Neural network based method for compensating model error

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Abstract: Two traditional methods for compensating function model errors, the method of adding systematic parameters and the least-squares collection method, are introduced. A proposed method based on a BP neural network (called the H-BP algorithm) for compensating function model errors is put forward. The function model is assumed as $y = f(x_1, x_2, \ldots, x_n)$, and the special structure of the H-BP algorithm is determined as $(n+1) \times p \times 1$, where $(n+1)$ is the element number of the input layer, and the elements are $x_1, x_2, \ldots, x_n$ and y' (y' is the value calculated by the function model); p is the element number of the hidden layer, and it is usually determined after many tests; 1 is the element number of the output layer, and the element is $\Delta y = y_0 - y'$ (y_0 is the known value of the sample). The calculation steps of the H-BP algorithm are introduced in detail. And then, the results of three methods for compensating function model errors from one engineering project are compared with each other. After being compensated, the accuracy of the traditional methods is about ±19 mm, and the accuracy of the H-BP algorithm is ±4.3 mm. It shows that the proposed method based on a neural network is more effective than traditional methods for compensating function model errors.

Key words: model error; neural network; BP algorithm; compensating

The model error is the difference between the mathematical model (for example, the functional model) and objective reality. It can be expressed in formula as[1]

$$\boldsymbol{F}_1 = \boldsymbol{M}_0 - \boldsymbol{W} \tag{1}$$

where $\boldsymbol{F}_1$ is the real model error; $\boldsymbol{M}_0$ is the mathematical model; $\boldsymbol{W}$ is the unknown objective reality; $\boldsymbol{M}_0 \neq \boldsymbol{W}$. Because the objective reality $\boldsymbol{W}$ is not known, the precision model $\boldsymbol{M}$ is usually used instead of $\boldsymbol{W}$[1],

$$\boldsymbol{F}_2 = \boldsymbol{M}_0 - \boldsymbol{M} \tag{2}$$

where $\boldsymbol{F}_2$ is the verisimilitude model error. In fact, it is difficult to study $\boldsymbol{F}_1$ or $\boldsymbol{F}_2$ in a fixed quantity, because there are so many factors that make up the model error. However, once the function model is established, no matter how precisely you expand the model, the function model error inevitably exists.

In 1968, Baarda, a geodetic scientist from the Netherlands, published a technical paper named *A testing procedure for use in geodetic networks*[2]. He was the first scholar in the world to study the model error. Since then, the model error has been regarded as a hot issue by other scholars all over the world. Based on the theory and the method of data snooping advanced by Baarda, the methods of identifying, testing and estimating model errors developed rapidly; method such as smoothing noisy data with a spline function[3], residual and influence in regression[4], geodetic parameter estimation through iterative weighting[5], L1 norm minimization for surveying applications[6], multiple outlier detection by evaluating redundancy contributions of observations[7], the collection filtering and nonparametric regression method[8], the theory and application of an adjustment model with additional systematical parameters[1], and the neural network method for detecting the model error [9]. At present, neural networks are widely applied in the data processing of surveying engineering[10–12].

How to compensate the model error has been a hard issue ever since 1968. The research has been done by many scholars in the world, and many research achievements have been made. At present, the chief methods for compensating the function model error are as follows: the method of adding systematic parameters and the least-squares collection method etc[1]. According to the usage of the methods above for compensating function model errors in some projects, the authors found that the methods do not change the structure of the original fitting model, and the methods are ineffective and insufficient. In this paper, a proposed method based on neural networks for compensating function model errors is put forward. Having been used in some projects, the proposed method is validated as being very effective for compensating function model errors[13].

1 Introduction of General Methods for Compensating Function Model Errors

In this paper, formulae of two general methods for compensating function model errors are introduced as follows.

1.1 The method of adding systematic parameters

The idea behind the method of adding systematic parameters is that: the items that have been affected by systematic errors are added to the function model, and then the unknown coefficients in the items are used as additional parameters to be adjusted together with the chief parameters. The adjustment model with additional (systemic) parameters is supposed as[1]

$$\left.\begin{aligned} \boldsymbol{V}_{n\times 1} &= \boldsymbol{A}_{n\times t}\hat{\boldsymbol{X}}'_{t\times 1} + \boldsymbol{B}_{n\times u}\hat{\boldsymbol{Y}}_{u\times 1} - \boldsymbol{L}_{n\times 1} \\ \boldsymbol{D}(\boldsymbol{L}) &= \sigma_0^2\boldsymbol{Q} = \sigma_0^2\boldsymbol{P}^{-1} \end{aligned}\right\} \tag{3}$$

where $R(A) = t$; $R(B) = u$; $R(Q) = n$; $R(A)$ means the rank

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of matrix $\boldsymbol{A}$; $\hat{\boldsymbol{X}}'$ means the chief parameters; $\hat{\boldsymbol{Y}}$ means the additional (systematic) parameters. According to the adjustment principle $\boldsymbol{V}^{\mathrm{T}}\boldsymbol{PV}=\min$, the results can be calculated by[1]

$$\begin{bmatrix}\hat{\boldsymbol{X}}'\\ \hat{\boldsymbol{Y}}\end{bmatrix}=\begin{bmatrix}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PA} & \boldsymbol{A}^{\mathrm{T}}\boldsymbol{PB}\\ \boldsymbol{B}^{\mathrm{T}}\boldsymbol{PA} & \boldsymbol{B}^{\mathrm{T}}\boldsymbol{PB}\end{bmatrix}^{-1}\begin{bmatrix}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PL}\\ \boldsymbol{B}^{\mathrm{T}}\boldsymbol{PL}\end{bmatrix} \tag{4}$$

$$\hat{\sigma}_0^2=\frac{\boldsymbol{V}^{\mathrm{T}}\boldsymbol{PV}}{n-(t+u)} \tag{5}$$

1.2 The least-squares collection method

The functional model and the randomicity model are supposed as[1]

$$\boldsymbol{L}=\boldsymbol{AX}+\boldsymbol{BY}+\boldsymbol{\Delta},\quad \boldsymbol{V}=\boldsymbol{A}\hat{\boldsymbol{X}}+\boldsymbol{B}\hat{\boldsymbol{Y}}-\boldsymbol{L} \tag{6}$$

where $\boldsymbol{L}_{n\times1}$ is the observation vector; $\boldsymbol{\Delta}_{n\times1}$ is the observation error vector; $\boldsymbol{X}$ is the chief parameters; $\boldsymbol{Y}$ is the random parameters for filtering the error. It is known as $\boldsymbol{D}_{\boldsymbol{\Delta}}=\sigma_0^2\boldsymbol{Q}=\sigma_0^2\boldsymbol{P}^{-1}$; $\boldsymbol{D}_Y=\sigma_0^2\boldsymbol{Q}_{YY}=\sigma_0^2\boldsymbol{P}_Y^{-1}$; $\boldsymbol{D}_{\Delta Y}=0$; $E(Y)=\mu_Y$; $E(\Delta)=0$. According to the theory of the broad sense adjustment, the mathematical expectation of random parameters acts as dummy observations, and the weight matrix is supposed as $\boldsymbol{P}_Y$, so, the least-squares collection model can be translated into the indirect adjustment model. Based on the adjustment principle $\boldsymbol{V}^{\mathrm{T}}\boldsymbol{PV}+\boldsymbol{V}_Y^{\mathrm{T}}\boldsymbol{P}_Y\boldsymbol{V}_Y=\min$, the results can be calculated by[1]

$$\left.\begin{aligned}\boldsymbol{X}&=[\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{BD}_Y\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{D}_{\boldsymbol{\Delta}})^{-1}\boldsymbol{A}]^{-1}\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{BD}_Y\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{D}_{\boldsymbol{\Delta}})^{-1}\boldsymbol{L}\\ \boldsymbol{Y}&=\boldsymbol{D}_Y\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{BD}_Y\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{D}_{\boldsymbol{\Delta}})^{-1}(\boldsymbol{L}-\boldsymbol{AX})\end{aligned}\right\} \tag{7}$$

$$\hat{\sigma}_0^2=\frac{\boldsymbol{V}^{\mathrm{T}}\boldsymbol{PV}}{n-t} \tag{8}$$

2 Method for Compensating Function Model Errors Based on Neural Network

2.1 Algorithm of back propagation

The neural network method is an adaptive mapping method. The feed-forward back propagation (BP) network is a very popular model in the neural network[13-14]. In multilayer feed forward networks, the processing elements are arranged in layers and only the elements in adjacent layers are connected (see Fig. 1). All the calculation formulae of the BP network can be found in Refs. [13 - 14].

2.2 Special structure of BP network for compensating model errors (H-BP algorithm)

A new particular structure of the BP network is put forward to compensate function model errors. The function model is assumed as $y=f(x_1,x_2,\dots,x_n)$, and the special structure of the BP network for compensating the function model errors is determined as $(n+1)\times p\times1$ (see Fig. 1).

1) The element number of the input layer is taken as $(n+1)$. The elements are $x_1,x_2,\dots,x_n$ and y'. Among them, y' is the value calculated by the function model $y=f(x_1,x_2,\dots,x_n)$.

2) The element number of the hidden layer is taken as p.

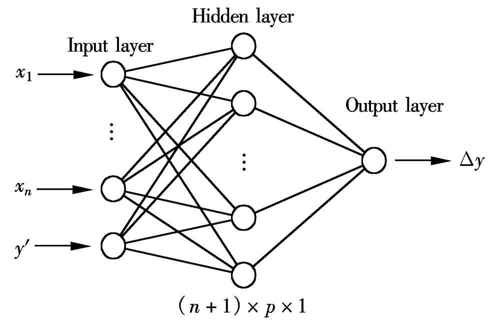

Fig. 1 The special structure of BP network

At present, the value of p is usually determined after many tests. (Note: In the project in this paper, the values of p are taken to be from 15 to 20 after being tested for many times.)

3) The element number of the output layer is taken as 1. The element is $\Delta y=y_0-y'$, where y_0 is the known value of the sample. Hence, Δy means the bias value of y' calculated by the function model, and it can be considered as the function model error.

Due to its high efficiency, the proposed method above is named as the H-BP algorithm in the following text, and it is convenient for describing.

2.3 Principle analysis of the H-BP algorithm for compensating function model errors

The H-BP algorithm constructed in this paper can be used for compensating function model errors. This can be explained by the BP network structure: one parameter y' calculated by the function model $y=f(x_1,x_2,\dots,x_n)$ is in the input layer, and the parameter of the output layer is Δy, the bias value of y' ($\Delta y=y_0-y'$, y' is the calculated value by the function model, y_0 is the true value). So, the proposed method based on a neural network is to simulate the difference between y' and y_0, and it is to compensate the function model errors of f.

After Δy is calculated, the formula for compensating the function model error of f is as follows:

$$y=y'+\Delta y \tag{9}$$

where y' is calculated by the function model f; Δy means the function model error of f, and it is simulated by the BP neural network above. If the function f we selected is different, the values of y' and Δy in Eq. (9) will be changed.

2.4 Calculating steps of the H-BP algorithm

Taking the elevation anomaly ξ fitting for example, the calculating steps of the proposed method based on a neural network for compensating function model errors are as follows:

1) Assume that the function between the elevation anomaly ξ and the coordinates (x,y) of the point is $\xi=f(x,y)$.

2) Assume that n points are in an area, of which n_1 point's values of the elevation anomaly ξ are already known, while n_2 ($n_2=n-n_1$) point's values of ξ need to be calculated. This method requires that n_1 must be greater than the numbers of the unknown parameters in the function model $\xi=$

$f(x, y)$.

3) Based on the function model $\xi = f(x, y)$ and the n_1 points, all the unknown parameters in the model can be calculated by the least squares network adjustment. Thus, after the adjustment, the elevation anomaly ξ' can be calculated for all points by the function model $\xi = f(x, y)$. That is

$$\xi_i' = f(x_i, y_i) \qquad i = 1, 2, \ldots, n \tag{10}$$

4) Calculate the difference $\Delta\xi$ between the elevation anomaly ξ' and the known value ξ_0 for all n_1 points, as

$$\Delta\xi_i = \xi_{0i} - \xi_i' \qquad i = 1, 2, \ldots, n_1 \tag{11}$$

where ξ_{0i} is the known value of the elevation anomaly (point i), and ξ_i' is the elevation anomaly of point i calculated by the function model $\xi = f(x, y)$.

5) The BP network is trained by a set of studying samples which include all the information of the prior n_1 points ($x_i, y_i, \xi_i'; \Delta\xi_i; i = 1, 2, \ldots, n_1$). The BP structure is $(2+1) \times p \times 1$. What needs to be explained is that x_i, y_i and ξ_i' are the three units in the input layer, and $\Delta\xi_i$ is in the output layer (see Fig. 1).

6) The differences in the elevation anomaly $\Delta\xi$ can be calculated through all n_2 points by the trained BP network. The elevation anomaly ξ can be calculated by

$$\xi_i = \xi_i' + \Delta\xi_i \qquad i = 1, 2, \ldots, n_2 \tag{12}$$

where ξ' is calculated by the function model and $\Delta\xi$ by the trained BP neural network.

3 Analysis of Results by the Proposed Method

3.1 Introduction of the project

The E-order GPS network of a city (about 280 km^2) has altogether 112 points, among which 56 GPS points have normal heights obtained by the third-order geodetic leveling. The accuracy of leveling is ±10 mm ($\sigma_0 = \pm 0.01$ m). So, the elevation anomaly ξ of the 56 points can be calculated. Among the 56 points, 18 evenly scattered points are selected as a study group to train the neural network, while the other 38 points form a test group to check the effectiveness of the trained neural network. The effectiveness of different methods can be checked by the test group (38 points).

The mean square error (MSE) acts as a guideline for estimating the effectiveness of different methods. The mean square error is calculated by

$$\sigma = \pm\sqrt{\frac{[vv]}{n}} \qquad v_i = \xi_i - \xi_{0i}; i = 1, 2, \ldots, 38 \tag{13}$$

where $n = 38$; v_i is the residual of point i in the test group; ξ_{0i} is the known elevation anomaly of point i; and ξ_i is the elevation anomaly of point i calculated by different methods.

3.2 Different methods for compensating model errors

To compare the effectiveness of the three methods above for compensating function model errors, four fitting methods are selected for the project.

1) Method A The plane fitting method (one function model)

The equation of the plane fitting method is given as

$$\xi_i' = b_0 + b_1x_i + b_2y_i, \quad v_i = \xi_i' - \xi_{0i} = b_0 + b_1x_i + b_2y_i - \xi_{0i}$$
$$i = 1, 2, \ldots, 18 \tag{14}$$

where ξ_{0i} is the known elevation anomaly of point i. According to the adjustment principle $\boldsymbol{V}^{\mathrm{T}}\boldsymbol{V} = \min$, the values of coefficients b_0, b_1, b_2 can be calculated, and the results are given in Tab. 1.

Tab. 1 The results of different compensating methods

Parameter	Method A	Method B	Method C	Method D
b_0	7.270	16.765	14.281	
b_1	-1.111×10^{-6}	-3.799×10^{-6}	-3.095×10^{-6}	
b_2	4.181×10^{-5}	4.199×10^{-5}	4.190×10^{-5}	
MSE of test group (38 points)/mm	±20.7	±19.5	±19.3	±4.3

2) Method B The plane fitting method + Adding systematic parameters

All the unknown coefficients in the model can be calculated by Eqs. (4) and (5). The values of the parameters after compensating this function model error are given in Tab. 1.

3) Method C The plane fitting method + The least-squares collection method

Being calculated by Eq. (7) and Eq. (8), the values of the parameters after compensating this model error are given in Tab. 1.

4) Method D The plane fitting method + The H-BP algorithm (neural network)

In Eqs. (10), (11) and (12), ξ' is calculated by Eq. (14). This method is to compensate the plane fitting model error based on the H-BP algorithm.

The results summary of the four methods is given in Tab. 1.

3.3 Analyzing the effectiveness of different methods

Now, let us analyze the effectiveness of the different methods in Tab. 1.

1) From Tab. 1, it can be seen that the MSE of method A is about ±20.7 mm. After being checked, the plane fitting method has function model errors.

2) Methods B, C and D are three different methods for compensating the function model error in method A. The MSE values of methods B, C and D are about ±19.5, ±19.3 and ±4.3 mm, respectively. Method B or C has some improvement compared with method A, so does method D (H-BP algorithm) when compared with method A. For the project, the general compensating methods have just reduced a little of the function model error. Fig. 2 shows the residuals v_i of the test group (38 points) by methods A, B, C and D.

3) Now, we compare methods A, B and C in Tab. 1. The values of the coefficients in method B and C are different from those in method A. But method B or C is still the plane fitting model. In fact, because the geoid is not a plane surface, the effectiveness of method B or C is not satisfying. After compensating the function model error, method D is no longer the plane fitting model. The proposed method changes the structure of the fitting model.

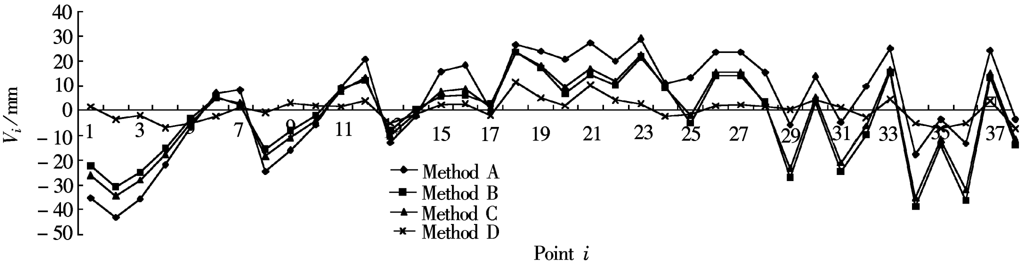

Fig. 2 The residuals v_i of the test group (38 points) by methods A, B, C and D

4 Conclusions

1) The two commonly used methods for compensating model errors are the method of adding systematic parameters and the least-squares collection method. The drawback of these two methods is that they do not change the structure of original function model, and they only modify the values of the parameters. Therefore, the two methods are not effective in compensating function model errors.

2) The proposed method based on neural networks is a better way to compensate function model errors. Being more efficient than traditional methods in compensating model errors, the proposed method has been applied in many projects, and all the results using the H-BP algorithm are satisfactory.

References

[1] Tao Benzao. *Statistical analyses of surveying data* [M]. Beijing: Surveying and Mapping Press, 1992: 55 − 87. (in Chinese)

[2] Baarda W. A testing procedure for use in geodetic networks [J]. *Neth Geod Comm Publ on Geodesy*, 1968, **2**(5): 27 − 55.

[3] Craven P, Wahba G. Smoothing noisy data with spline functions [J]. *Numer Math*, 1979, **31**(2): 377 − 403.

[4] Cook R D, Weisberg S. *Residuals and influence in regression* [M]. London: Chapmanand Hall Press, 1982: 23 − 41.

[5] Aduol F W O. Geodetic parameter estimation through iterative weighting [J]. *Survey Review*, 1994, **32** (252): 359 − 367.

[6] Marshall J, Bethel J. Basic concepts of L1 norm minimization for surveying applications [J]. *Journal of Surveying Engineering*, 1996, **122**(4): 168 − 179.

[7] Ding X, Colemen R. Multiple outlier detection by evaluating redundancy contributions of observations [J]. *Journal of Geodesy*, 1996, **70**(8): 489 − 498.

[8] Fischer B, Hegland M. Collocation filtering and nonparametric regression [J]. *Zeitschrift für Vermessungswesen*, 1999, **124**(1): 17 − 24.

[9] Hu Wusheng, Sha Yuejin, Kuang Shanlong. New method for transforming global positioning system height into normal height based on neural network [J]. *Journal of Surveying Engineering*, 2004, **130**(1): 36 − 39.

[10] Kuhar M, Stopar B, Turk G, et al. The use of artificial neural network in geoid surface approximation [J]. *Allgemeine Vermessungs-Nachrichten*, 2001, **108**(1): 22 − 27.

[11] Kavzoglu T, Saka M H. Modeling local GPS/leveling geoid undulations using artificial neural networks [J]. *J Geodesy*, 2005, **78**(9): 520 − 527.

[12] Lin Laosheng. Application of a back-propagation artificial neural network to regional grid-based geoid model generation using GPS and leveling data [J]. *Journal of Surveying Engineering*, 2007, **133**(2): 81 − 89.

[13] Hu Wusheng. *The theory of neural network and its applications in engineering* [M]. Beijing: Surveying and Mapping Press, 2006: 63 − 92. (in Chinese)

[14] Rao M A, Srinivas J. *Neural networks: algorithms and applications* [M]. Pangbourne: Alpha Science International Ltd, 2003: 17 − 113.

基于神经网络方法的模型误差补偿

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摘要:介绍了 2 种补偿模型误差的传统方法:附加系统参数方法和最小二乘配置法. 提出了一种基于 BP 算法的补偿模型误差的神经网络方法,简称为 H-BP 算法. 假设函数模型为 $y = f(x_1, x_2, \cdots, x_n)$,则 H-BP 算法的神经网络结构为$(n+1) \times p \times 1$,$(n+1)$是输入层元素个数,具体为 $x_1, x_2, \cdots, x_n$ 和 y',其中 y'是函数模型计算值;p 为隐含层节点数,一般通过大量试验得到;1 是输出层元素个数,具体为 $\Delta y = y_0 - y'$,其中 y_0 是样本真值. 然后,详细介绍了 H-BP 算法的具体计算步骤. 最后,结合一个工程实例,对 3 种补偿方法的结果进行了详细对比分析. 传统方法补偿之后的精度约为 ±19 mm,H-BP 算法补偿之后的精度为 ±4. 3 mm. 结果表明,与传统方法相比,新方法对模型误差的补偿效果更好.

关键词:模型误差;神经网络;BP 算法; 补偿

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