Journal of Southeast University (English Edition) Vol. 25, No. 2, pp. 267 – 270 June 2009 ISSN 1003—7985

Impact of box ratio and pinwheel pattern on the pallet loading problem

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Abstract: The pinwheel pattern as a suitable and advantageous alternative for the loading implementation of the pallet loading problem (PLP) is identified after a survey on the loading pattern. The definitions, elements, categories, generating algorithms of the pinwheel pattern are discussed and a uniform symmetric pinwheel notation is proposed. Based on the forming geometry of a pinwheel, the pinwheel structure is analyzed in terms of the innate box ratio, the box/block orientation and the box number by combinatorial and geometrical methods. A revised data set for the PLP with an area ratio range from 1 to 76 and a box ratio range from 1 to 10 is proposed. All pinwheel instances with this data set are calculated, and box ratio range is obtained for each possible pinwheel pattern, which can be found for all non-prime numbers of boxes. And a high box ratio makes an optimal pinwheel pattern more likely appear. Results identify the impact of the above pinwheel pattern and the box ratio on the pallet loading problem.

Key words: packing; pallet loading problem; loading pattern; box ratio; pinwheel pattern

The pallet loading problem (PLP) is essentially a two-dimensional problem of packing a pallet with the maximum number of small identical boxes[1-3]. Over 30 years, heuristics have prevailed in searching for PLP solutions; however, the optimal number of boxes certainly remains to be solved, and their loading pattern should also be worked out as verification. In a PLP problem, boxes loaded on the pallet should be orthogonal; hence, the boxes can only lie in two orientations: H-box and V-box. If a box lies horizontally with its longer edge parallel to the longer edge of the pallet, then it is an H-box; otherwise a V-box. The one-layer layout of H-boxes and V-boxes on the pallet constructs the pallet loading pattern, which may consist of blocks (rectangles with several boxes of the same orientation) and/or boxes.

Recently Martins and Dell[2] have solved the optimality of all PLP instances with an area ratio of pallet to box less than 101, and their paper and many other papers have paid much attention to the algorithms. However, since the box ratio is only from 1 to 4 and the pallet loading pattern is inadequately addressed, we argue that the pallet loading pattern and the box ratio need to be reexamined before completely solving the pallet loading problem.

1 Pallet Loading Pattern and Box Ratio

Given the pallet, the loading pattern will be determined by the box figure, i. e. form, size, and orientation. However, the box ratio (length to width) and its impact on the PLP loading patterns are seldom addressed.

1.1 Pallet loading pattern

Pallet loading patterns can be classified as guillotine, first-order non-guillotine and superior-order non-guillotine[4-5]. A guillotine pattern is a pattern that results from successive guillotine cuts, and it has two types: one-block homogeneous pattern and two-block pattern. A first-order non-guillotine pattern has more than two homogeneous blocks, and a cut is a first-order if it produces five new rectangles without any guillotine cutting pattern[4]. A superior-order non-guillotine pattern is a pattern that cannot be obtained by successive guillotine cuts and/or first-order non-guillotine cuts.

Early block heuristics can generate optimal patterns for first-order non-guillotine problems, and many complex algorithms are based on the superior order non-guillotine pattern[1]. Among these patterns, the pinwheel pattern is worth mentioning since it is a sound, textured and easy-to-implement pattern proposed by practitioners[6].

1.2 Pinwheel pattern

A pinwheel consists of four leaves and a central hole; the leaves are arranged successively head-to-tail in a closed-loop style. A pinwheel pattern is a rigid textured pattern and it is anti-sliding. Its rigid rectangular contour gives the pinwheel pattern a 180° rotational symmetry by the centroid. According to Ref. [7], the suitability of pallet-loading patterns should be determined with respect to a variety of criteria affecting stability and clampability. As an alternative solution of the pallet loading problem, the pinwheel pattern trades optimality for practical suitability, so it can satisfy real-life aspects such as load stability, maneuverability, and transportability.

For a comprehensive understanding of this pinwheel pattern, we can define the leaf on the left lower corner as the first leaf which lies horizontally (1H). Consequently, there are 2V, 3H and 4V in an anticlockwise sequence (see Fig. 1 (a)). Based on the leaf conformation, pinwheel patterns can be classified into three categories: simple pinwheel (see Fig. 1(a)), block pinwheel (see Fig. 1(b)), and nested pinwheel (see Figs. 1(c) and (d)).

1) Simple pinwheel In a simple pinwheel, there are two H-boxes and two V-boxes which form 1H, 2V, 3H and 4V leaves as shown in Fig. 1(a); both the contour of the pinwheel and its only central hole are squares. The simple pinwheel pattern is 90° rotationally symmetric and is the first-order non-guillotine pattern.

2) Block pinwheel In a block pinwheel, there is at least a pair of leaves which is no longer a single box, but block of $m \times n$ H-boxes (or $p \times q$ V-boxes), where integers $m, n, p, q \geqslant 1$. There are two styles of such a pattern: P1 and P3. If 1H

Received 2008-11-20.

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Foundation item: The National Natural Science Foundation of China (No. 70571033, 70831002).

Citation: Yi Junmin, Wang Dupeng, Zhou Jing. Impact of box ratio and pinwheel pattern on the pallet loading problem[J]. Journal of Southeast University (English Edition), 2009, 25(2): 267 – 270.

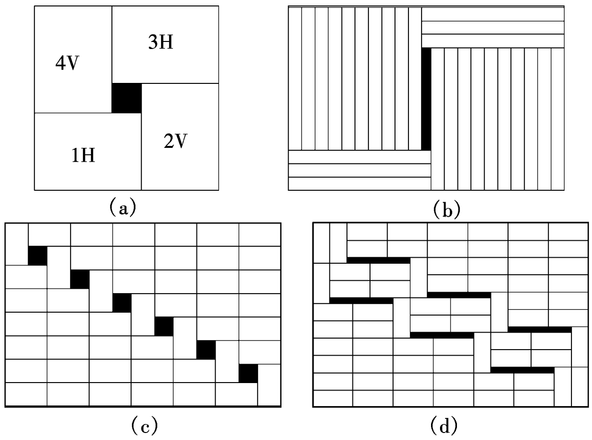

Fig. 1 Pinwheel patterns by category. (a) Simple pinwheel(4 boxes); (b) Block pinwheel(620 mm × 58 mm, 26 boxes); (c) Nested pinwheel(183 mm × 102 mm, 49 boxes); (d) Nested pinwheel (175 mm × 75mm, 70 boxes) on 1 200 mm × 800 mm pallet

is lower than 2V in height, then it is a P1 style(see Fig. 1 (b)); otherwise it is a P3 style. The contour of a block pinwheel is usually a rectangle; and only if the four leaves are the same block but with different orientations($m = q$, $n = p$), then the pinwheel has a square contour. A block pinwheel is also a first-order non-guillotine pattern; early block algorithms can form such patterns.

3) Nested pinwheel In a nested pinwheel pattern, one or more leaves of a pinwheel loop are further shared by other pinwheel loop(s), and more boxes/blocks are added to complete a rectangular contour. A nested pinwheel can be nested simple or nested block, and leaf sharing pinwheel can be H and/or V style. For example, in an H style, the $m \times n$ leaf (H-leaf) is shared by two adjacent loops with all holes located along the secondary diagonal. If more than one leaf in a loop are further shared by adjacent loops, then there is a lattice of holes rather than a line of holes along the diagonal (see Fig. 1(d)). If the hole lattice is in k rows l columns(integers $k, l > 1$) along the secondary diagonal, then there will be k H-leaves and 1 V-leaves on the horizontal contour edge, with a length of $kna + lqb$. Likewise, the contour width is $kmb + lpa$. But the total box number is always $(k + l)(kmn + lpq)$. If one of the two parameters k, l is equal to 1, then the nested pinwheel pattern is a first-order non-guillotine pattern, as shown in Fig. 1(c); otherwise the nested pinwheel pattern with a k-row l-column hole lattice is a superior-order non-guillotine pattern, as shown in Fig. 1(d).

In general, we propose a generic notation for pinwheel patterns:

$$[k, l-]\, m \times n - p \times q \tag{1}$$

where k rows l columns represent a hole lattice; $m \times n$ denotes $m \times n$ H-boxes in an H-leaf and $p \times q$ denotes $p \times q$ V-boxes in a V-leaf . For example, Fig. 1(d) can be noted as 3, 2 − 2 × 2 − 1 × 1. If there is no nested loop, the item in the square bracket can be ignored; this is a block pinwheel(3 × 1 − 1 × 10)(see Fig. 1(b)). Especially, 1 × 1 − 1 × 1(see Fig. 1 (a)) is a simple pinwheel.

1.3 Box ratio and dataset

Dowsland[8] gave the constraints of the pallet length to width ratio as between 1 and 2, the box ratio between 1 and 4, and the pallet to box area ratio from 1 to 51 as set Cover Ⅰ and 51 to 101 as set Cover Ⅱ. However, the range of box ratio from 1 to 4 is derived from a box range with a common length of greater than or equal to 200 mm and less than or equal to 600 mm, and a width of greater than or equal to 150 mm and less than or equal to 450 mm[9]. In practical situations, especially in factories, the box range may vary to exceed the limit significantly; thus the box ratio can exceed 4. Also Alvarez-Valdes et al. [1] pointed out that the definitions of sets cover Ⅰ and cover Ⅱ are subject to some ambiguity. And the higher the area ratio, the more possible one-block optimal patterns; but the one-block pattern tends to be unstable and lack practical value[7]. Thus it is time to reexamine the range of box ratio and area ratio constraints. Hereby, we propose a unique dataset with the range of 1 to 10 for the box ratio and the range of 1 to 76 for the area ratio.

2 Pinwheel Model and Calculation

An instance of the PLP is given as (X, Y, a, b), where X, Y, a, b are the pallet length, the pallet width, the box length and the box width, respectively; moreover, they are assumed to be all positive integers and $X \geqslant Y \geqslant a \geqslant b$[8]. For convenience, the three ratios are defined as follows:

Pallet ratio

$$\alpha = \frac{X}{Y}$$

Box ratio

$$\beta = \frac{a}{b}$$

Pallet to box area ratio

$$\gamma = \left\lfloor \frac{XY}{ab} \right\rfloor$$

2.1 Hole constraint

In a pinwheel loop, the central hole should be small enough not to allow any box to fill in, so

$$|pa - mb|\,|na - qb| \leqslant ab \tag{2}$$

By the pinwheel definition, if the first leaf (1H) is lower than the second leaf (2H), then it is a P1 style with $pa - mb > 0$ and $na - qb > 0$; otherwise, it is a P3 style with $pa - mb < 0$ and $na - qb < 0$.

Hence, there is always

$$(pa - mb)(na - qb) \leqslant 0 \tag{3}$$

Substituting $a = b\beta$ into (3), it is changed to

$$pn\beta^2 - (mn + pq + 1)\beta + mq \leqslant 0 \tag{4}$$

Then the range of β is

$$\frac{mn + pq + 1 - \sqrt{\Delta}}{2pn} \leqslant \beta \leqslant \frac{mn + pq + 1 + \sqrt{\Delta}}{2pn} \tag{5}$$

where $\Delta = b^2 - 4ac = (mn + pq + 1)^2 - 4mnpq = (mn - pq + 1)^2 + 4pq \geqslant 0$, because m, n, p, q are all integers no less than 1. Then the box ratio range for a given pinwheel pattern can be determined.

2.2 Contour shape

For a $k, l - m \times n - p \times q$ pinwheel with $k \times l$ pinwheel loops, its rectangular contour has a length of $(kna + lqb)$ and a width of $(kmb + lpa)$. Thus the contour shape ratio δ is

$$\delta = \frac{kna + lqb}{lpa + kmb} = \frac{kn\beta + lq}{lp\beta + km} \tag{6}$$

δ should be no less than 1, since the length is always greater than the width. If $\delta = \alpha$, it is preferable since the whole pallet deckboard can be fully filled.

While the box number $(k + l)(kmn + lpq)$ ranges from 4 to 76, there are 10 860 combination instances of k, l, m, n, p, q. We calculate all the possible instances to form pinwheels, while β is obtained from the average of its lower and upper ranges and δ is calculated from Eq. (6). In the instance of the Euro pallet, with $0.9\alpha \leqslant \delta \leqslant 1.1\alpha$, there are 419 classes of pinwheels (every $k, l - m \times n - p \times q$ instance is a class, and many classes may have the same box number), and the computed β range is within [1, 32.27]. If the β range is restricted to [1, 10], then there are 267 classes of possible pinwheels (see Fig. 2); however, if the β range is within [4, 10], then there are 79 classes of pinwheels. It shows that the classes with higher box ratios are nearly one-third (79/267) of the total range. In Tab. 1, there are some pinwheel examples for loading on the common pallets, such as the ISO 1 200 mm × 1 000 mm pallet, the Euro 1 200 mm × 800 mm pallet and the Japan 1 140 mm square pallet. Note that the pinwheel pattern is a near-optimal alternative solution; the optimal pattern may be found in the ranges as shown in Fig. 2. Compared with the optimal results (see the last column in Tab. 1) from Martins and Dell's[2] methods, there are four optimal solutions in Tab. 1, which are in rows 1, 3, 4, 5; and others are near optimal with differences of 1 to 3. Furthermore, the examples in rows 5, 7, 9 are shown in Figs. 1(b), (c) and (d), respectively.

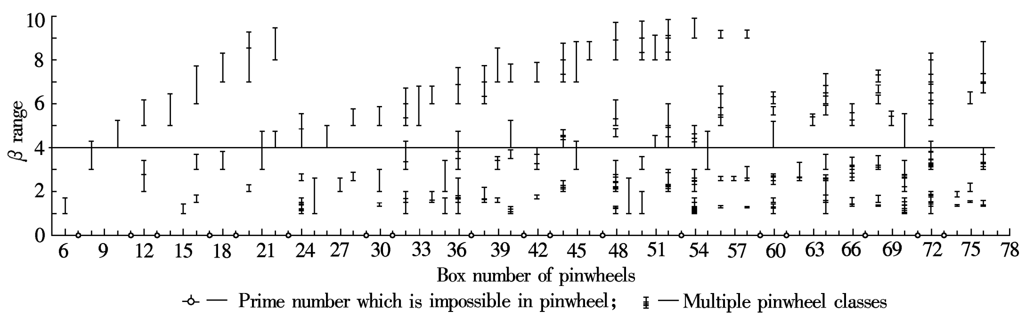

Fig. 2 Box number and β range for pinwheels on the Euro pallet

Tab. 1 Examples of pinwheel patterns as optimal solution on some pallets

Row	k	l	m	n	p	q	Box number	β range	Example							Optimal number
									β	X	Y	a	b	δ	γ	
1	2	1	1	1	1	1	9	1.000 to 2.618	1.900	1 200	1 000	475	250	1.231	10	9
2	1	2	1	2	1	1	12	1.000 to 1.707	1.604	1 200	1 000	369	230	1.238	14	13
3	1	1	3	1	1	3	12	3.000 to 5.303	4.157	1 140	1 140	661	159	1.000	12	12
4	1	2	1	1	1	2	15	2.000 to 3.414	2.126	1 200	1 000	404	190	1.168	15	15
5	1	1	3	1	1	10	26	10.000 to 11.359	10.690	1 200	800	620	58	1.512	27	26
6	2	1	4	1	1	4	36	4.000 to 6.561	5.294	1 140	1 140	413	78	1.096	40	39
7	6	1	1	1	1	1	49	1.000 to 2.618	1.794	1 200	800	183	102	1.518	51	50
8	3	1	2	1	2	4	56	4.000 to 4.637	4.338	1 200	1 000	295	68	1.158	59	58
9	3	2	2	2	1	1	70	2.000 to 2.618	2.307	1 200	800	173	75	1.493	74	73
10	1	3	4	4	1	1	76	4.000 to 4.266	4.200	1 200	1 000	252	60	1.191	79	78

3 Discussion

The pinwheel pattern can be constructed by any number of boxes except for those prime numbers. For each k, l, m, n, p, q combination, there is a β range for the hole smaller than a box. Only when its contour ratio is close to the target pallet ratio, may it be an optimal pattern. One special case is when the box ratio is an integer, a zero-hole pattern may be denser and optimal, but it loses the hole characteristics of a pinwheel.

We set the hole constraint of not allowing a box in it; nevertheless, there is a bigger hole which can embed another pinwheel or block in it, such as those patterns given by the five block algorithm[10]. However, only a square-contoured pinwheel embedded seamlessly into a square hole is recommended for anti-sliding.

A higher box ratio is not rare in real-life box dimensions. By our calculations, some block pinwheel patterns are preferably optimal for longer boxes (see Fig. 1(b)). So, the PLP needs to be reexamined when $\beta > 4$.

4 Conclusion

With simple, block and nested pinwheel patterns and their uniform notation, we identify the pinwheel pattern as a suitable and advantageous alternative for the PLP loading implementation. To cover the varieties of real-life situations, all pinwheel instances with an area ratio of 1 to 76 and a box ratio of 1 to 10 are calculated, and it shows that each pinwheel pattern has a specific range of box ratios, which may be optimal. And a higher box ratio makes an optimal pinwheel pattern more likely appear. Our results identify the impact of the above pinwheel pattern and the box ratio on the pallet loading problem, and it suggests that previous PLP algorithms need to be reexamined with regard to box ratio and loading pattern.

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货物长宽比及转轮样式对托盘装箱问题的影响

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摘要: 在对托盘装箱问题的装箱样式进行调查后,研究了解决托盘装箱问题的一种具有优势的转轮装箱样式. 对该样式的定义、组成要素、分类和产生的算法进行了讨论,并提出一种对称式转轮样式的统一命名方式. 基于转轮样式的几何形成条件,运用组合与几何学的方法,就货物本身的长宽比特性、货物取向和单层货物数量对转轮样式的结构影响进行了讨论,提出了托盘装箱问题的修改数据集: 面积比范围为 1 ~ 76,货物长宽比范围为 1 ~ 10. 并计算了这一数据集下所有箱数为非质数的转轮样式实例,以及每一可能转轮样式的货物长宽比范围. 计算结果显示货物长宽比越大越有可能获得转轮样式的最优解,表明转轮样式和货物长宽比的确影响托盘装箱问题.

关键词: 装箱;托盘装箱问题;装箱样式;长宽比;转轮样式

中图分类号: O221. 7;O157. 3