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Oriented quantum coalgebra structure on the tensor product of an oriented quantum coalgebra with itself

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Abstract: Oriented quantum algebras (coalgebras) are generalizations of quasitriangular Hopf algebras (coquasitriangular Hopf algebras) and account for regular isotopy invariants of oriented 1-1 tangles, oriented knots and links. Let (H, σ, D, U) be an oriented quantum coalgebra over the field k. Then $(H\otimes H, \varphi, D\otimes D, U\otimes U)$ is a trivial oriented quantum coalgebra structure on the tensor product of coalgebra H with itself, where $\varphi(a\otimes b, c\otimes d) = \sigma(a, c)\sigma(b, d)$. This paper presents the oriented quantum coalgebra structure $(H\otimes H, \tilde{\sigma}, D\otimes D, U\otimes U)$ on coalgebra $H\otimes H$, where $\tilde{\sigma}(a\otimes b, c\otimes d) = \sigma^{-1}(d_1, a_1)\sigma(a_2, c_1)\sigma^{-1}(d_2, b_1)\sigma(b_2, c_2)$. So a nontrivial oriented quantum coalgebra structure is obtained and it is dual to Radford's results in the paper "On the tensor product of an oriented quantum algebra with itself" published in 2007. Theoretically, the results of this paper are important in constructing the invariants of oriented knots and links.

Key words: oriented quantum coalgebra; twisted oriented quantum coalgebra; knot invariant

Since the advent of quantum groups[1], many algebraic structures have been described which are related to invariants of 1-1 tangles, knots, links or 3-manifolds. Quantum algebras and coalgebras give rise to regular isotopy invariants of unoriented 1-1 tangles. Oriented quantum (co) algebras give rise to most known regular isotopy invariants of oriented links; for that reason, oriented quantum algebras are important. The invariants that account for (oriented) quantum (co) algebras are studied in Refs. [2 – 6]. Especially, Radford[2] gave the oriented quantum algebra structure on the tensor product of an oriented quantum algebra with itself. In this paper, we present the oriented quantum coalgebra structure on the tensor product of an oriented quantum coalgebra with itself. Theoretically, the results of this paper are important in constructing the invariants of oriented knots and links.

Throughout this paper, we assume that k is a field. Let (H, Δ, ε) be a coalgebra over the field k. If a tensor product is written without an index, then it is assumed to be taken over k, that is $\otimes = \otimes_k$.

Generally, objects are represented by their underlying vector spaces. For $a \in H$, we write $\Delta(a) = a_1\otimes a_2$, which is the usual variation of the Heyneman-Sweedler notation[7] for the coproduct.

1 Definitions and Lemmas

Definition 1[6] A strict oriented quantum coalgebra over k is a quadruple (H, σ, D, U), where H is a coalgebra over k, $\sigma: H\otimes H\rightarrow k$ is an invertible bilinear form, and D and U are commuting coalgebra automorphisms, such that

$$\text{(OQC1)}\quad \sigma(a_1, U(b_2))\sigma^{-1}(D(a_2), b_1) = \varepsilon(a)\varepsilon(b) = \sigma^{-1}(D(a_1), b_2)\sigma(a_2, U(b_1))$$

$$\text{(OQC2)}\quad \sigma(a, b) = \sigma(D(a), D(b)) = \sigma(U(a), U(b))$$

$$\text{(OQC3)}\quad \sigma(a_1, b_1)\sigma(a_2, c_1)\sigma(b_2, c_2) = \sigma(a_2, b_2)\sigma(a_1, c_2)\sigma(b_1, c_1)$$

for all $a, b, c \in H$.

A twisted oriented quantum coalgebra over k is a tuple (H, σ, D, U, G), where (H, σ, D, U) is a strict oriented quantum coalgebra over k, and $G \in H^*$ (the dual of linear space) is invertible, such that $D^*(G) = U^*(G) = G$ and $(D\circ U)(h) = G^{-1}\rightharpoonup h\leftharpoonup G$ for all $h \in H$, where $h^*\rightharpoonup h = h_1\langle h^*, h_2\rangle$ and $h\leftharpoonup h^* = h_2\langle h^*, h_1\rangle$ for all $h^* \in H^*$.

Remark 1 Let (H, σ, D, U) and (H', σ', D', U') be strict oriented quantum coalgebras over k. Then $(H\otimes H', \sigma'', D\otimes D', U\otimes U')$ is a strict oriented quantum coalgebra over k, called the tensor product of (H, σ, D, U) and (H', σ', D', U'), where $\sigma''(a\otimes a', b\otimes b') = \sigma(a, b)\sigma'(a', b')$ for $a, b \in H$ and $a', b' \in H'$.

Remark 2 If (H, σ, D, U) is a strict oriented quantum coalgebra over k, then (H, σ, U, D) is also a strict oriented quantum coalgebra over k, because D and U are commuting coalgebra automorphisms.

Lemma 1 If (H, σ, D, U) is a strict oriented quantum coalgebra over k, then for all $a, b, c \in H$, we have

$$\sigma^{-1}(a_1, b)\sigma^{-1}(a_2, c) = \sigma^{-1}(b_1, c_1)\sigma^{-1}(a_1, c_2)\sigma^{-1}(a_2, b_2)\sigma(b_3, c_3) \tag{1}$$

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$$\sigma^{-1}(a_1, c)\sigma^{-1}(a_2, b) = \sigma(b_1, c_1)\sigma^{-1}(a_1, b_2)\sigma^{-1}(a_2, c_2)\sigma^{-1}(b_3, c_3) \tag{2}$$

$$\sigma(a, c_1)\sigma(b, c_2) = \sigma^{-1}(a_1, b_1)\sigma(b_2, c_1)\sigma(a_2, c_2)\sigma(a_3, b_3) \tag{3}$$

$$\sigma(b, c_1)\sigma(a, c_2) = \sigma(a_1, b_1)\sigma(a_2, c_1)\sigma(b_2, c_2)\sigma^{-1}(a_3, b_3) \tag{4}$$

Proof It is obvious.

2 Oriented Quantum Coalgebra Structure on Tensor Product of Oriented Quantum Coalgebra with Itself

Theorem 1 Suppose that (H, σ, D, U) is a strict oriented quantum coalgebra over k. Then $(H\otimes H, \tilde{\sigma}, \tilde{D}, \tilde{U})$ is a strict oriented quantum coalgebra over k, where

$$\tilde{\sigma}(a\otimes b, c\otimes d) = \sigma^{-1}(d_1, a_1)\sigma(a_2, c_1)\sigma^{-1}(d_2, b_1)\sigma(b_2, c_2)$$

$$\tilde{D} = D\otimes D, \quad \tilde{U} = U\otimes U$$

$$\tilde{\sigma}^{-1}(a\otimes b, c\otimes d) = \sigma^{-1}(b_1, c_1)\sigma(d_1, b_2)\sigma^{-1}(a_1, c_2)\sigma(d_2, a_2)$$

for all $a, b, c, d \in H$.

Proof We will take four steps to finish the proof.

Step 1 We claim that σ is invertible with an inverse

$$\tilde{\sigma}^{-1}(a\otimes b, c\otimes d) = \sigma^{-1}(b_1, c_1)\sigma(d_1, b_2)\sigma^{-1}(a_1, c_2)\sigma(d_2, a_2)$$

In fact, for all $a, b, c, d \in H$, we have

$$\tilde{\sigma}(a_1\otimes b_1, c_1\otimes d_1)\tilde{\sigma}^{-1}(a_2\otimes b_2, c_2\otimes d_2) = \sigma^{-1}(d_1, a_1)\sigma(a_2, c_1)\sigma^{-1}(d_2, b_1)\sigma(b_2, c_2)\sigma^{-1}(b_3, c_3)\sigma(d_3, b_4)\sigma^{-1}(a_3, c_4)\cdot$$
$$\sigma(d_4, a_4) = \sigma^{-1}(d_1, a_1)\sigma(a_2, c_1)\sigma^{-1}(d_2, b_1)\sigma(d_3, b_2)\sigma^{-1}(a_3, c_2)\sigma(d_4, a_4) = \sigma^{-1}(d_1, a_1)\cdot$$
$$\sigma(a_2, c_1)\varepsilon(b)\sigma^{-1}(a_3, c_2)\sigma(d_2, a_4) = \sigma^{-1}(d_1, a_1)\varepsilon(b)\varepsilon(c)\sigma(d_2, a_2) = \varepsilon(a)\varepsilon(b)\varepsilon(c)\varepsilon(d)$$

Similarly, $\tilde{\sigma}^{-1}(a_2\otimes b_2, c_2\otimes d_2)\tilde{\sigma}(a_1\otimes b_1, c_1\otimes d_1) = \varepsilon(a)\varepsilon(b)\varepsilon(c)\varepsilon(d)$.

Step 2 Since (H, σ, D, U) is a strict oriented quantum coalgebra over k, we have

$$\tilde{\sigma}^{-1}(\tilde{D}(\tilde{a}_1), \tilde{b}_2)\tilde{\sigma}(\tilde{a}_2, \tilde{U}(\tilde{b}_1)) = \sigma^{-1}(D(b_1), c_3)\sigma(d_3, D(b_2))\sigma^{-1}(D(a_1), c_4)\sigma(d_4, D(a_2))\sigma^{-1}(U(d_1), a_3)\sigma(a_4, U(c_1))\cdot$$
$$\sigma^{-1}(U(d_2), b_3)\sigma(b_4, U(c_2)) = \sigma^{-1}(D(b_1), c_3)\sigma^{-1}(D(a_1), c_4)\sigma(d_2, D(a_2))\sigma^{-1}(U(d_1), a_3)\cdot$$
$$\sigma(a_4, U(c_1))\sigma(b_2, U(c_2)) = \sigma^{-1}(D(b_1), c_3)\sigma^{-1}(D(a_1), c_4)\varepsilon(d)\sigma(a_2, U(c_1))\sigma(b_2, U(c_2)) =$$
$$\sigma^{-1}(D(a_1), c_2)\varepsilon(d)\sigma(a_2, U(c_1))\varepsilon(d) = \varepsilon(a)\varepsilon(b)\varepsilon(c)\varepsilon(d)$$

Likewise, $\tilde{\sigma}(\tilde{a}_1, \tilde{U}(\tilde{b}_2))\tilde{\sigma}^{-1}(\tilde{D}(\tilde{a}_2), \tilde{b}_1) = \varepsilon(a)\varepsilon(b)\varepsilon(c)\varepsilon(d)$. So OQC1 is satisfied.

Step 3 For OQC2, one can compute it as follows:

$$\tilde{\sigma}(\tilde{D}(\tilde{a}), \tilde{D}(\tilde{b})) = \tilde{\sigma}(C\otimes D(b), D(c)\otimes D(d)) = \sigma^{-1}(D(d_1), D(a_1))\sigma(D(a_2), D(c_1))\cdot$$
$$\sigma^{-1}(D(d_2), D(b_1))\sigma(D(b_2), D(c_2)) = \tilde{\sigma}(\tilde{a}, \tilde{b})$$

and $\tilde{\sigma}(\tilde{U}(\tilde{a}), \tilde{U}(\tilde{b})) = \tilde{\sigma}(\tilde{a}, \tilde{b})$ is proved similarly.

Step 4 Finally we will verify that OQC3 holds for $\tilde{\sigma}$.

$$\tilde{\sigma}(\tilde{a}_1, \tilde{b}_1)\tilde{\sigma}(\tilde{a}_2, \tilde{c}_1)\tilde{\sigma}(\tilde{b}_2, \tilde{c}_2) = \sigma^{-1}(d_1, a_1)\sigma(a_2, c_1)\sigma^{-1}(d_2, b_1)\sigma(b_2, c_2)\sigma^{-1}(f_1, a_3)\sigma(a_4, e_1)\sigma^{-1}(f_2, b_3)\sigma(b_4, e_2)\cdot$$
$$\sigma^{-1}(f_3, c_3)\sigma(c_4, e_3)\sigma^{-1}(f_4, d_3)\sigma(d_4, e_4) = \sigma^{-1}(d_1, a_1)\sigma(a_2, c_1)\sigma^{-1}(d_2, b_1)\sigma(b_2, c_2)\sigma^{-1}(f_1, a_3)\sigma(a_4, e_1)\sigma^{-1}(b_{31}, c_{31})\cdot$$
$$\sigma^{-1}(f_2, c_{32})\sigma^{-1}(f_3, b_{32})\sigma(b_{33}, c_{33})\sigma(b_4, e_2)\sigma(c_4, e_3)\sigma^{-1}(f_4, d_3)\sigma(d_4, e_4) = \quad \text{(by Eq. (1))}$$
$$\sigma^{-1}(d_1, a_1)\sigma(a_2, c_1)\sigma^{-1}(d_2, b_1)\sigma^{-1}(f_1, a_3)\sigma(a_4, e_1)\sigma^{-1}(f_2, c_2)\sigma^{-1}(f_3, b_2)\sigma(b_3, c_3)\sigma(b_4, e_2)\sigma(c_4, e_3)\sigma^{-1}(f_4, d_3)\cdot$$
$$\sigma(d_4, e_4) = \sigma^{-1}(d_1, a_1)\sigma(a_2, c_1)\sigma^{-1}(d_2, b_1)\sigma^{-1}(a_{31}, c_{21})\sigma^{-1}(f_1, c_{22})\sigma^{-1}(f_2, a_{32})\sigma(a_{33}, c_{23})\sigma(a_4, e_1)\sigma^{-1}(f_3, b_2)\cdot$$
$$\sigma(b_3, c_3)\sigma(b_4, e_2)\sigma(c_4, e_3)\sigma^{-1}(f_4, d_3)\sigma(d_4, e_4) = \quad \text{(by Eq. (1))}$$
$$\sigma^{-1}(d_1, a_1)\sigma^{-1}(d_2, b_1)\sigma^{-1}(f_1, c_1)\sigma^{-1}(f_2, a_2)\sigma(a_3, c_2)\sigma(a_4, e_1)\sigma^{-1}(f_3, b_2)\sigma(b_3, c_3)\sigma(b_4, e_2)\sigma(c_4, e_3)\sigma^{-1}(f_4, d_3)\cdot$$
$$\sigma(d_4, e_4) = \sigma^{-1}(d_1, a_1)\sigma^{-1}(d_2, b_1)\sigma^{-1}(f_1, c_1)\sigma^{-1}(f_2, a_2)\sigma(a_3, c_2)\sigma(a_4, e_1)\sigma(d_{31}, b_{21})\sigma^{-1}(f_3, d_{32})\sigma^{-1}(f_4, b_{22})\cdot$$
$$\sigma^{-1}(d_{33}, b_{23})\sigma(b_3, c_3)\sigma(b_4, e_2)\sigma(c_4, e_3)\sigma(d_4, e_4) = \quad \text{(by Eq. (2))}$$
$$\sigma^{-1}(d_1, a_1)\sigma^{-1}(f_1, c_1)\sigma^{-1}(f_2, a_2)\sigma(a_3, c_2)\sigma(a_4, e_1)\sigma^{-1}(f_3, d_2)\sigma^{-1}(f_4, b_1)\sigma^{-1}(d_3, b_2)\sigma(b_3, c_3)\sigma(b_4, e_2)\sigma(c_4, e_3)\cdot$$
$$\sigma(d_4, e_4) = \sigma^{-1}(d_1, a_1)\sigma^{-1}(f_1, c_1)\sigma(d_{21}, a_{21})\sigma^{-1}(f_2, d_{22})\sigma^{-1}(f_3, a_{22})\sigma^{-1}(d_{23}, a_{23})\sigma(a_3, c_2)\sigma(a_4, e_1)\sigma^{-1}(f_4, b_1)\cdot$$
$$\sigma^{-1}(d_3, b_2)\sigma(b_3, c_3)\sigma(b_4, e_2)\sigma(c_4, e_3)\sigma(d_4, e_4) = \quad \text{(by Eq. (2))}$$
$$\sigma^{-1}(f_1, c_1)\sigma^{-1}(f_2, d_1)\sigma^{-1}(f_3, a_1)\sigma^{-1}(d_2, a_2)\sigma(a_3, c_2)\sigma(a_4, e_1)\sigma^{-1}(f_4, b_1)\sigma^{-1}(d_3, b_2)\sigma(b_3, c_3)\sigma(b_4, e_2)\sigma(c_4, e_3)\cdot$$
$$\sigma(d_4, e_4) = \sigma^{-1}(f_1, c_1)\sigma^{-1}(f_2, d_1)\sigma^{-1}(f_3, a_1)\sigma^{-1}(d_2, a_2)\sigma(a_3, c_2)\sigma(a_4, e_1)\sigma^{-1}(f_4, b_1)\sigma^{-1}(d_3, b_2)\sigma(b_3, c_3)\cdot$$
$$\sigma^{-1}(b_{41}, c_{41})\sigma(c_{42}, e_2)\sigma(b_{42}, e_3)\sigma(b_{43}, c_{43})\sigma(d_4, e_4) = \quad \text{(by Eq. (3))}$$

$\sigma^{-1}(f_1,c_1)\sigma^{-1}(f_2,d_1)\sigma^{-1}(f_3,a_1)\sigma^{-1}(d_2,a_2)\sigma(a_3,c_2)\sigma(a_4,e_1)\sigma^{-1}(f_4,b_1)\sigma^{-1}(d_3,b_2)\sigma(c_3,e_2)\sigma(b_3,e_3)\sigma(b_4,c_4)\cdot$
$\sigma(d_4,e_4)=\sigma^{-1}(f_1,c_1)\sigma^{-1}(f_2,d_1)\sigma^{-1}(f_3,a_1)\sigma^{-1}(d_2,a_2)\sigma(a_3,c_2)\sigma^{-1}(a_{41},c_{31})\sigma(c_{32},e_1)\sigma(a_{42},e_2)\cdot$
$\sigma(a_{43},c_{33})\sigma^{-1}(f_4,b_1)\sigma^{-1}(d_3,b_2)\sigma(b_3,e_3)\sigma(b_4,c_4)\sigma(d_4,e_4)=$ (by Eq. (3))
$\sigma^{-1}(f_1,c_1)\sigma^{-1}(f_2,d_1)\sigma^{-1}(f_3,a_1)\sigma^{-1}(d_2,a_2)\sigma(c_2,e_1)\sigma(a_3,e_2)\sigma(a_4,c_3)\sigma^{-1}(f_4,b_1)\sigma^{-1}(d_3,b_2)\sigma(b_3,e_3)\sigma(b_4,c_4)\cdot$
$\sigma(d_4,e_4)=\sigma^{-1}(f_1,c_1)\sigma^{-1}(f_2,d_1)\sigma^{-1}(f_3,a_1)\sigma^{-1}(d_2,a_2)\sigma(c_2,e_1)\sigma(a_3,e_2)\sigma(a_4,c_3)\sigma^{-1}(f_4,b_1)$
$\sigma(a_{41},b_{31})\sigma(d_{42},e_3)\sigma(b_{32},e_4)\sigma^{-1}(d_{43},b_{33})\sigma(b_4,c_4)=$ (by Eq. (4))
$\sigma^{-1}(f_1,c_1)\sigma^{-1}(f_2,d_1)\sigma^{-1}(f_3,a_1)\sigma^{-1}(d_2,a_2)\sigma(c_2,e_1)\sigma(a_3,e_2)\sigma(a_4,c_3)\sigma^{-1}(f_4,b_1)\sigma(d_3,e_3)\sigma(b_2,e_4)\sigma^{-1}(d_4,b_3)\cdot$
$\sigma(b_4,c_4)=\sigma^{-1}(f_1,c_1)\sigma^{-1}(f_2,d_1)\sigma^{-1}(f_3,a_1)\sigma^{-1}(f_4,b_1)\sigma^{-1}(d_2,a_2)\sigma(c_2,e_1)\sigma(d_{31},a_{31})\sigma(d_{32},e_2)\cdot$
$\sigma(a_{32},e_3)\sigma^{-1}(d_{33},a_{33})\sigma(a_4,c_3)\sigma(b_2,e_4)\sigma^{-1}(d_4,b_3)\sigma(b_4,c_4)=$ (by Eq. (4))
$\sigma^{-1}(f_1,c_1)\sigma^{-1}(f_2,d_1)\sigma^{-1}(f_3,a_1)\sigma^{-1}(f_4,b_1)\sigma(c_2,e_1)\sigma(d_2,e_2)\sigma(a_2,e_3)\sigma(b_2,e_4)\sigma^{-1}(d_3,a_3)\sigma(a_4,c_3)\sigma^{-1}(d_4,b_3)\cdot$
$\sigma(b_4,c_4)=\widetilde{\sigma}(\tilde{a}_2,\tilde{b}_2)\widetilde{\sigma}(\tilde{a}_1,\tilde{c}_2)\widetilde{\sigma}(\tilde{b}_1,\tilde{c}_1)$

Thus, $(H\otimes H,\widetilde{\sigma},\widetilde{D},\widetilde{U})$ is a strict oriented quantum coalgebra over k.

Remark 3 Theorem 1 is the dual of theorem 4. 1 in Ref. [2].

Corollary 1 Suppose that (H,σ,D,U,G) and (H,σ,D,U,G') are twisted oriented quantum coalgebras over k. Then $(H\otimes H,\widetilde{\sigma},\widetilde{D},\widetilde{U},\widetilde{G})$ is a twisted oriented quantum coalgebra over k, where

$$\widetilde{\sigma}(a\otimes b,c\otimes d)=\sigma^{-1}(d_1,a_1)\sigma(a_2,c_1)\sigma^{-1}(d_2,b_1)\sigma(b_2,c_2)$$

$$\widetilde{D}=D\otimes D,\quad \widetilde{U}=U\otimes U,\quad \widetilde{G}=G\otimes G'$$

$$\widetilde{\sigma}^{-1}(a\otimes b,c\otimes d)=\sigma^{-1}(b_1,c_1)\sigma(d_1,b_2)\sigma^{-1}(a_1,c_2)\sigma(d_2,a_2)$$

for all $a,b,c,d\in H$, where $(G\otimes G')\rightharpoonup(h\otimes h')=G\rightharpoonup h\otimes G'\rightharpoonup h'$ and $(h\otimes h')\leftharpoonup(G\otimes G')=h\leftharpoonup G\otimes h'\leftharpoonup G'$ for all $h,h'\in H$.

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定向量子余代数与其自身张量积上的定向量子余代数结构

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摘要:定向量子代数(定向量子余代数)是拟三角 Hopf 代数(余拟三角 Hopf 代数)的推广并且可以得到定向 1-1 缠绕、定向扭结和连接的正则合痕不变量. 令(H,σ,D,U)为域 k 上的定向量子余代数,则$(H\otimes H,\varphi,D\otimes D,U\otimes U)$为余代数 H 与其自身张量积上一个平凡的定向量子余代数结构,其中 $\varphi(a\otimes b,c\otimes d)=\varphi(a,c)\varphi(b,d)$. 本文给出余代数 $H\otimes H$ 上的一种定向量子余代数结构$(H\otimes H,\widetilde{\sigma},D\otimes D,U\otimes U)$,其中 $\widetilde{\sigma}(a\otimes b,c\otimes d)=\sigma^{-1}(d_1,a_1)\cdot\sigma(a_2,c_1)\sigma^{-1}(d_2,b_1)\sigma(b_2,c_2)$. 进一步得到定向量子余代数与其自身张量积上的一种非平凡的定向量子余代数结构,这一结果对偶于 Radford 发表于 2007 年的《On the tensor product of an oriented quantum algebra with itself》一文中的结论. 理论上本文的结果对于构造定向扭结和连接不变量是非常重要的.

关键词:定向量子余代数;扭曲定向量子余代数;扭结不变量

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