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Method for integer ambiguity resolution in GPS network RTK

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Abstract: A method for integer ambiguity resolution in the global positioning system (GPS) multi-reference station network real time kinematic (RTK) is proposed. First, the barycenter of the triangle of reference stations for ambiguity resolution is taken as a reference point. The satellite which has the largest elevation angle with the reference point is selected as a reference satellite. The parameters for constructing the weight matrix of carrier phase observation and the criteria for checking the correctness of integer ambiguity resolution of a network are obtained. Then, the wide ambiguity is calculated by a linear combination method of dual-band observation. And the L1 ambiguity is obtained by a non-ionosphere combination method. The Kalman filter is introduced to refine the floating-point solution of ambiguity and estimate the real-time tropospheric delay. Finally, the cofactor matrix of ambiguity is de-correlated by Z-transformation to reduce the searching space of the integer ambiguity solution and improve the efficiency of the least-squares ambiguity decorrelation adjustment (LAMBDA) algorithm. The experimental results show that this method can reliably obtain the integer ambiguity solution among multi-reference stations with 40 epochs.

Key words: network real time kinematic; ambiguity; troposphere; reference satellite; least-squares ambiguity decorrelation adjustment

Network RTK is an effective method for precisely positioning in GNSS, among which the key step is to quickly determine the integer ambiguity resolution between reference stations[1]. So far, a two-step method has been widely studied; that is, the long-wavelength characteristics of the wide-lane combination is used for fixing the wide ambiguity in the first step, and then the non-ionosphere combination method is utilized for estimating the L1 ambiguity in the second step[2]. In this paper, a new method is proposed for selecting the reference satellite. Based on the conventional two-step method, the Kalman filter is proposed to estimate the float ambiguity resolution, and the LAMBDA algorithm is used to fix the integer ambiguity resolution. The reliability and efficiency of the method for integer ambiguity resolution are significantly improved.

1 New Criteria for Determining Reference Satellite

Network RTK processes GNSS data in a unit composed of multiple reference stations. Because the angles between a satellite and diverse reference stations are different, the disunity of the reference satellites in a resolution unit can lead to some uncertain errors of correction direction. Consequently, the RTK positioning is difficult to be realized. Furthermore, the determination of the reference satellite is a key factor for determining the weight matrix in the LAMBDA algorithm[3]. Therefore, a reasonable choice of the reference satellite is an important step before ambiguity resolution.

In this paper, a new criterion for determining a reference satellite is presented. The satellite which has the largest elevation angle to the barycenter of the resolution unit is taken as a reference satellite. The spatial relationship between the reference satellite and reference stations is shown in Fig. 1.

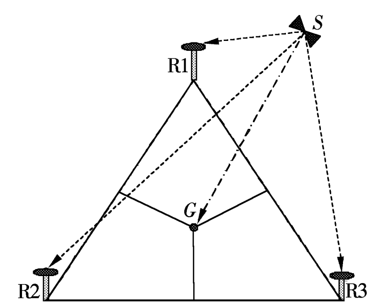

Fig. 1 The spatial relationship between reference stations and the reference satellite

In the standard space coordinate WGS-84, the space coordinates of the reference stations R1, R2, R3 are taken as (X_1, Y_1, Z_1), (X_2, Y_2, Z_2) and (X_3, Y_3, Z_3), respectively. The coordinate of the reference satellite is taken as (X_S, Y_S, Z_S). G is the barycenter of the triangle-net. Then, the coordinates of G can be written as

$$\left.\begin{aligned} X_G &= \frac{X_1 + X_2 + X_3}{3} \\ Y_G &= \frac{Y_1 + Y_2 + Y_3}{3} \\ Z_G &= \frac{Z_1 + Z_2 + Z_3}{3} \end{aligned}\right\} \tag{1}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_H = \boldsymbol{H}\left(\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_S - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_G \right) \tag{2}$$

where

$$\boldsymbol{H} = \begin{bmatrix} -\sin B_G \cos L_G & -\sin B_G \sin L_G & \cos B_G \\ -\sin L_G & \cos L_G & 0 \\ \cos B_G \cos L_G & \cos B_G \sin L_G & \sin B_G \end{bmatrix} \tag{3}$$

where B_G and L_G are the latitude and the longitude of the triangle-net's barycenter, respectively. Then, the elevation angle between the satellite and the barycenter can be computed as

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$$E = \arctan \frac{Z_H}{(X_H^2 + Y_H^2)^{1/2}} \tag{4}$$

According to Eq. (4), the reference satellite can be determined when maximizing the value of E. It can be ensured that the reference satellites for all reference stations are consistent in the triangle-net. Moreover, the new criterion for determining the reference satellite plays an important role in integer ambiguity resolution between the reference stations, which can be interpreted as follows:

1) The criterion can be used for checking the accuracy of the ambiguity resolution between the reference stations. As shown in Fig. 1, the accuracy of the ambiguity resolution for a triangle-net can be checked by

$$\Delta\nabla N_{\mathrm{R1R2}} + \Delta\nabla N_{\mathrm{R2R3}} + \Delta\nabla N_{\mathrm{R3R1}} = 0 \tag{5}$$

2) The criterion can be utilized for obtaining the inverse integer ambiguity solution of the difficult baseline because of the multipath effect, lower satellite elevation angle, overlong baseline, etc. If the ambiguity of baseline R1R2 cannot be fixed, it can be calculated by

$$\Delta\nabla N_{\mathrm{R1R2}} = -\Delta\nabla N_{\mathrm{R2R3}} - \Delta\nabla N_{\mathrm{R3R1}} \tag{6}$$

2 Integer Ambiguity Resolution

2.1 Wide-lane ambiguity resolution

There are two approaches to resolving the wide-lane ambiguity, i.e., the double P-code pseudo-range method and the double frequency linear combination method. The latter has better precision than the former because of carrier observation[3]. Therefore, the double frequency linear combination method is applied in this study, and the wide-lane integer ambiguity can be fixed by

$$\Delta\nabla \bar{\boldsymbol{N}}_{\mathrm{w}} - \frac{1}{\sqrt{n}}\boldsymbol{Z}_{\alpha/2} < \Delta\nabla \boldsymbol{N}_{\mathrm{w}} < \Delta\nabla \bar{\boldsymbol{N}}_{\mathrm{w}} + \frac{1}{\sqrt{n}}\boldsymbol{Z}_{\alpha/2} \tag{7}$$

where $\Delta\nabla \bar{\boldsymbol{N}}_{\mathrm{w}}$ is the double difference (DD) wide-lane integer ambiguity; $\Delta\nabla \boldsymbol{N}_{\mathrm{w}}$ is the DD wide-lane float ambiguity; n is the number of the epochs; $\boldsymbol{Z}_{\alpha/2}$ is the z-score of the hypothesis test. When the confidence level is set at 95%, any element in $\boldsymbol{Z}_{\alpha/2}$ is 1.96.

2.2 L1 ambiguity floats resolution

The float ambiguity $\Delta\nabla \boldsymbol{N}_1$ can be resolved by a non-ionosphere combination[4] of L1 and L2; that is,

$$\Delta\nabla \boldsymbol{N}_1 = \frac{1}{\lambda_{\mathrm{n}}}\{\lambda_{\mathrm{w}}\left(\frac{f_1}{f_1+f_2}\Delta\nabla \boldsymbol{\phi}_1 - \frac{f_2}{f_1+f_2}\Delta\nabla \boldsymbol{\phi}_2\right) - \Delta\nabla \boldsymbol{\rho} - \Delta\nabla \boldsymbol{O} - \Delta\nabla \delta\boldsymbol{\rho}_{\mathrm{trop}} - \Delta\nabla \boldsymbol{M}\} - \frac{f_2}{f_1-f_2}\Delta\nabla \boldsymbol{N}_{\mathrm{w}} \tag{8}$$

where λ_{n} is the narrow-lane combination wavelength; λ_{w} is the wide-lane combination wavelength; f_1 and f_2 are the frequencies of L1 and L2, respectively; $\Delta\nabla \boldsymbol{\phi}_1$ and $\Delta\nabla \boldsymbol{\phi}_2$ are the DD carrier observations of L1 and L2, respectively; $\Delta\nabla \boldsymbol{\rho}$ is the DD distance between the reference station and the satellite; $\Delta\nabla \boldsymbol{O}$ is the DD multipath; $\Delta\nabla \delta\boldsymbol{\rho}_{\mathrm{trop}}$ is the DD troposphere delay; $\Delta\nabla \boldsymbol{M}$ is the DD orbit error. The impact of the ionosphere can be effectively eliminated by Eq. (8). Since the orbit error $\boldsymbol{O}$ and the multipath effect are usually ignored[4], the main error effect on the accuracy of $\Delta\nabla \boldsymbol{N}_1$ is the tropospheric delay error[5].

With respect to the troposphere mapping function[6], Eq. (8) can be rewritten as

$$\lambda_{\mathrm{w}}\left(\frac{f_1}{f_1+f_2}\Delta\nabla \boldsymbol{\phi}_1 - \frac{f_2}{f_1+f_2}\Delta\nabla \boldsymbol{\phi}_2\right) - \Delta\nabla \boldsymbol{\rho} - \frac{f_2\lambda_{\mathrm{n}}}{f_1-f_2}\Delta\nabla \boldsymbol{N}_{\mathrm{w}} = [\Delta\boldsymbol{F}_{\mathrm{trop}}(\theta_{\mathrm{Q}}) \quad \Delta\boldsymbol{F}_{\mathrm{trop}}(\theta_{\mathrm{P}}) \quad \lambda_{\mathrm{n}}]\begin{bmatrix} D_{\mathrm{Q}} \\ D_{\mathrm{P}} \\ \Delta\nabla \boldsymbol{N}_1 \end{bmatrix} + \boldsymbol{\varepsilon} \tag{9}$$

where θ_{P} and θ_{Q} are the satellite elevation angle of P station and Q station, respectively; $\Delta\boldsymbol{F}_{\mathrm{trop}}(\cdot)$ are the single differential (SD) mapping function; D_{Q} and D_{P} are the tropospheric zenith delay of P station and Q station, respectively; $\boldsymbol{\varepsilon}$ is the residual error.

In order to solve Eq. (9) to obtain the tropospheric zenith delay and the L1 float ambiguity resolution, the Kalman filter is introduced when considering the main delay error of the zenith direction as a random walk process[6]. The initial value of the state transition noise variance σ_{trop}^2 is set as 10 to 90 cm^2/h which is obtained by the Niell model[7], and that of the ambiguity state transition noise variance σ_N^2 is set as zero[8]. Then, the state equation of the corresponding discrete system can be established as

$$\left.\begin{aligned} D(t_n) &= D(t_{n-1}) + W_D(t_n) & W_D(t_n) &\sim N(0,\ \Delta t\sigma_{\mathrm{trop}}^2) \\ \Delta\nabla \boldsymbol{N}_1(t_n) &= \Delta\nabla \boldsymbol{N}_1(t_{n-1}) + W_N(t_n) & W_N(t_n) &\sim N(0,\ \Delta t\sigma_N^2) \end{aligned}\right\} \tag{10}$$

Combining Eqs. (9) and (10), the Kalman filter equation can be established as

$$\left.\begin{aligned} \boldsymbol{X}_n &= \boldsymbol{X}_{n-1} + \boldsymbol{W}_n & \boldsymbol{W}_n &\sim N(0,\ \boldsymbol{U}_n) \\ \boldsymbol{L}_n &= \boldsymbol{B}_n\boldsymbol{X}_n + \boldsymbol{V}_n & \boldsymbol{V}_n &\sim N(0,\ \boldsymbol{Q}_n) \end{aligned}\right\} \tag{11}$$

where

$$\boldsymbol{B}_n = [\Delta\boldsymbol{F}(\theta_{\mathrm{Q}}) \quad \Delta\boldsymbol{F}(\theta_{\mathrm{P}}) \quad \lambda_{\mathrm{n}}]$$

$$\boldsymbol{X}_n = [D_{\mathrm{P}} \quad D_{\mathrm{Q}} \quad \Delta\nabla \boldsymbol{N}_1]^{\mathrm{T}}$$

$$\boldsymbol{L}_n = \lambda_{\mathrm{w}}\left(\frac{f_1}{f_1+f_2}\Delta\nabla \boldsymbol{\phi}_1 - \frac{f_2}{f_1+f_2}\Delta\nabla \boldsymbol{\phi}_2\right) - \Delta\nabla \boldsymbol{\rho} - \frac{f_2\lambda_{\mathrm{n}}}{f_1-f_2}\Delta\nabla \boldsymbol{N}_{\mathrm{w}}$$

where $\boldsymbol{U}_n$ is the state noise variance matrix; $\boldsymbol{Q}_n$ is the observation error variance matrix.

The tropospheric zenith delay and the float ambiguity can be estimated by the above filter, and then the integer ambiguity can be fixed by the LAMBDA algorithm based on the float ambiguity and the cofactor matrix. The comparison results between the Kalman filter and the single-epoch algorithm show that the error between the tropospheric delay estimated by the Kalman filter and that calculated by the posterior data is less than 2 cm (see Fig. 2). The L1 ambiguity becomes stable after 80 epochs by the Kalman filter while it keeps fluctuating by the single-epoch algorithm (see Fig. 3).

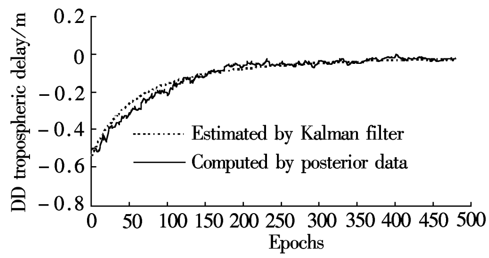

Fig. 2 Comparison of DD tropospheric delay error

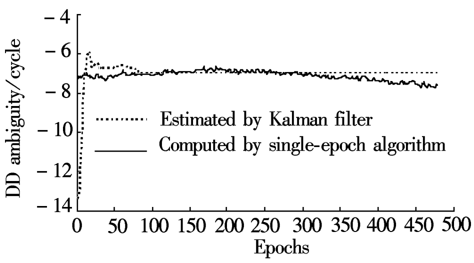

Fig. 3 Comparison of DD ambiguity

2.3 L1/L2 integer ambiguity resolution

To apply the LAMBDA algorithm for fixing the integer ambiguity, a cofactor matrix for the DD ambiguity is constructed[9]. The weight matrix is constructed by the satellite elevation, which can be expressed as

$$\boldsymbol{P}=\begin{bmatrix} 1+\dfrac{\sin E_{k1}}{\sin E_{\mathrm{r}}} & 1 & \cdots & 1 \\ 1 & 1+\dfrac{\sin E_{k2}}{\sin E_{\mathrm{r}}} & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1+\dfrac{\sin E_{k(n-1)}}{\sin E_{\mathrm{r}}} \end{bmatrix} \tag{12}$$

where E_{r} is the reference satellite elevation; E_{k1}, …, $E_{k(n-1)}$ are the other $n-1$ satellite elevations. According to the error propagation law[10], the cofactor matrix of the float ambiguity can be calculated by

$$\boldsymbol{Q}_{\hat{N}}=\lambda_{\mathrm{w}}^{2}\left(\frac{f_1}{f_1+f_2}\right)^{2}\boldsymbol{P}^{-1}+\lambda_{\mathrm{w}}^{2}\left(\frac{f_2}{f_1+f_2}\right)^{2}\boldsymbol{P}^{-1} \tag{13}$$

Therefore, the integer ambiguity can be obtained by integer searching, where the searching space is defined as

$$(\hat{\boldsymbol{N}}-\boldsymbol{N})^{\mathrm{T}}\boldsymbol{Q}_{\hat{N}}^{-1}(\hat{\boldsymbol{N}}-\boldsymbol{N})\leqslant\chi^{2} \qquad \boldsymbol{N}\in\mathbf{Z}^{n} \tag{14}$$

where $\hat{\boldsymbol{N}}$ is the float ambiguity; $\boldsymbol{N}$ is the integer ambiguity. The searching space is an n-dimensional ellipsoid centered at $\hat{\boldsymbol{N}}$, whose shape and size are determined by $\boldsymbol{Q}_{\hat{N}}$ and χ^2. With the increase in the ambiguity space, the searching space expands and the search time extends[11]. In order to improve the efficiency, the searching space is transformed into an approximate round ball by the Z-transformation matrix, which can be expressed as

$$\boldsymbol{Q}_{\hat{Z}}=\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{Q}_{\hat{N}}\boldsymbol{Z}, \quad \hat{z}=\boldsymbol{Z}^{\mathrm{T}}\hat{\boldsymbol{N}} \tag{15}$$

The spatial distributions of the cofactor matrix before and after transformation are shown in Fig. 4. The peak of the ellipse in Fig. 4(a) is smaller than that in Fig. 4(b), indicating that the correlation of the cofactor matrix is significantly reduced by Z-transformation.

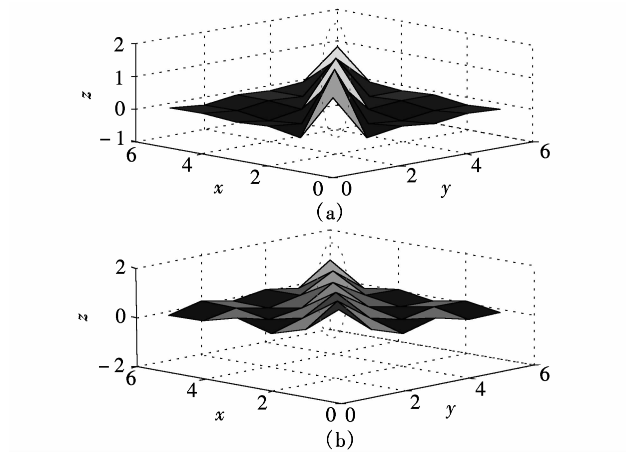

Fig. 4 Spatial distribution of cofactor matrix. (a) Before Z-transformation; (b) After Z-transformation

After Z-transformation, the searching space can be defined as

$$(\hat{z}-z)^{\mathrm{T}}\boldsymbol{Q}_{\hat{z}}^{-1}(\hat{z}-z)\leqslant\chi^{2} \qquad z\in\mathbf{Z}^{n} \tag{16}$$

Since $\boldsymbol{Q}_{\hat{Z}}$ makes the searching space close to spherical, its search tree is less than that before transformation. Therefore, the searching efficiency of the integer ambiguity is improved. After the integer resolution of z is obtained by Eq. (16), the integer ambiguity can be resolved by inverse Z-transformation.

When $\Delta\nabla\overline{N}_1$ is fixed by the LAMBDA algorithm, $\Delta\nabla\overline{N}_2$ can be calculated by

$$\Delta\nabla\overline{\boldsymbol{N}}_2=\Delta\nabla\overline{\boldsymbol{N}}_{\mathrm{w}}-\Delta\nabla\overline{\boldsymbol{N}}_1 \tag{17}$$

3 Case Study

A case study is presented for checking the proposed method, and a triangle-net is established by using four reference stations in a Tianjin continuous operational reference system. The location information for the four reference stations is shown in Fig. 5.

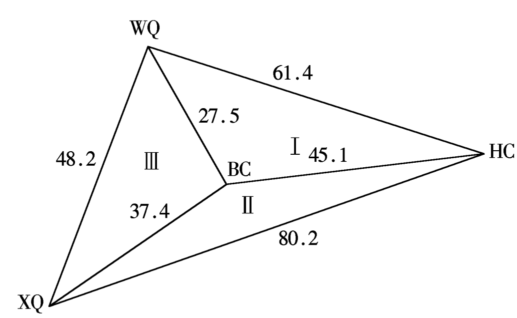

Fig. 5 Distribution of reference stations (unit: km)

3.1 DD ambiguity resolution

The results of the DD ambiguity resolution of PRN 08 are shown in Fig. 6. It can be seen that the DD ambiguity resolution converges gradually and is close to an integer after 60 to 80 epochs for the three midrange baselines, BC-WQ, BC-XQ and HC-BC. The proposed method is proved to be efficient in quickly and accurately obtaining the integer ambiguity resolution. However, for the long baseline XQ-HC, it is difficult to obtain the integer ambiguity resolution in a short time. For this case, Eq. (6) is applied for solving the integer ambiguity of the difficult baseline.

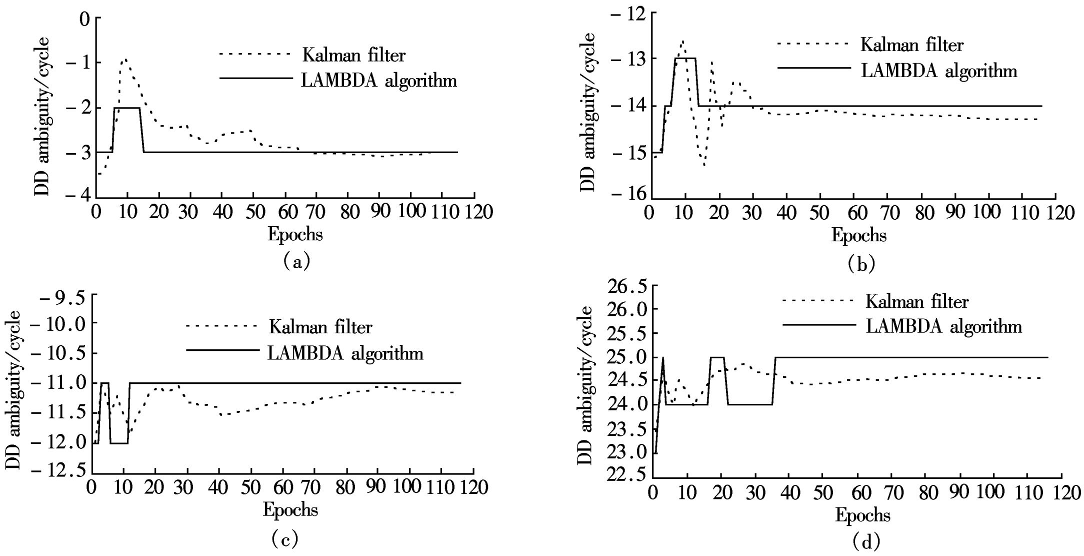

Fig. 6 DD ambiguity of PRN 08. (a) BC-WQ; (b) BC-XQ; (c) HC-BC; (d) XQ-HC

3.2 Quality checking

The validity of the DD ambiguity is checked by Eq. (5) for describing the relationships between the baselines in the network. Tab. 1 shows the closure error of the double integer ambiguity of the network Ⅱ in Fig. 4. Considering PRN 28 as a reference satellite, the integer ambiguity resolution is calculated with 30 epochs by the Kalman filter and the LAMBDA algorithm. Meanwhile, the correct DD integer ambiguity solutions are obtained by the professional GPS software Bernese for comparison.

Tab. 1 Closure error of DD ambiguity in network Ⅱ cycle

Satellite number	DD ambiguity			Error
	BC-XQ	XQ-HC	HC-BC	
11	−15(−15)	23(23)	−8 (−8)	0
20	−33(−33)	26(26)	7(7)	0
08	−14(−14)	24(25)	−11(−11)	−1
27	−23(−23)	9(10)	13(13)	−1
17	−2(−2)	16(16)	−14(−14)	0

Note: The data in the parentheses are the correct DD ambiguity solutions obtained by the professional GPS software Bernese.

As shown in Tab. 1, the closure error of PRN 08 and PRN 27 are equal to zero due to the false DD ambiguity of the long baseline XQ-HC. If the integer ambiguity of two shorter baselines in a unit is correctly resolved, the longer baseline can be determined by the integer ambiguity resolutions of the other baselines according to Eq. (5). For example, the DD integer ambiguity resolution of PRN 08 can be corrected to 25 by the true integer ambiguity resolution of BC-XQ and HC-BC. The rest may be deduced by analogy. So the DD ambiguity of the long baseline can be determined quickly and accurately.

4 Conclusion

Integer ambiguity resolution is regarded as the key technology of the GPS network RTK. Based on the conventional two-step method, the Kalman filter is proposed to estimate the float ambiguity resolution and the LAMBDA algorithm is used to fix the integer ambiguity resolution. The reliability and efficiency of the method for integer ambiguity resolution are significantly improved. And the proposed method has been already applied in the network RTK system self-developed by Southeast University.

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一种 GPS 网络 RTK 整周模糊度解算方法

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摘要： 提出了一种适用于 GPS 多参考站网络的 RTK 模糊度固定方法. 首先，以参与模糊度解算的参考站三角形重心为参考点，将与其形成高度角最大的卫星选作参考卫星，进而提供了载波观测值权阵构造的参数以及模糊度网解正确性的检验条件. 然后，通过双频线性观测数据组合计算双差宽巷模糊度，并利用无电离层观测数据组合计算 L1 模糊度浮点解，引入卡尔曼滤波进行精化，并实时估计对流层延迟参数. 最后，通过 Z 变换对模糊度协因素阵进行去相关，从而减小模糊度整数解的搜索空间，提高 LABMDA 算法固定模糊度的效率. 实验结果表明，该方法能在 40 个历元内可靠实现多个参考站间整周模糊度解算.

关键词： 网络实时动态(RTK)；模糊度；对流层；参考站；最小二乘模糊度降相关方法

中图分类号： P225