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Study on compressibility of traffic flow

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Abstract: In order to describe the compressibility of traffic flows and determine the compression factors, the Mach number of gas dynamics is introduced, and the concept and the formula of the compression factor are obtained. According to the concept of the compression factor and its differential equation, a stop-wave model is built. The theoretical value and the observed one are obtained by the survey data in Changchun city. The relative error between the two values is 20. 3% . The accuracy is improved 39% compared with the result from the traditional stop-wave model. The results show that the traffic flow is compressible, and the methods of research on gas compressibility is also applicable to the traffic flow. The stop-wave model obtained by the compression factor can better describe the phenomenon of the stop wave at a signalized intersection when compared with the traditional stop-wave model.

Key words: compressibility of traffic flow; Mach number; compression factor; stop wave

In early times, people considered the traffic flow as a stream in traffic flow theory, and vehicles also ran in regular directions. There is interference between vehicles when the traffic density is high enough, and the traffic flow shows stringy characteristics. Vehicles stop and go, or run slowly and rapidly due to the restrictions imposed by traffic signals and accidents, and this disturbance will spread in the form of a traffic wave. This phenomenon embodies the compressibility of the traffic flow. In all, the traffic flow possesses the main characteristics of a liquid.

Just because of the comparability between the traffic flow and liquid, hydrodynamics theory for traffic flow studies emerged. In 1955, the kinematic wave model, established by Lighthill and Whitham[1], is considered as the marked thesis and a start of traffic flow dynamics theory. Soon, a similar theory was proposed by Richards[2]. So, the traffic flow dynamics theory is called the LW theory or the LWR theory. In later research, the application was at its zenith. In 2004, the dynamic traffic distribution a model based on moving wave theory was proposed by Lee et al[3]. Carey and Semmens[4] assessed motor vehicle division port-of-entry performance using traffic wave theory as the guideline in 2005.

The one-dimensional pipe flow model which applies in low speed mixed traffic was developed by Wu[5]. Traffic press and traffic index are introduced, and the form and the dissipation process of traffic congestion were analyzed by the numerical simulation method. The experimental results were close to the facts. The anisotropic traffic flow dynamics model is proposed and traffic audio-velocity was introduced in Ref. [6]. On the basis of foreign research, deceleration probability[7] and the driver sensitivity model were analyzed by Xue et al. [8]. A cellular automaton for a single lane of multi-speed mixed lanes was researched by Kuang et al. [9], and it infused a new idea for studying the traffic flow model.

In making a comprehensive view of traffic flow dynamics models, traffic flow characteristics are abstracted from liquid flow characteristics. The liquid compressibility is small enough to ignore, so it is not appropriate for the research of traffic compressibility. Furthermore, traditional traffic models are developed on the basis of continuous models. There is little quantificational analysis of traffic compressibility. In fact, traffic compressibility is not neglected especially with the obvious change in traffic flow density. So, a traffic flow hydrodynamics model describing traffic compressibility is built applying gas dynamics. The model is brief, and parameters are calibrated easily. Meanwhile, it is proved that the methods of gas dynamics can be applied to research traffic flow characteristics, and a new method to describe traffic waves is provided.

1 Model

1.1 Basic theory

Compared with liquid, gas has obvious compressibility. From a physics perspective, it is considered that gas is disturbed if parameters such as pressure and density are changed. The disturbance is strong or light based on the change degree of parameters, and it is called a light or a strong disturbance. The disturbance spreads around in the form of waves. The speed of the disturbance and that of gas particles are different because they result from two different movement forms—wave movement and the mechanical movement of particles[10].

In a traffic flow, traffic parameters are changed due to a disturbance called a traffic wave. For example, a traffic flow is disturbed while a car in this moving flow stops or decelerates, and the car is a disturbance source and the disturbance spreads along the traffic flow by influencing vehicles behind. Also, the vehicles speed is different from the spreading traffic wave speed. The traffic wave speed is the spreading speed of a traffic wave in a traffic flow medium(vehicles); namely, it is the moving speed of the density change in the traffic flow.

Although there is a difference between the characteristics

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of gas particles going everywhere and vehicles running in a directed orientation, the situation under the influence of outside disturbance is similar. If the gas particle is restricted as is the vehicle on the lane, then the gas particle will go in a directed orientation.

The compressibility of gas is relative to the speed of the airflow and the sound in it. The sound speed is closely related to the compressibility medium. For the sake of considering two factors at the same time, the Mach number is introduced to describe the compressibility of gas. The concept of the Mach number is

$$M_a = \frac{v}{c} \tag{1}$$

where M_a is the Mach number, v is the gas speed, and c is the spreading speed of disturbance in the gas flow.

The Mach number is the ratio of a particle moving speed at the speed of sound[10]. In the traffic flow, v is the traffic flow speed, c is the spreading speed of a disturbance in the traffic flow. The relationship between the gas compressibility and the Mach number can be described by the Euler equation:

$$-M_a^2 \frac{\mathrm{d}v}{v} = \frac{\mathrm{d}\rho}{\rho} \tag{2}$$

where $\mathrm{d}v/v$ and $\mathrm{d}\rho/\rho$ represent the relative change values in speed and the density of gas, respectively. The Euler equation indicates the relationship of density change and gas compressibility. It is a basic theory to quantitatively analyze traffic flow compressibility.

1.2 Basic model

Based on above theories, the Mach number is the foundation of researching traffic flow compressibility and establishing a compression factor model. To better describe traffic flow compressibility, a compression factor is introduced to represent the traffic flow compressibility according to Eq. (2). The model is built to describe the relationship of the traffic flow compressibility and the traffic flow speed and density. It is shown by the differential equation as follows:

$$-a^2 \frac{\mathrm{d}u}{u} = \frac{\mathrm{d}k}{k} \tag{3}$$

where $\mathrm{d}u/u$ and $\mathrm{d}k/k$ are the relative change value of speed and the density of the traffic flow, respectively.

The difference form of Eq. (3) is

$$-a^2 \frac{u_2 - u_1}{u_1} = \frac{k_2 - k_1}{k_1} \tag{4}$$

where u_1 and u_2 are the speed of the traffic flow before and after compression, respectively; k_1 and k_2 are the density of the traffic flow before and after compression, respectively.

Eq. (4) shows the relationship of the traffic flow compressibility and the relative change value of density due to the change in speed. If the speed-density relationship can be obtained, the speed can be replaced by the density in Eq. (4), so the compression factor can be calculated by the function involving the traffic flow density only.

1.3 Extended model

1.3.1 Especial situation

At the intersection, the vehicles are waiting for the green light behind the stop line when the light is red. The traffic flow density increases soon, and the change spreads in the traffic flow in the form of a traffic wave. A traffic flow compression phenomenon is very obvious. So a special situation is studied first.

The traffic flow is running on the road at a certain speed. When the vehicles meet a red light, early arrival vehicles stop and wait, and later arrivals decelerate subsequently. The compression of the traffic flow is shown. The traffic flow density is the jam density and the speed drops to zero finally. Therefore, the final density is k_j and the final speed is zero. Eq. (4) becomes

$$a^2 = \frac{k_j - k_1}{k_1} \tag{5}$$

where a represents the degree of compression of the traffic flow, and it is a positive number,

$$a = \sqrt{\frac{k_j - k_1}{k_1}} \tag{6}$$

The actual bicycle density is 0.08 bike/m^2 from investigated data in Shijiazhuang city. The jam density of the bicycle flow at the intersection is 0.65 bike/m^2 according to the results from the project team[11]. The bicycle compression factor can be calculated by Eq. (6), and the result is 2.67.

According to the traffic flow compression model, the traffic wave model is discussed only. Eq. (1) can be changed to $c = v/M_a$. Then the traffic wave speed model is obtained by substituting corresponding parameters for variables of the traffic flow. So the traffic wave speed can be calculated

$$u_w = -\frac{u}{a} \tag{7}$$

where u_w is the speed of the traffic wave, u is the speed of the traffic flow, and a is the traffic flow compression factor calculated by Eq. (6).

The minus sign in Eq. (7) represents that the direction of the traffic flow is opposite that of the traffic wave. Eq. (6) is introduced into Eq. (7), and then the stop wave speed at the intersection can be calculated by

$$u_w = -u \Big/ \sqrt{\frac{k_j - k_1}{k_1}} = -u\sqrt{\frac{k_1}{k_j - k_1}} \tag{8}$$

1.3.2 Common situation

The above theory and equations are only for the stop wave of the traffic flow at the intersection, but there is a disturbance in the traffic flow on the road. The traffic flow is compressed resulting from the change in the traffic flow density. The final speed is not zero, and the final density is not the jam density. If the speed-density relationship is obtained, then the relationship of the compression factor and density is built by Eq. (3). The speed-density relationship is supposed as $u = f(k)$; then the equation as $\frac{\mathrm{d}u}{\mathrm{d}k} = f'(k)$ is existent. Moreover, Eq. (3) can be transformed to $-a^2 = \frac{\mathrm{d}k}{\mathrm{d}u}\frac{u}{k}$. Then, Eq. (9) is obtained as

$$-a^2 = \frac{f(k)}{f'(k)k} \tag{9}$$

Namely,

$$a = \sqrt{-\frac{f(k)}{f'(k)k}} \tag{10}$$

The traffic wave model is also obtained as

$$|u_w| = \left|\frac{u}{a}\right| = \left|f(k)\sqrt{-\frac{f'(k)k}{f(k)}}\right| \tag{11}$$

where $|u_w|$ is the numerical value of the traffic wave.

The direction of the traffic wave may be forward or backward. Relative to the source of the disturbance, the traffic influence resulting from it(vehicle) spreads backward. In other words, front vehicles only affect those behind them. So the traffic wave spreads backward relative to the traffic flow. But from the point of view of a fixed point, the traffic wave spreads onward when $|u|$ is greater than $|u_w|$, otherwise backward. To show the traffic wave direction, the value of the traffic wave speed is calculated first, and then its direction is determined by comparing it with the traffic flow speed.

In traffic flow theory, the relationship of speed and density is different in different traffic conditions. Under common conditions, the relation is linear. When the traffic flow density is very big or very small, the logarithm and exponential model can be used. The traffic flow compression factor and the traffic wave can be calculated in different situations as follows:

1) Linearity model

The Greenshield model is $u = u_f\left(1 - \frac{k}{k_j}\right)$, then the differential equation is $f'(k) = \frac{\mathrm{d}u}{\mathrm{d}k} = -\frac{u_f}{k_j}$. According to Eq. (10), the traffic flow compression factor $a = \sqrt{\frac{k_j - k}{k}}$ can be obtained. And then the traffic wave value calculated by the traffic wave model (Eq. (11)) is shown as

$$|u_w| = \left|u_f\sqrt{\frac{(k_j - k)k}{k_j^2}}\right| \tag{12}$$

where u_f is the free speed of the traffic flow.

2) Logarithm model

The Greenberg model is $u = u_m \ln\left(\frac{k_j}{k}\right)$. The traffic flow compression factor is $a = \sqrt{\ln\left(\frac{k_j}{k}\right)}$ and the traffic wave can be obtained by the same method as

$$|u_w| = \left|u_m\sqrt{\ln\left(\frac{k_j}{k}\right)}\right| \tag{13}$$

where u_m is the optimal speed of vehicles.

3) Exponential model

The Underwood model is $u = u_f\exp\left(-\frac{k}{k_m}\right)$. By the same method, the compression factor is $a = \sqrt{\frac{k_m}{k}}$ and the traffic wave speed value is

$$|u_w| = \left|u_f\exp\left(-\frac{k}{k_m}\right)\sqrt{\frac{k}{k_m}}\right| \tag{14}$$

where k_m is the optimal density of vehicles.

2 Model Validation

2.1 Traffic survey

In order to validate the speed model of the stop wave at the intersection, a traffic survey is conducted at Renmin Street in Changchun city. The road condition for the survey is shown in Fig. 1.

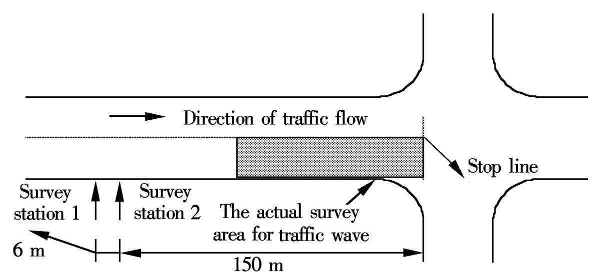

Fig. 1 Sketch map of stop wave survey

There are some notes during the survey. A intersection without the interference of pedestrians and bicycle but with special straight lane is chosen. A 150-meter stretch is selected from survey station 1 to the stop line because the speed is not disturbed by the intersection. The distance is 6 m between survey station 1 and station 2 which are set with white adhesive tape as the sign in order to obtain the spot speed of vehicles. The hourly traffic volume of a lane is obtained in station 1. The choice of the actual traffic wave survey area must be less than the maximal queue length in order to obtain the stop wave speed. In the survey area, the ruler is put along the lane from the stop line for registering the stop distance from the stop line. At the same time, the stop instant is registered using a stopwatch. Then the actual value of the stop wave speed can be calculated.

According to the video tape, in order to calculate the actual value of the stop wave speed, the survey table is given (see Tab. 1).

Tab. 1 Stop instant and stop location survey

Cycle number	The *n*-th vehicle			The *m*-th vehicle		
	n	Stop instant	Stop location	*m*	Stop instant	Stop location
1	1	1.58	0.7	4	6.19	22.1
2	3	12.34	12.3	5	20.82	25.7
3	1	2.44	2.4	2	6.27	8.5
⋮	⋮	⋮	⋮	⋮	⋮	⋮

2.2 Parameter demarcation

From Eq. (8), u, k and k_j need to be known. These parameters can be obtained by the following process. The spot speed is calculated by

$$u_i = \frac{s_{12}}{t_2 - t_1} \tag{15}$$

where u_i is the spot speed of the i-th vehicle; s_{12} is the distance between survey station 1 and station 2, which is 6 m here; t_1 and t_2 are the arriving instants at survey station 1 and station 2, respectively.

The time mean speed is calculated by

$$\bar{u}_t = \sum_{i=1}^{N} \frac{u_i}{N} \tag{16}$$

where $\bar{u}_t$ is the time mean speed on the road segment and N is the number of observed vehicles.

The space mean speed is calculated by

$$\bar{u}_s = \bar{u}_t + \frac{\sigma_t^2}{\bar{u}_t} \tag{17}$$

where $\bar{u}_s$ is the space mean speed on the road segment and ${\sigma_t}^2$ is the observed value variance of the time mean speed.

The traffic flow density is calculated by

$$k = \frac{Q}{\bar{u}_s} \tag{18}$$

where k is the traffic flow density of the road section and Q is the hourly traffic volume of a lane.

The jam density is calculated by

$$k_j = \frac{(m-n) \times 1\ 000}{s_m - s_n} \tag{19}$$

where k_j is the jam density of the road section; s_m and s_n are the distance of the m-th and the n-th vehicle in the observation area from stop position to stop line, respectively.

The actual observed value of the stop wave speed is calculated by

$$u_{wo} = \frac{s_m - s_n}{t_m - t_n} \tag{20}$$

where u_{wo} is the observed value of the stop wave speed; t_n and t_m are the n-th and the m-th stop time in the observation area, respectively.

2.3 Results and analysis

From an observation of more than 30 cycles, the results are obtained. The average density is 13 vehicle/km, the average speed is 48.4 km/h and the jam density is 163 vehicle/km. According to Eq. (8), the calculated value of the stop wave speed is 14.2 km/h. The observed value stop wave speed from observation is 11.8 km/h.

In order to compare the calculated value and the observed value of the stop wave speed, the relative error is calculated by

$$e_r = \frac{|\bar{u}_{wc} - \bar{u}_{wo}|}{\bar{u}_{wo}} \tag{21}$$

where e_r is the relative error between the calculated and the observed value of the stop wave speed and $\bar{u}_{wc}$ is the calculated mean value. From the above data, the relative error is 20.3%.

The calculated value of the stop wave speed using the traditional stop-wave model, which is $u_w = -u_f k/k_j$ at the intersection, is 4.8 km/h, and the relative error is 59%. So compared with the traditional stop-wave model, the precision of the traffic wave model based on the compressive factor, is improved to a certain extent.

3 Conclusion

Traffic flow theory is introduced and the traditional traffic flow model is concluded in this paper. The Mach number is introduced for traffic flow research according to gas dynamics and the formula of the traffic flow compression factor is obtained by using a gas dynamics method. Therefore, the compressibility of the traffic flow can be analyzed quantificationally. Based on the compression factor, traffic wave models are established for common and special situations. The stop-wave model at an intersection is validated. The results show that the error between the actual value and the theoretical value is 20.3%. The precision is improved to some degree compared with the traditional stop-wave model. It indicates that the traffic flow is compressible. The stop-wave model resulting from the compressibility of the traffic flow can obviously reflect wave phenomena of the traffic flow at intersections. The stop-wave model at an intersection is only validated in this paper. The validation for the common traffic wave model should be done in the future.

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交通流压缩特性研究

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摘要: 为了描述交通流的压缩性并确定交通流压缩系数,将气体动力学中的 Mach 数引入交通流研究中,得到交通流压缩系数的定义及计算公式. 根据交通流压缩系数的定义及其微分方程,建立了停车波模型. 根据长春市实际调查数据,对模型进行计算及验证,得到停车波波速的理论值,该值与观测值之间的相对误差为 20. 3% ,与传统停车波模型的计算结果相比,精度提高了 39% . 结果表明:交通流具有可压缩性,研究气体流压缩性的方法同样适用于交通流;与传统停车波模型相比,由交通流压缩系数计算公式得到的停车波模型能更好地描述信号交叉口的停车波现象.

关键词: 交通流压缩性;Mach 数;压缩系数;停车波
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