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Entropy function optimization for radar imaging

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Abstract: The convergence performance of the minimum entropy auto-focusing (MEA) algorithm for inverse synthetic aperture radar (ISAR) imaging is analyzed by simulation. The results show that a local optimal solution problem exists in the MEA algorithm. The cost function of the MEA algorithm is not a downward-convex function of multi-dimensional phases to be compensated. Only when the initial values of the compensated phases are chosen to be near the global minimal point of the entropy function, the MEA algorithm can converge to a global optimal solution. To study the optimal solution problem of the MEA algorithm, a new scheme of entropy function optimization for radar imaging is presented. First, the initial values of the compensated phases are estimated by using the modified Doppler centroid tracking (DCT) algorithm. Since these values are obtained according to the maximum likelihood (ML) principle, the initial phases can be located near the optimal solution values. Then, a fast MEA algorithm is used for the local searching process and the global optimal solution can be obtained. The simulation results show that this scheme can realize the global optimization of the MEA algorithm and can avoid the selection and adjustment of parameters such as iteration step lengths, threshold values, etc.

Key words: radar signal processing; inverse synthetic aperture radar (ISAR) imaging; auto-focusing

Inverse synthetic aperture radar (ISAR) imaging is a powerful tool for obtaining two-dimensional scattering characteristics of moving targets. It provides high range resolution by transmitting wide-band signals and high cross-range resolution by coherently accumulating the rotational component of moving targets. ISAR imaging consists of two stages. The first stage is motion compensation, which can be applied for transforming the relative motion between a radar and a moving target into the radar's rotating motion around the target on the turntable. The second stage is range-Doppler (R-D) image formation of the target.

Motion compensation, including range alignment and phase compensation, is a fundamental stage in ISAR imaging. Phase compensation is referred to the compensation of the phase error due to the translation component of the moving target. When the phase error compensation is well completed, the image focusing effect is good. Therefore, phase compensation is also called auto-focusing. Recently, various phase compensation algorithms for image focusing have been presented[1-5], where phase compensation components are determined by the focusing effect. The functions in these algorithms, such as the image contrast function[1] and the image entropy function[4], are multi-dimensional ones of phase compensation components. There is a local optimal solution problem in these algorithms. To solve this problem, some progress has been made on performance analysis and optimization modification for the auto-focusing algorithm oriented to the maximal image contrast[2-3]. However, there are few studies on the optimal solution of the minimum entropy auto-focusing (MEA) algorithm[4], which hypothesizes that the entropy function only has one single-point extreme value. In this paper, it is proved that the extreme-value solution of the entropy function of multi-dimensional images is not unique and the solution of the MEA algorithm may not be the global optimal solution. Subsequently, the global optimal solution of the MEA algorithm is given.

1 Characteristics of Multiple Extreme Values for MEA Algorithm

1.1 Theory of MEA algorithm

In ISAR imaging, 1-by-K complex vector $\boldsymbol{G}(n)=\{G(n,1), G(n,2), \ldots, G(n,K)\}$ denotes an aligned range profile, where $n=1,2,\ldots,N$ is the cross range bin number. The phase compensation is to compensate $\boldsymbol{G}(n)$ with $\exp[-\mathrm{j}\hat{\theta}(n)]$, where $\hat{\theta}(n)$ is the estimation of the translational phase error $\theta(n)$. The phase error $\theta(1)$ of the first echo is usually assumed to be zero. Then the ISAR image $I(n,k)$ can be obtained by the image formation[6-7] which is a Fourier transformation with respect to n of the phase compensated $\boldsymbol{G}(n)$. Thus,

$$I(n,k)=\sum_{m=1}^{N}[G(m,k)\exp(-\mathrm{j}\hat{\theta}(m))]\cdot \exp\left[-\mathrm{j}\frac{2\pi}{N}(m-1)(n-1)\right] \tag{1}$$

where $k=1,2,\ldots,K$ is the range bin number. The two-dimensional entropy function of an ISAR image is defined as[8]

$$E(\boldsymbol{I})=-\sum_{n=1}^{N}\sum_{k=1}^{K}\frac{|I(n,k)|^2}{s(\boldsymbol{I})}\ln\left[\frac{|I(n,k)|^2}{s(\boldsymbol{I})}\right] \tag{2}$$

where $s(\boldsymbol{I})=\sum_{n=1}^{N}\sum_{k=1}^{K}|I(n,k)|^2$ is the energy function of the image.

In the MEA algorithm, Eq. (2) is used as the cost-func-

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tion to estimate the focusing property of an ISAR image. The entropy of an image with uniformly-distributed brightness is large while that of a good focused image is relatively small. Therefore, an ISAR image can be well-focused by selecting $\hat{\theta}(n)$ appropriately and making the entropy smaller. $\hat{\theta}(n)$ can be calculated by

$$\hat{\theta}(n) = \underset{\hat{\theta}(n)}{\arg\min}[E(\boldsymbol{I})] \tag{3}$$

It is a $(N-1)$-dimensional optimization problem for the estimation of $\hat{\theta}(n)$, where $n = 2, 3, \ldots, N$. The searching process is described in Ref. [4].

1.2 Multiple extreme values of MEA algorithm

The MEA algorithm hypothesizes that the entropy function $E(\boldsymbol{I})$ has a unique minimum value in a global region because the optimal solution of $\theta(n)$ is iteratively obtained by searching a minimum value in the local region. However, this hypothesis may not be true in practice. The critical optimal solution of phase error estimation, i. e., the global minimum value, can only be achieved under some special conditions which probably are not satisfied all the time. Fig. 1 is a simple illustration of the iteration processing for searching the optimal solution of the MEA algorithm. The optimal solution of $\theta(n)$ is assumed to be a one-dimensional problem. θ^0 is the initial value for iteration. θ^1 and θ^2 are the searched minimum values for the first and the second iteration time, respectively. Considering the limitation of the iteration step length and the entropy function property, the final searched minimum value θ^n is converged to a point of θ^* which is misunderstood as the optimal solution of the phase error rather than the global minimum value θ^{**}.

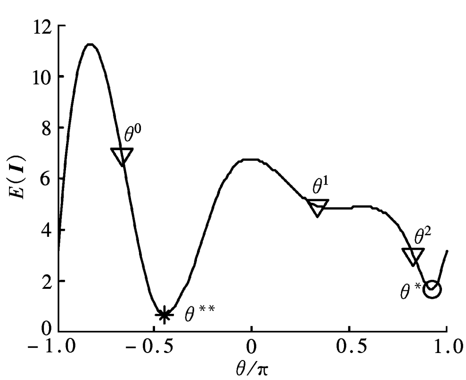


Fig. 1 An example of iteration processing for searching for a global optimal solution of MEA algorithm

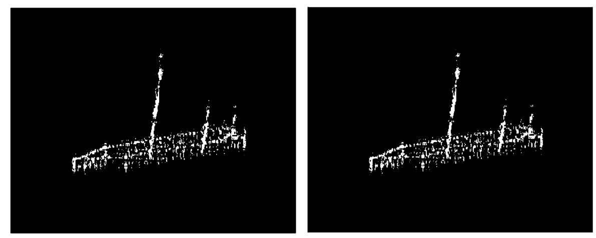
$E(\boldsymbol{I})$ is usually regarded as a multi-dimensional function $f(\boldsymbol{\theta})$ of variables $\boldsymbol{\theta} = \{\theta(n),\ n = 1, 2, \ldots, N\}$. $f(\boldsymbol{\theta})$ can be written as

$$f(\boldsymbol{\theta}) = -\sum_{n=1}^{N}\sum_{k=1}^{K}\frac{|I(n,k)|^2}{s(\boldsymbol{I})}\ln\left[\frac{|I(n,k)|^2}{s(\boldsymbol{I})}\right] \tag{4}$$

The radar target in the simulation is a ship model which is made up of multiple equal-strength scatters. And the radar imaging is under the condition that the range alignment is already completed. Supposing that $\boldsymbol{\theta}^0$ is the compensated phase error, the condition of the downward-convex function in the N-dimensional phase error domain should be expressed as

$$f[\alpha\boldsymbol{\theta}^1 + (1-\alpha)\boldsymbol{\theta}^2] \leqslant \alpha f(\boldsymbol{\theta}^1) + (1-\alpha)f(\boldsymbol{\theta}^2)$$
$$0 < \alpha < 1 \tag{5}$$

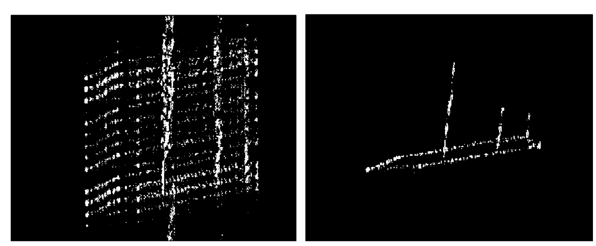
Case 1 The estimated phase error $\boldsymbol{\theta}^1$ is assumed to be located near $\boldsymbol{\theta}^0$. The initial image is a compensated result by the Doppler centroid tracking (DCT) algorithm (see Fig. 2(a)). Fig. 2(b) presents the ISAR imaging result by using the MEA algorithm. It can be seen that when the searching length is selected to be large enough, the auto-focusing processing may almost not be useful and the focusing effect of the initial image cannot be improved.



(a) (b)

Fig. 2 ISAR images of a ship model by using MEA algorithm under the condition that $\boldsymbol{\theta}^1$ is located near $\boldsymbol{\theta}^0$ and threshold parameters are not selected appropriately. (a) Initial image; (b) Final image

Case 2 The estimated phase error $\boldsymbol{\theta}^1$ is assumed to be located far away from $\boldsymbol{\theta}^0$. The initial image is an uncompensated result (see Fig. 3(a)). Fig. 3(b) presents the ISAR imaging result by using the MEA algorithm. It can be seen that when the searching length is selected appropriately, the auto-focusing processing is useful and the focusing effect of the initial image can be effectively improved.



(a) (b)

Fig. 3 ISAR images of a ship model by using MEA algorithm under the condition that $\boldsymbol{\theta}^1$ is located far away from $\boldsymbol{\theta}^0$ and searching parameters are selected appropriately. (a) Initial image; (b) Final image

In case 1, $f[\alpha\boldsymbol{\theta}^1 + (1-\alpha)\boldsymbol{\theta}^2]$ in Eq. (5) is smaller than $\alpha f(\boldsymbol{\theta}^1) + (1-\alpha)f(\boldsymbol{\theta}^2)$ so that $f(\boldsymbol{\theta})$ is satisfied with the downward-convex condition. On the contrary, the downward-convex condition is not satisfied in case 2. Generally, the entropy function $f(\boldsymbol{\theta})$ as a multiple-dimension function of $\boldsymbol{\theta}$ is not a downward-convex function, which means that the minimum value of the image entropy is not unique. Therefore, the solution of the MEA algorithm[4] is not certain to be a global optimal solution in the whole region.

1.3 Relationship between MEA and ML algorithms

As for artificial targets such as airplanes and warships, the images of the radar scattering points are considered to be focused, and thus the maximum likelihood (ML) phase compensation algorithm and the MEA algorithm are consistent with each other from the point of the focusing effect[8]. When ignoring the influence of the rotational component on the phase compensation, the DCT algorithm is similar to the ML phase compensation algorithm. However, since the rotational component of a target scatterer always exists in a real case, the DCT algorithm is generally different from the ML algorithm. Though the imaging result in Fig. 2(a) is basically focused by using phase compensation of the DCT algorithm, appropriate searching is still needed to approach the global minimum entropy. Otherwise, it is hard to improve the focusing effect (see Fig. 2(b)).

2 Optimization Solution of MEA Algorithm

Excepting the MEA algorithm, other phase compensation algorithms such as contrast-based phase compensation algorithm[9] are virtually used to solve a multi-dimensional optimization problem. Currently in the implementation of these algorithms, the steepest descent algorithm is often utilized for improving the searching efficiency. However, though the optimal value can be obtained by the MEA algorithm, the results are still local extreme values.

In applied mathematics, some approaches have been proposed to solve the global-extreme-value searching problem, such as the annealing simulated algorithm, the genetic algorithm and the ant algorithm. But for the multiple-dimensional optimization problem, the computation of the ant colony algorithm and the genetic algorithm is usually extremely huge[10]. It is almost impossible to overcome the huge computation of searching works when employing these two optimization algorithms in phase compensation of ISAR imaging. However, by using the annealing simulating algorithm, the steepest descend processing with a small random disturbance can be realized. This can result in the searching process away from the local extreme value, and gradually approach the global extreme value when the system temperature is dropped. Therefore, it is important to select an appropriate random disturbance and control the varying trace of the system temperature. But it is still a problem on how to adjust the parameters in annealing processing.

A fast MEA algorithm is designed in cooperation with the ML phase compensation algorithm to avoid selecting parameters such as iteration step lengths. First, the radar image obtained by the modified DCT phase compensation algorithm[11] is selected as an initial image. In this phase using the compensation algorithm, the rotational component of the initial image is eliminated and the phase estimation is made. Only a small iteration is needed to realize the ML phase estimation and compensation. Because the ML algorithm is virtually consistent with the MEA algorithm, the modified DCT algorithm can overcome the problem of the DCT algorithm because the initial value of the estimated phase is not in the region of the global optimal solution. Then, a fast MEA algorithm is used to search the local extreme value of the compensated phase. The principle of the fast MEA algorithm can be described as follows.

Substitute Eqs. (1) and (2) into $\frac{\partial E(\boldsymbol{I})}{\partial \theta(n)}=0$, and then

$$\exp[-\mathrm{j}\theta(n)] = \frac{w^*(n)}{|w(n)|} \tag{6}$$

where

$$w(n) = \sum_{k=1}^{K} \boldsymbol{G}(n, k)\left\{\sum_{q=1}^{N} \ln(|I(q, k)|) I^*(q, k) \cdot \exp\left[-\mathrm{j}\frac{2\pi}{N}(q-1)(n-1)\right]\right\} \tag{7}$$

The operation in $\{\cdot\}$ in Eq. (7) can be fast implemented by the FFT. Because $I(q, k)$ in Eq. (7) is related to $\{\theta(n), n=1, 2, \ldots, N\}$, the variables $\{\theta(n), n=1, 2, \ldots, N\}$ in Eq. (6) can not be solved explicitly. This can be solved by the following iteration:

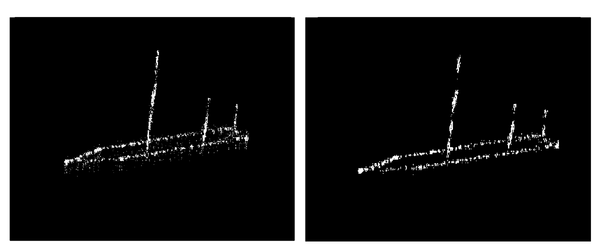
$$\exp[-\mathrm{j}\theta_{l+1}(n)] = \frac{w_l^*(n)}{|w_l(n)|} \tag{8}$$

where

$$w_l(n) = \sum_{k=1}^{K} \boldsymbol{G}(n, k)\left\{\sum_{q=1}^{N} \ln(|I_l(q, k)|) I_l^*(q, k) \cdot \exp\left[-\mathrm{j}\frac{2\pi}{N}(q-1)(n-1)\right]\right\} \tag{9}$$

It can be found from Eq. (8) that the compensated phase information of the l-th iteration is transferred to the phase estimation of the $(l+1)$-th iteration. Because this relationship is derived from the extreme value equation $\frac{\partial E(\boldsymbol{I})}{\partial \theta(n)}=0$, the estimated phase can be gradually converged to the local extreme value in the neighboring region of the initial phase. Therefore, the adjustment of the parameters can be realized.

An estimating result from the modified DCT algorithm is selected as the initial value of the compensated phase, as shown in Fig. 4(a). The local searching is done by the fast MEA algorithm according to Eq. (8). The result shown in Fig. 4(b) exhibits that this algorithm can obtain the global optimal solution of the MEA algorithm. Moreover, it can avoid the selection and adjustment of parameters such as iteration step lengths, threshold values, etc.



(a) (b)

Fig. 4 ISAR images of a ship model by using the modified DCT algorithm under the condition that $\boldsymbol{\theta}^1$ is located near $\boldsymbol{\theta}^0$. (a) Initial image; (b) Final image

3 Conclusion

The entropy function of an ISAR image may not be a downward-convex function of the phase error. A global optimal solution of the MEA algorithm is presented, where the estimating result from a modified DCT algorithm is first selected as the initial value of the compensated phase and then the local searching is executed by a fast MEA algorithm. The results show that this process can obtain the global optimal solution of the MEA algorithm.

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基于雷达成像的熵函数优化方法

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摘要：对 ISAR 成像的最小熵自聚焦(MEA)算法进行了收敛性分析．仿真结果表明，MEA 算法存在局部最优问题，作为其代价函数的 ISAR 像熵函数并非多维补偿相位的下凸函数．只有当该补偿相位矢量的初值选取合适，使其处于像熵函数的全局最小点附近时，MEA 算法才能收敛到全局最优解．针对 MEA 算法的最优化问题，给出了一种基于雷达成像的熵函数优化方法．该方法首先采用改进的多普勒中心跟踪法估计补偿相位初值．该初值是最大似然准则下的估计结果，可以使初始相位位于最优解附近．然后，利用快速 MEA 算法进行局部搜索，得到全局最优解．仿真结果表明，该算法不仅实现了 MEA 算法的全局最优求解，还可避免步长、阈值等参数的选择与调整．

关键词：雷达信号处理；逆合成孔径雷达(ISAR)成像；自聚焦

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