame following the 1995 NBCC code, all the beams are 250 mm × 500 mm, and all the columns are 400 mm × 400 mm with a 1.5% reinforcement ratio. The reinforcement ratios of all the beams are illustrated in Fig. 5. These two buildings are considered to represent ductile and non-ductile moment-resisting frames, respectively.

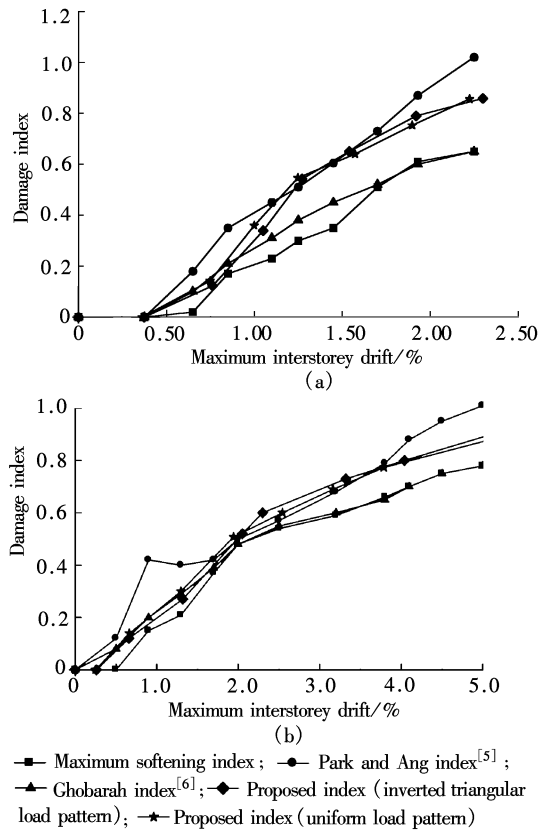


**Fig. 5** Basic information of the three-storey building<sup>[6]</sup>. (a) 1963 ACI frame; (b) 1995 NBCC frame (unit: m)

### 2.2 Results and discussion

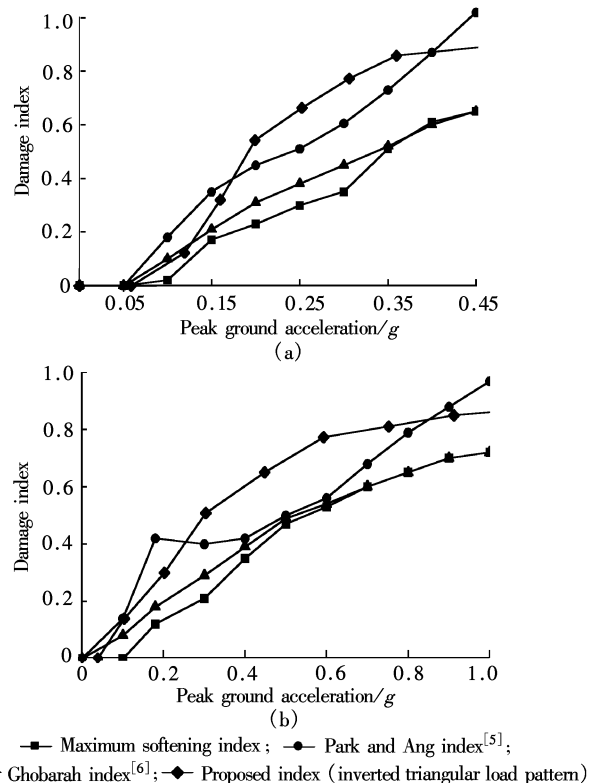
The selection of lateral load patterns is a critical point in the pushover analysis. Different load patterns yield different performances of buildings, or even erroneous predictions<sup>[10–11]</sup>. In this case, two types of load patterns (inverted triangular and uniform) are considered in the pushover analysis of both frames. Different indices vs. maximum interstorey drift are presented in Fig. 6. The data of the final soften index, the Park index and the Ghobarah index of frames, undergoing El-Centro earthquake that scaled to different PGAs, are cited from Ref. [6] in diagrams.

From Fig. 6, a better consistency between the proposed damage index and other indices is observed. The initial part of the proposed index curve with low damage values is close to that of the Ghobarah index, while the middle part is close to the Park index and the final part with high damage values is in the middle of them. Fig. 6 also indicates that the proposed damage index is not greatly influenced by different load patterns since it represents the relative values of stiffness, which are the same as for the Ghobarah index. As there is no conclusive verdict on the selection of lateral load patterns, the proposed damage index is rational in seismic evaluation with the non-linear static analysis.



**Fig. 6** Damage indices variation with the maximum interstorey drift. (a) 1963 ACI frame; (b) 1995 NBCC frame

Through the transformation strategy discussed in section 1.2.2, the relationship between the proposed damage index  $D$  and the PGA can be obtained without a series of dynamic analyses. Fig. 7 shows the proposed index curve rising and



**Fig. 7** Relationship between damage indices and PGA. (a) 1963 ACI frame; (b) 1995 NBCC frame

deviating from others after application of the transformation method. This phenomenon is reasonable. The curves of other indices are obtained by a specific earthquake record (El-Centro) but those of the proposed index are obtained by the design response spectrum according to the proposed approach. Design response spectra are the statistical results and used for seismic design, which implies that there is a high non-exceeding probability<sup>[12]</sup> in design response spectra to insure enough degrees of safety. Therefore, the analysis results with the proposed approach are conservative.

The results of the pushover analysis also indicate that the frame designed by the 1963 ACI code is a typical non-ductile one forming the storey yielding mechanism at the first storey, while the other one designed by the 1995 NBCC code is a ductile frame forming a satisfactory structure yielding mechanism. Therefore, the maximum interstorey drift of the NBCC frame is much greater (4% vs. 2.25%). This information can be observed from Fig. 8. The profiles of the storey damage index are far from each other in the non-ductile frame and much closer in the ductile frame. Thus, the proposed damage index for the storey is useful in determining the distribution of damage to various storeys and identifying which storey controls the performance of the whole frame.

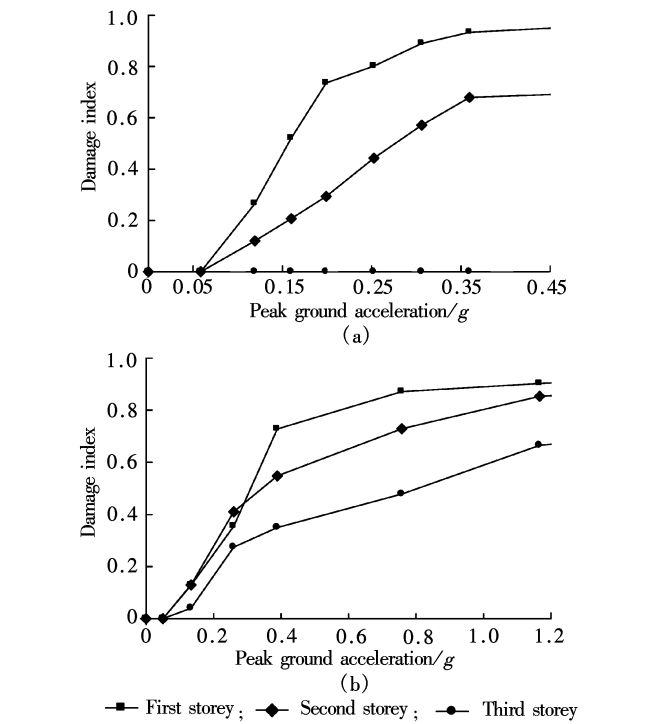


Fig. 8 Proposed damage indices for each storey. (a) 1963 ACI frame; (b) 1995 NBCC frame

2.3 Proposed damage index for different performance levels

The statistical relationship between the proposed index and the Ghobarah index with various PGA levels is illustrated in Fig. 9. After comparison with the damage states defined by Ghobarah et al.<sup>[6]</sup> for reinforced concrete structures, the conservative suggested values of the proposed damage index for different performance levels are listed in Tab. 1.

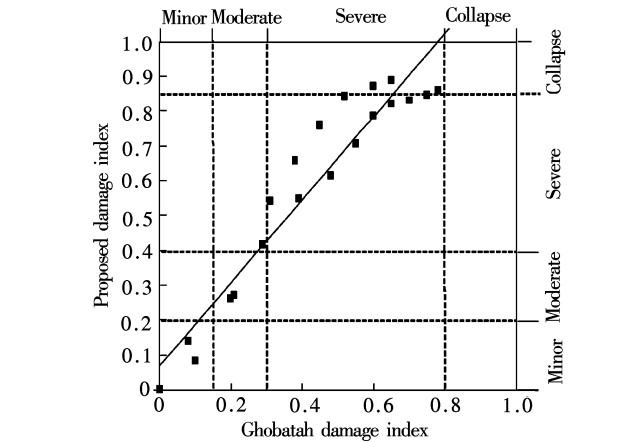


Fig. 9 Relationship between damage indices and performance levels

Tab. 1 Proposed damage index values for different performance levels

Performance level	Damage state	Proposed damage index
Level 1	Minor	0 to 0.2
Level 2	Moderate	0.2 to 0.4
Level 3	Severe	0.4 to 0.85
Level 4	Collapse	0.85 to 1

3 Summary and Conclusions

A simplified stiffness-based damage index and approach for seismic damage analysis of moment-resisting frames have been elaborated on. It can be applied to seismic performance evaluation of moment-resisting frames subjected to earthquakes of different intensities. Through the analysis of two low-rise frames, some conclusions can be drawn:

- 1) There is a satisfactory correlation among various damage indices. These results are not conclusive but significant because they increase the confidence in relying on damage indices as performance indicators for structures.
- 2) The calculation of the proposed damage index is convenient and the results of the assessments emphasize safety with the proposed approach.
- 3) The proposed damage index is not sensitive to various load patterns. So it is suitable for seismic performance evaluation based on non-linear static analysis. The storey damage index defined in Eq. (4) is useful in identifying the distribution of damage to various storeys. Thus, it reflects the yielding mechanism of frames to a certain extent.
- 4) However, these results are for the simple cases of three-storey frames. The reliability may not be as good as in other cases of more complex structures due to the limitations of the traditional pushover method<sup>[10]</sup>. For high-rise structures, some improvements should be introduced to the proposed method to consider the contribution of high modals<sup>[13]</sup> and changeable modal properties<sup>[14]</sup>.

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## 基于刚度的框架结构抗震能力评估简化方法

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**摘要:**针对结构物在地震作用下的灾害评估问题,提出了一种基于刚度退化概念的框架结构整体与层间损伤指标.该指标通过结构静力弹塑性分析方法进行计算,利用塑性铰考虑结构的地震损伤.同时,根据能力谱方法建立了该损伤指标与抗震设防等级的关系.然后,将建议指标应用于2个3层钢筋混凝土框架结构,并与其他损伤指标进行了对比.结果表明:建议的损伤指标偏于安全,且对静力弹塑性分析的水平荷载模式不敏感;层间损伤指标能够清晰地反映各楼层的损伤情况,从而判断薄弱层的位置.最后,通过统计分析给出了结构不同性能水准与损伤指标的对应关系,为基于性能的框架结构抗震评估提供参考.

**关键词:**损伤指标;抗震评估;能力谱法;框架;水平荷载模式;pushover分析;性能水准;刚度退化

**中图分类号:**TU313.3;TU375.4