Journal of Southeast University (English Edition) Vol. 25, No. 3, pp. 381 – 384 Sept. 2009 ISSN 1003—7985

A general method for silo classification using rupture plane

Yuan Fang[1] Wang Hui[1] Hu Yuxia[2] Zhang Songsong[1]

([1] School of Civil Engineering and Architecture, Henan University of Technology, Zhengzhou 450052, China)
([2] School of Civil and Resource Engineering, University of West Australia, Perth 6009, Australia)

Abstract: A new method is presented for silo classification using the rupture plane in the storage. The proposed method is based on the formula of rupture plane orientation determined by the aspect ratio of specified silos and involves the effects of the conical angle of the storage surface, so it can deal with two cases: the conical angle of the storage surface and the flat storage surface. Compared with the two popular methods based on the silo-aspect ratio and the silo rupture plane, respectively, in current engineering design, the new method can give a range of critical aspect ratios to classify deep silos and squat silos according to the conical angle of the storage surface or the flat storage surface. Moreover, the range covers the critical aspect ratios obtained from the existing classification methods including the Chinese code based on silo aspect ratios, the German code, Reimbert brothers' definition and Liang et al.'s definition, so it is a more general method to classify silos and the existing methods can be viewed as special cases of the presented method.

Key words: silo; aspect ratio; bulk solid; rupture plane; rupture angle

Storage bins are commonly classified into two types. One is the deep bin, and the other is the shallow bin or bunker. For the convenience of readers, in this paper, the circular deep bin is termed as (deep) silo, and the circular bunker is termed as squat silo. The classification of storage bins is important since the lateral pressure applied on walls is usually calculated differently for silos and squat silos and finally affects the silo design. At present, the two classifications used in the Chinese code are based on the aspect ratio and Airy's definition concerning the orientation of the rupture plane in the bulk solid within the silo. These two types of classification have been studied by many researchers and are widely used in most practical designs. However, there is lack of consistency between these two methods.

In the classification based on the aspect ratio, the aspect ratio is defined as the ratio of silo height to its diameter (h/d) for circular bins or the ratio of silo height to its width for square bins. However, the critical silo aspect ratios, which divide the bins into silos and squat silos, are different in the Chinese code and the German code. In the Chinese code[1], the bins with aspect ratios less than 1.5 are squat silos (bunkers), while in the German code[2], the bins with aspect ratios less than 0.8 are squat silos (bunkers). The difference between the two codes is rather great. Moreover, another critical value was proposed by Liang et al.[3], in which the circular bin is referred to as a squat silo when the silo aspect ratio is less than or equal to 0.7.

In the classification based on the rupture plane, there are also various definitions. In the Chinese code, the deep bin and the shallow bin are divided according to Airy's definition[4], in which the bins are termed as deep bins when the rupture plane starts at one side wall of the silo and reaches the opposite side wall of the bin. For a shallow bin, the rupture plane reaches the top surface of the storage. In Reimbert brothers' definition[5], the shallow bins are named by the definition that the rupture plane does not cut through the central line of the bin.

In this paper, a new classification method based on the rupture plane in the storage is proposed, and a comparison between the proposed approach and the existing methods is made.

1 Benchmark Formula Used to Determine Rupture Plane

A circular silo with stored-bulk-solid is taken into consideration and shown in Fig. 1(a). The top of the bulk solid in Fig. 1(a) has a conical shape, and the angle of the top cone is denoted by β, which is always regarded as the surcharge angle of the bulk solid. The central line of the silo is O_1O_2. The dotted line CM is the rupture plane with an angle θ to the vertical. The planes from the rupture point C to the cone

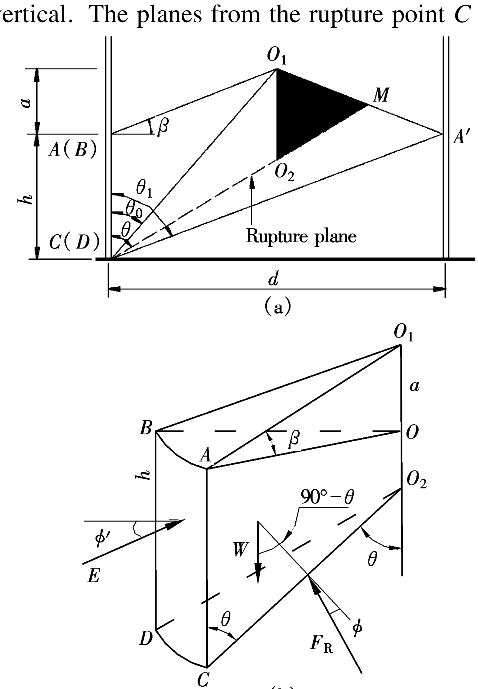

Fig. 1 Mechanical model setup in the new method. (a) A silo model; (b) FBD

Received 2009-03-10.

Biography: Yuan Fang (1964—), female, doctor, professor, fangyuan64@163.com.

Foundation items: The National Key Technologies R&D Program during the 11th Five-Year Plan Period (No. 2006BAD08B01), the Program for New Century Excellent Talents in Universities of Henan Province (No. 2005HANCET-13).

Citation: Yuan Fang, Wang Hui, Hu Yuxia, et al. A general method for silo classification using rupture plane [J]. Journal of Southeast University (English Edition), 2009, 25(3): 381 – 384.

tip O_1 and to the opposite cone bottom A' have angles θ_0 and θ_1, respectively, which are measured from the vertical in a clockwise direction and can be calculated by

$$\cot\theta_0 = 2\delta + \tan\beta \tag{1}$$

$$\cot\theta_1 = \delta = \frac{h}{d} \tag{2}$$

where δ is the aspect ratio.

In order to determine the orientation of the rupture plane, the benchmark formula derived by Yuan[6], which was based on the limit-equilibrium theory, is used to calculate the rupture angle here.

In Fig. 1(a), a silo with a conical top storage is plotted. We assume that the rupture plane in the storage reaches the opposite side of the cone surface with an angle θ to the vertical (see the dotted line in Fig. 1(a)). The free body diagram (FBD) of a wedge for further analysis is plotted in Fig. 1(b), being formed by means of the storage body above the rupture plane with a unit arc length AB. However, it is necessary to point out that the shaded part of the storage above the rupture plane in Fig. 1(a) is not included in the wedge FBD due to the characteristics of circular silos.

The volume of the wedge in Fig. 1(b) can be calculated by

$$V = \frac{1}{24\delta^2}h^2(6\delta + \tan\beta - \cot\theta) \tag{3}$$

from which the weight of the wedge can be obtained by

$$W = \gamma V = \frac{\gamma}{24\delta^2}h^2(6\delta + \tan\beta - \cot\theta) \tag{4}$$

where γ is the unit weight of the bulk solid. It is clearly seen that W can be evaluated subsequently once the plane of rupture (i. e. rupture angle θ) is determined.

There are three forces acting on the sliding wedge (see Fig. 1(b)). They are the weight of wedge W, the reaction force across the plane of rupture F_R, and the force from the silo wall E. Force F_R is acting at an angle ϕ to the normal direction of the rupture plane, which corresponds to the inter-particle friction of the bulk solid, and ϕ is the internal friction angle of the bulk material; force E is acting at an angle ϕ' to the normal direction of the arc face, and ϕ' is the (external) frictional angle between the silo wall and the bulk solid.

The equilibrium of the block by eliminating F_R can be written as

$$E = \frac{W\cos(\theta + \phi)}{\sin(\theta + \phi + \phi')} \tag{5}$$

Subsequently, substituting the expression of the gravity W into Eq. (5) gives

$$E = \frac{\gamma}{6}R^2\left(\frac{3h}{R} + \tan\beta - \cot\theta\right)\frac{\cos(\theta + \phi)}{\sin(\theta + \phi + \phi')} \tag{6}$$

Since the force E is a function of the rupture plane angle θ, the rupture plane will occur with the inclination angle θ when the force E reaches its critical value, that is,

$$\frac{dE}{d\theta} = 0 \tag{7}$$

which yields

$$\cot\theta = -\frac{\cos(2\phi + \phi') + \cos\phi'}{2\cos\phi\sin(\phi + \phi')} + \frac{\sqrt{[\cos(2\phi + \phi') + \cos\phi']^2 + 4\cos\phi\sin(\phi + \phi')\left[\sin\phi\cos(\phi + \phi') + \left(\tan\beta + \frac{3}{\delta}\right)\cos\phi'\right]}}{2\cos\phi\sin(\phi + \phi')} \tag{8}$$

By substituting θ determined by Eq. (8) into Eq. (6), the maximum force from the silo wall E_{max} can be obtained. It is proved that Eq. (6) provides better estimation on lateral pressures applied on silos[6-7].

2 New Method for Silo Classification

2.1 New method and existing methods

Following the parameters θ and θ_0 calculated by Eq. (8) and Eq. (1) respectively, this paper presents a new method for silo classification. In this new method, a definition is given that when the rupture plane angle reaches $\theta \geqslant \theta_0$, the bin is a deep silo, and when $\theta < \theta_0$, the bin is a squat silo.

It is worth pointing out that the proposed method follows the same reference line, the central line of the silo, as the one in Reimbert Brothers' theory. However, a different formula is employed to determine the rupture angle θ in our work.

For comparison, the existing methods are introduced here. Reimbert Brothers' definition of the deep bin and shallow bin is shown in Fig. 2, in which the rupture plane is defined by the internal friction angle of the storage. For example, the shallow bin is defined as

$$d \geqslant 2h\tan\left(\frac{\pi}{4} - \frac{\phi}{3}\right) \tag{9}$$

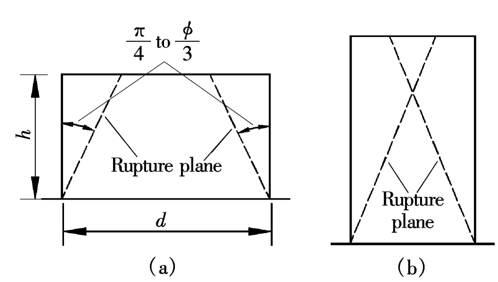

Fig. 2 Reimbert brothers' definition of deep bin and shallow bin. (a) Bunker; (b) Silo

Unlike the proposed method and Reimbert brothers' definition, Airy's theory[3] determines the deep bin and the shallow bin according to the rule of whether the rupture plane reaches the opposite wall of the bin (see Fig. 3). For example, when $\theta > \theta_1$, it is a deep bin; otherwise, it is a shallow bin. Whereas, according to Reimbert brothers' definition, when $\theta_0 < \theta < \theta_1$, it can be a deep bin.

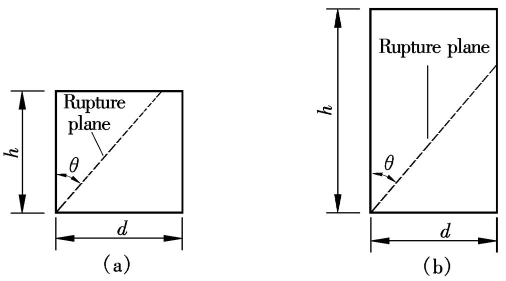

Fig. 3 Airy's definition on deep bin and shallow bin. (a) Bunker; (b) Silo

2.2 Calculation and analysis

For wheat of Henan, China, the parameters tested by Xu[8] are as follows: $\phi = 28.06°$, $\phi' = 23.46°$ (to concrete), $\beta = 28.8°$. The angles θ_0, θ_1 and θ are calculated using Eqs. (1), (2) and (8), respectively. The three angles θ, θ_0 and θ_1 vs. the storage aspect ratio δ are plotted in Fig. 4 for the storage with a conical top surface ($\beta = 28.8°$) and a flat surface ($\beta = 0°$).

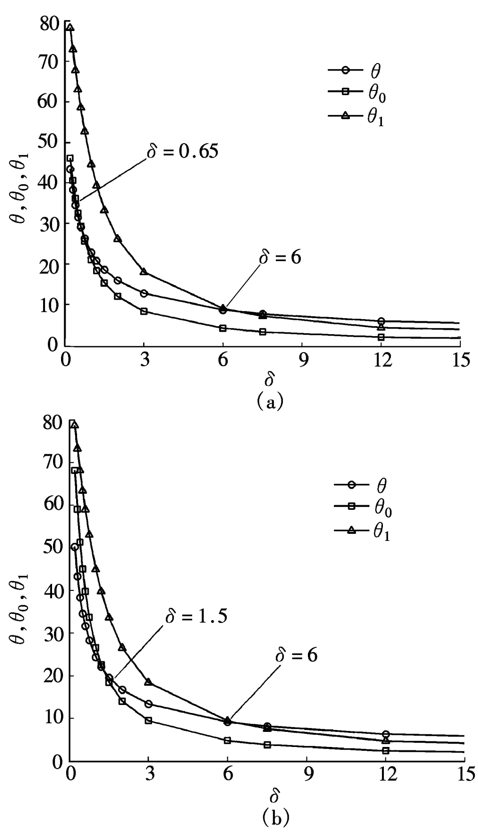

Fig. 4 The relationship between $\theta(\theta_0, \theta_1)$ and δ. (a) $\beta = 28.8°$; (b) $\beta = 0°$

From Fig. 4 (a), we can see that when $\delta < 0.65$, $\theta < \theta_0$, and when $\delta > 6$, $\theta > \theta_1$. At the same time, Fig. 4(b) shows that when $\delta < 1.5$, $\theta < \theta_0$ and when $\delta > 6$, $\theta > \theta_1$. Thus, in the proposed method, the parameter $\delta = 0.65$ can be regarded as the critical value to define the deep silo and the squat silo for the case $\beta = 28.8°$, and $\delta = 1.5$ for the case $\beta = 0°$.

Reimbert brothers' definition (Eq. (9)) gives the critical value for the squat silo as $\delta \leqslant 0.697$. This value 0.697 is close to the value 0.65 under the condition $\beta = 28.8°$.

It is obvious that the critical aspect ratios of $\delta = 1.5$ and 0.65 with different conical angles cover the range in the previous methods based on aspect ratios (1.5 in the Chinese code[1], 0.8 in the German code[2] and 0.7 by Liang et al.'s definition[3]) and 0.697 in Reimbert brothers' definition of the rupture plane.

For the methods based on rupture planes and Airy's definition which is also employed in the Chinese code, the critical value of the parameter δ is equal to 6, regardless of the angle β. This result agrees well with the one shown in Fig. 4 for the condition $\theta > \theta_1$ with both angles $\beta = 28.8°$ and 0°. However, it is much greater than the critical aspect ratio 0.65 in the new method and 1.5 in the Chinese code based on aspect ratios.

3 Conclusions

A new silo classification method based on the rupture plane formed in the storage is proposed. The new method is assessed against the existing classification methods in terms of critical aspect ratio defining silos and squat silos. Some conclusions can be drawn as follows:

1) The new classification method can include the effects of the conical angle of the storage surface, so it can deal with two cases: the conical angle of the storage surface and the flat storage surface.

2) The critical values of the aspect ratios δ obtained with the two conical angles of the storage surface in this paper cover the wide range found in the existing methods. It is obvious that the wide range is due to the conical angles of the storage surface.

3) There is a large deviation between the new rule and Airy's definition which is also used in the Chinese code. That is to say, the two classifications in the Chinese code do not match well.

Overall, the new classification method proposed in this paper can provide a range of critical aspect ratios depending on the conical angle of the storage surface. This range covers the values in the existing classification methods with a sound theoretical base. The existing methods can only provide critical aspect ratios for some special cases. Therefore, the new method proposed in this paper is a more general method used in silo classification, and can be used in engineering design.

References

[1] Cui Yuanrui, Gui Hengshi, Yuan Hailong, et al. GB 50077—2003 Chinese code for design of reinforced concrete silo[S]. Beijing: China Planning Press, 2004.

[2] German standard DIN1055 design loads for buildings: loads on silo bins, part 6 [S]. Berlin: German Standards Institute, 1986.

[3] Liang Chuanzhen, Liang Yongmao, Xing Meimei. On the load of squat silo[J]. *Grain Distribution Technology*, 1999 (3): 3 − 4. (in Chinese)

[4] Airy W. The pressure of grain[C]//*Minutes of Proceedings of the Institute of Civil Engineers*. London, 1897: 347 − 358.

[5] Reimbert M, Reimbert A. Pressure and overpressure vertical and horizontal silos[C]//*Proceedings of the International Conference on Design of Silos for Strength and Flow*.

London, 1980: 11 −28.
[6] Yuan Fang. Analysis of bulk-solid pressures on curve walls and its engineering application [D]. Dalian: School of Civil and Hydraulic Engineering of Dalian University of Technology, 2004. (in Chinese)
[7] Yuan Fang, Wang Fuming. An approach for calculation of the bulk material wall pressures in squat silos[J]. *Journal of Dalian University of Technology*, 2006, **46**(1): 75 −79. (in Chinese)
[8] Xu Q K. Test of physical mechanics property for granular grain [D]. Zhengzhou: School of Civil Engineering and Architecture of Henan University of Technology, 2007. (in Chinese)

筒仓分类方法评价

原　方[1]　王　辉[1]　胡玉霞[2]　张松松[1]

([1] 河南工业大学土木建筑学院, 郑州 450052)
([2] 西澳大学土木和资源学院, 澳大利亚珀斯 6009)

摘要: 提出了一种新的基于破裂面的深浅仓分类方法. 该方法考虑了贮料顶面倾角的影响, 可以处理贮料顶面平堆和锥堆 2 种工况. 以此为依据, 对目前工程设计中常用的 2 类深浅仓分类方法(即按高径比分类方法和按破裂面分类方法)进行评价. 结果表明: 该新方法能够根据贮料顶面锥堆或平堆给出区分深浅仓高径比的临界值范围, 该范围覆盖中国规范按高径比分类方法、德国规范、Reimbert 兄弟、梁传珍等现有方法所给出的区分深浅仓的高径比临界值. 因此, 该方法是一种更一般性的深浅分类方法, 而现有的深浅仓分类方法只是它的特例.

关键词: 筒仓; 高径比; 散粒体; 破裂面; 破裂角
中图分类号: TU249. 2