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FNM method for estimating reliability of existing bridges

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Abstract: Combined with current specifications and stress characteristics of concrete filled steel tubular (CFST) arch bridges, the determination principle of safe-middle-failure three-stage mode is given. Accordingly, damage probability and failure probability and the corresponding reliability indices are calculated; a direct relationship between reliability indices and three-stage working status is made. Based on the three-stage working mode, a combined FNM (finite element-neural network-Monte-Carlo simulation) method is put forward to estimate the reliability of existing bridges. According to time-dependent reliability theory, subsequent service time is divided into several stages; minimum samples required by the Monte-Carlo method are generated by random sampling; training samples are calculated by the finite element method, and the training samples are extended by the neural network; failure probability and damage probability are calculated by the Monte-Carlo method. Thus, time dependent reliability indices are obtained, and the working status is judged. A case study is investigated to estimate the reliability of an actual bridge by the FNM method. The bridge is a CFST arch bridge with an 83.6 m span and it has been in operation for 10 years. According to analysis results, in the tenth year, the example bridge is still in safe status. This conclusion is consistent with the facts, which proves the feasibility of the FNM method for estimating the reliability of existing bridges.

Key words: existing bridges; time-dependent reliability; three-stage working mode; Monte-Carlo simulation; FNM method

Deterioration mechanisms of existing structures have been identified and studied over the last few decades. Uncertainties associated with mechanical loading and environmental stressors make it difficult to accurately predict the life-cycle reliability of these structures[1-2]. Recently, scholars at home and abroad have done much research on this aspect. Based on reliability theory, this paper deals with health monitoring, bridge management, strengthening decision-making and so on[3-5].

At present, reliability assessments of structural members, structures and engineering systems are based on the safety-failure two-stage working mode. Academician Wang[6] pointed out that the working status of members, structures and engineering systems can be divided into three stages, which are collectively called the safe-middle-failure three-stage working mode. This is to say that there is a status between safe status and failure status; that is, a structure is working with damage. Undoubtedly, the three-stage working mode is in even more conformity with actual situations. According to relevant literature and current specifications, a judgment criterion of the three-stage working mode is given in this paper. The concept of time-dependent reliability indices based on the three-stage working mode is given. And the working status of structures can be directly determined by time-dependent reliability indices. It is significant for strengthening decision-making.

As to the majority of reliability assessment methods needed by numerical integration, an explicit expression of the structural performance function is required. The FNM (finite element-neural network-Monte-Carlo simulation) method for estimating the reliability of existing bridges proposed in this paper is new and independent of existing literature. Without the need of the explicit expression of the structural performance function, combined with time variant reliability theory, the FNM method can quantitatively analyze reliability indices of existing bridges.

1 Risk Identification

In the aging stage, the process that resistance varies with time is very complicated and irreversible. The risk factors can be divided into three hierarchies, which are loading, environmental effects and material internal causes. First, an analytic hierarchy process is used to calculate the weights of risk factors. According to the weight of each factor, main factors are selected, and minor factors are ignored. Secondly, samples of main factors are obtained by bridge testing. However, because of test methods, working environments and so on, the existence of abnormal samples is inevitable. These abnormal samples will affect the study function of the neural network. So these abnormal samples should be rejected. A fuzzy clustering analytical method is used to solve this problem. The steps are as follows:

1) Building a fuzzy similar matrix is the key of the fuzzy clustering analytical method. Let $S = \{x^1, x^2, \ldots, x^N\}$ be samples of fuzzy clustering. Each sample is made up of n effect factors: $x_i = \{x_1^i, x_2^i, \ldots, x_n^i\}$. Then a correlation coefficient method is used to obtain the correlation coefficient of any two samples x_i and x_k. Consequently, the fuzzy similar matrix is structured as

$$\underset{\sim}{\boldsymbol{R}} = (r_{jk})_{N\times N} \tag{1}$$

2) Reforming similarity relation into equivalence relation. In general case, $\underset{\sim}{\boldsymbol{R}}$ is a fuzzy similar matrix, satisfying symmetry and reflexivity, but not satisfying transitivity. So $\underset{\sim}{\boldsymbol{R}}$ needs to be reformed into a fuzzy equivalent matrix $\underset{\sim}{\boldsymbol{R}}^*$.

3) Clustering. A maximal tree method is used to reject the abnormal samples, which is satisfied with $r_{ij} < \lambda = 0.6$.

Based on the samples obtained from the above risk identification method, the random variable and its probability distribution can be determined by the probabilistic method.

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Then the input variable and the output variable are defined, and the performance function can be built.

2 Time-Variation of Main Factors

2.1 Vehicle load time-dependent model

The bridge subsequent service time (t_c, T_s) can be divided into m equal periods. The length of each period of time is $\tau = (T_s - t_c)/m$. And the stochastic process of the live load effect $S_Q(t)$ can be divided into m maximums of random variables denoted by S_{Qi}. Let S_{Qi} be subjected to an extreme distribution in τ; then the probability distribution function is[7]

$$F_{S_{Qi}}(x) = \exp\{-\exp[-\alpha_t(x-\mu_t)]\} \tag{2}$$

The maximum distribution function of the bridge subsequent service time is

$$F_{S'_{Qr}}(x) = [F_{S_{Qi}}(x)]^m \tag{3}$$

2.2 Resistance time-dependent model

The value of the load random variable statistical parameter is obtained by real bridge tests. Based on a lot of experiments and testing results, the time-dependent model of concrete intensity and its standard deviation in general atmospheric environments is given as follows[3]:

The concrete average intensity after t years is

$$\mu_f(t) = \eta(t)\mu_{f0} \tag{4}$$

where μ_{f0} is the average intensity of concrete after 28 d, and

$$\eta(t) = K e^{-0.0246(\ln t - 1.7154)^2} \tag{5a}$$

The standard deviation of concrete intensity after t years is

$$\sigma_f(t) = \xi(t)\sigma_{f0} \tag{5b}$$

where σ_{f0} is the standard deviation of concrete intensity after 28 d, and

$$\xi(t) = 0.0305t + 1.2368 \tag{5c}$$

This model shows that the average intensity of concrete after 50 years is still greater than the average intensity after 20 d. However, its standard deviation is much higher than that of 28 d.

The function relation of the elastic modulus and strength is[8]

$$E_c = \frac{10^5}{2.2 + 33/f_{cu}} \tag{6}$$

Concrete-filled steel tube (CFST) can be converted into the same material as follows[9]:

$$E_{sc} = 0.85[(1-\rho)E_c + \rho E_s] \tag{7}$$

3 Determination of Three-Stage Mode

The working status of the three-stage mode (see Fig. 1) is as follows:

The reliability probability is

$$P_s = P(R^s > S) = 1 - P_d \tag{8a}$$

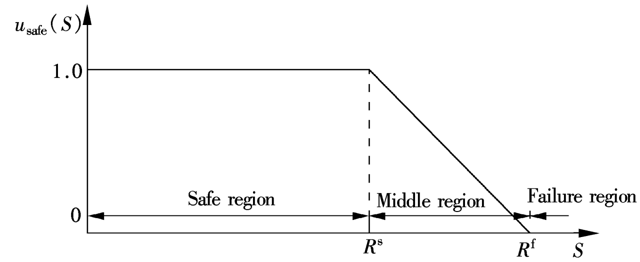

Fig. 1 Three-stage mode

where P_d is the damage probability.

The middle-status probability is

$$P_m = P(R^f \geqslant S \geqslant R^s) \tag{8b}$$

The failure probability (risk probability) is

$$P_f = P(S > R^f) \tag{8c}$$

The three working statuses are independent of each other and complete, and the relationship of their probabilities is

$$P_s + P_m + P_f = 1 \tag{9}$$

As to a CFST arch bridge, the maximum axial force is taken as a capability index. CFST is treated as reinforced concrete members in common structure design[9]. The carrying capacity can be calculated according to JTG D62—2004[10]:

$$N_1 = f_{cd}bx + f'_{sd}A'_s - \sigma_s A_s \tag{10}$$

When the confinement effect is considered, the carrying capacity can be calculated according to CESC28—90[11]

$$N_2 \leqslant \varphi_e \varphi_1 f_c A_c (1 + \sqrt{\theta} + 1.1\theta) \tag{11}$$

Obviously, $N_1 < N_2$. Because the improvement in capacity originating from the confinement effect is not taken into account. When $N_1 < N \leqslant N_2$, the load effect on CFST members will exceed design values. However, it still does not reach ultimate compressive bearing capacity. Under this condition, the structure is in the middle status; when $N > N_2$, the structure is in the failure status. In this way, the three-stage working mode of a CFST arch bridge can be expressed as follows:

The reliable probability is

$$P_s = P(N \leqslant N_1 \cap f \leqslant L/800) = 1 - P_d \tag{12a}$$

The middle status probability is

$$P_m = 1 - P_s - P_f \tag{12b}$$

The failure probability (risk probability) is

$$P_f = P(N > N_2 \cup f > L/800) \tag{12c}$$

4 FNM Method

Based on time-dependent reliability theory, the bridge subsequent service time (t_c, T_s) can be divided into m equal periods, such as one year, five years or ten years etc. In each period, based on the discrete resistance model, the value of time-dependent resistance $R(t_i)$ $(i = 1, 2, \ldots, m)$ is equal to the mid-value in each period. And the maximum value of load effect in each period is $S(t_i)$ $(i = 1, 2, \ldots, m)$. So the failure probability of subsequent service time is as follows:

$$P_f(T_1) = P[R(t_1) \leqslant S(t_1) \cup \ldots \cup R(t_m) \leqslant S(t_m)] \tag{13}$$

The damage probability $P_d(T_1)$ is similar to $P_f(T_1)$. $P_d(T_1)$ and $P_f(T_1)$ can be worked out by the discretization method; $\beta_d(T_1)$ and $\beta_f(T_1)$ are time-dependent reliability indices corresponding to $P_d(T_1)$ and $P_f(T_1)$.

The relationships of time-dependent reliability indices and safe-middle-failure three work statuses are as follows:

Safe status

$$\beta_d(t) > \beta_T \tag{14a}$$

Middle status(damage status)

$$\beta_d(t) \leqslant \beta_T \leqslant \beta_f(t) \tag{14b}$$

Failure status

$$\beta_T < \beta_f(t) \tag{14c}$$

After obtaining n groups of random samples by risk identification, the FNM method is used to calculate risk probability in this paper. n is the empirical lower limit of the Monte-Carlo simulation. In order to achieve upper precision, MCS requires that

$$n \geqslant \frac{100}{P_f} \tag{15}$$

Because the efficiency of the finite element method is relatively low, it is impossible to achieve all sample data by the finite element method alone. The neural network is used to solve this problem. Let the maximum limit of the neural network's training sample number be equal to N. These N groups of samples can be worked out by ANSYS PDS. The rest $n - N$ groups of samples can be achieved by the neural network. As for the neural network, training samples cannot be too small; otherwise, the precision of the neural network will be too low. However, training samples also cannot be too big; otherwise, the network will be in lack of reasoning ability. The empirical formula of the topological structure relationship is given by Hunter[12] as follows:

$$N = 1 + \frac{h(k + j + 1)}{j} \tag{16}$$

where k is the node number of the input layer; j is the node number of the output layer; and h is the node number of the hidden layer. Among n results, if there are M_2 failure samples and M_1 samples in the middle status, then three probabilities can be calculated as follows:

The reliability probability is

$$P_s = \frac{n - M_1 - M_2}{n} \tag{17a}$$

The failure probability(risk probability) is

$$P_f = \frac{M_2}{n} \tag{17b}$$

The damage probability is

$$P_d = \frac{M_1 + M_2}{n} \tag{17c}$$

The aforesaid is the main idea of the FNM method. The FNM method is realized by VB 6. 0 and ANSYS PDS in this paper.

5 Example

The Jiefang Bridge, built in 1998, is one of the super-large bridges across Zhujiang River in Guangzhou city. The main span is a three-span CFST through-tied arch with a span arrangement of 55 m + 83. 6 m + 55 m. The bridge has no wide bracing. The total width of the bridge is 25 m.

Through risk identification, the main factors affecting reliability are live loads, temperature effects, crowded loads, elastic modulis of steel pipe, steel pipe concrete strength, section loss of arch rib and foundation scouring. Because the antirust coat of the Jiefang Bridge is 75 mm inorganic zinc-rich primer + 25 mm epoxy sealer + 100 mm epoxy intermediate paint + 2 × 50 mm dion paint, the protection effect is very good, and the section loss is not considered in the calculation. To sum up, input random variables are the elastic modulis of CFST in side span, the elastic modulis of CFST in middle span, dead loads, crowded loads, live loads, temperature effects, and foundation scouring.

Where, both live load and steel pipe concrete strength are time-dependent random variables, and the time-dependent mode is created from the section 2 of this paper. The other factors are random variables, whose distributed parameters take values by statistical analysis results of test data. According to Eqs. (6) and (7), the concrete elastic modulus can replace the strength as an input variable in the finite element mode. According to statistical testing of χ^2, the distributions of the section loss of arch rib, the concrete elastic modulus and the section loss of main reinforcement are proved not to refuse to follow the lognormal distribution. Vehicle load effect is proved to be in the extreme distribution.

As to one variable X_i, disturbance quantity can be denoted by a random variable, so X_i can be expressed as a sum of a determined part and a random part,

$$X_i = \mu_{X_i}(1 + \alpha) \tag{18}$$

The above input method is used in this example.

A 3D finite element model is shown in Fig. 2. To sum up, there are 2 528 elements and 1 780 nodes in this model.

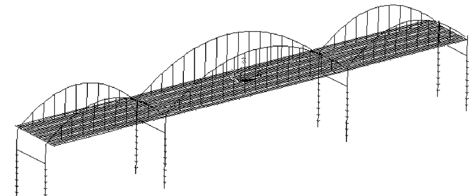

Fig. 2 3D finite element model

Thinking about double nonlinearity, the New-Raphson iterative method is used in the stochastic finite element solving process. If the solution is divergent, the structure will transform into an institution system, and the failure status of the structure can be determined directly in this case. If the solution is convergent, the working mode of the structure can be determined by Eq. (12).

As to a certain time $t = 10$, 10^7 samples are obtained by Monte-Carlo sampling. The subsequent service time is divided into nine periods. In each period, 1 050 samples are calculated by PDS (see Tab. 1). The remaining $(10^7/9 - 1\,050) \times 9$ samples are analyzed by an optimized BP neural network. The failure probability and the damage probability are shown in Fig. 3. And time-dependent reliability indices and $\beta_f(t)$ of the Jiefang bridge are calculated in this example (see Fig. 4).

Tab. 1 Samples of neural network (six groups of numbers chosen)

Input	Side span elastic modulus	1.302×10^{-1}	1.890×10^{-1}	-7.777×10^{-2}	-1.118×10^{-1}	1.092×10^{-1}	1.187×10^{-1}
	Mid span elastic modulus	1.065×10^{-2}	5.617×10^{-3}	-7.359×10^{-2}	1.407×10^{-1}	1.967×10^{-1}	-1.928×10^{-1}
	Dead load	8.951×10^{-3}	7.323×10^{-2}	-6.534×10^{-2}	1.374×10^{-1}	1.243×10^{-1}	1.990×10^{-1}
	Crowded load	1.620×10^{-1}	-3.382×10^{-1}	-1.642×10^{-1}	-9.500×10^{-1}	4.283×10^{-1}	-1.295×10^{-1}
	Live load	-9.806×10^{-3}	3.187×10^{-1}	3.337×10^{-1}	4.392×10^{-1}	2.939×10^{-1}	4.764×10^{-1}
	Temperature effect	4.022×10^{-1}	3.957×10^{-1}	-9.731×10^{-1}	6.128×10^{-1}	-4.670×10^{-1}	9.306×10^{-1}
	Foundation scouring	8.041×10^{-2}	-9.254×10^{-1}	-9.565×10^{-2}	2.272×10^{-2}	-6.821×10^{-1}	-3.255×10^{-1}
Output	Side span maximum axial force/N	-9.388×10^{6}	-1.113×10^{7}	-9.640×10^{6}	-1.195×10^{7}	-1.137×10^{7}	-1.305×10^{7}
	Mid span maximum axial force/N	-1.365×10^{7}	-1.587×10^{7}	-1.376×10^{7}	-1.711×10^{7}	-1.632×10^{7}	-1.857×10^{7}
	Side span maximum deflection/m	-3.298×10^{-2}	-4.580×10^{-2}	-4.911×10^{-2}	-5.145×10^{-2}	-3.724×10^{-2}	-7.022×10^{-1}
	Mid span maximum deflection/m	-5.202×10^{-2}	-7.928×10^{-2}	-8.151×10^{-2}	-8.451×10^{-2}	-6.126×10^{-2}	-1.053×10^{-1}
	N_1 in mid span/N	1.791×10^{7}	1.789×10^{7}	1.785×10^{7}	1.785×10^{7}	1.787×10^{7}	1.793×10^{7}
	N_2 in mid span/N	2.607×10^{7}	2.607×10^{7}	2.606×10^{7}	2.607×10^{7}	2.607×10^{7}	2.607×10^{7}
	N_1 in side span/N	1.313×10^{7}	1.310×10^{7}	1.311×10^{7}	1.312×10^{7}	1.311×10^{7}	1.312×10^{7}
	N_2 in side span/N	2.028×10^{7}	2.027×10^{7}	2.028×10^{7}	2.028×10^{7}	2.027×10^{7}	2.028×10^{7}
Judgement		Safe	Safe	Safe	Safe	Safe	Damage

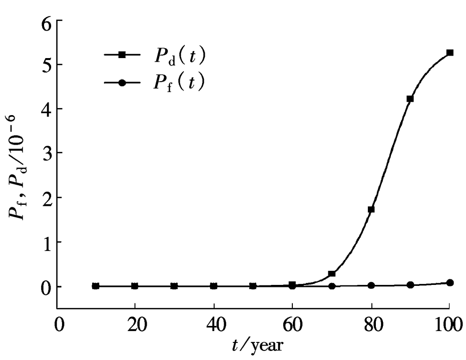

Fig. 3 Time-dependent failure probability and damage probability

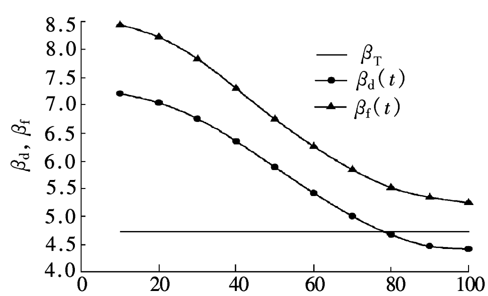

Fig. 4 Time-dependent reliability indices

6 Conclusion

Because the safety grade of the Jiefang Bridge is of the first order, according to GB/T 50283—1999[13], $\beta_T = 4.7$. According to analysis results (see Fig. 4), when $t = 10$, $\beta_f(t = 10) = 8.450\,9$, $\beta_d(t = 10) = 7.213\,2$. $\beta_f(t = 10) > \beta_d(t = 10) > \beta_T$. The Jiefang Bridge is in safe status in the tenth year. The conclusion is consistent with test results. Under normal conditions, when $t = 80$, $\beta_d(t) < \beta_T < \beta_f(t)$, the Jiefang Bridge will fall into the middle status after eighty years, and the structure needs maintenance and strengthening.

Combined with current specifications, the FNM method presented in this paper proposes the determination method of the three-stage working mode. Considering both time-dependent effects of resistance and load, the FNM method can work out bearing capability, reliability indices and remaining life of existing bridges in their service time. A case study is investigated using the FNM method to estimate the reliability of existing bridges. The example shows that the FNM method is reasonable, simple and practical, which is significant for the reliability analysis of a large structure.

However, because the test sample is very limited, the age of the example bridge is relatively young, and there is some initial damage as well, errors are inevitable. Our future work is further testing and more reasonable sampling.

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评价既有桥梁可靠性的 FNM 法

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摘要：结合现行规范及钢管混凝土拱桥的受力特点，给出钢管混凝土拱桥安全－中介－失效三级工作模式的确定准则. 据此计算结构的损伤概率与失效概率及对应的可靠度指标，建立可靠度指标与三级工作模式的直接关系. 基于三级工作模式，提出评估在役桥梁结构可靠度的有限元－神经网络－蒙特卡罗（FNM）法. 根据时变可靠度理论，将结构继续服役期离散化，随机抽样生成蒙特卡罗法所需的最少样本，由随机有限元法计算训练样本，用神经网络扩展训练样本，最后通过蒙特卡罗法计算失效概率和损伤概率，得到结构时变可靠度，判断其工作状态. 运用该方法对一座建成 10 年，主跨 83.6 m 的钢管混凝土拱桥进行分析，结果表明，该桥主拱处于安全状态，这一结论与实际情况相符. 说明 FNM 法对于评价在役桥梁的可靠性是可行的.

关键词：既有桥梁；时变可靠度；三级工作模式；蒙特卡罗模拟；FNM 法
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