Journal of Southeast University (English Edition) Vol. 25, No. 4, pp. 518 – 522 Dec. 2009 ISSN 1003—7985

Virtual fracture test of asphalt mixture based on discrete element method

Chen Jun Huang Xiaoming

(School of Transportation, Southeast University, Nanjing 210096, China)

Abstract: In order to study fracture behaviors of asphalt mixtures, virtual tests of the two-dimensional (2D) microstructure based on the discrete element method (DEM) are designed. The virtual structure of the 2D digital specimen of asphalt mixture is generated based on a particle generation program, in which the gradation and the irregular shapes of aggregates are considered. With the 2D digital specimens, a DEM-based mixture model is established and center-point beam fracture simulation tests are conducted by the DEM. Meanwhile, a series of calibration tests are carried out in laboratory to evaluate the DEM model and validate the methods of virtual fracture tests. The test results indicate that the fracture intensity of asphalt mixtures predicted by the DEM matches very well with the intensity obtained in laboratory. It is concluded that the microstructural virtual tests can be used as a supplemental tool to evaluate fracture properties of asphalt mixtures.

Key words: asphalt mixture; fracture; discrete element method; virtual test

Cracking has been pointed out as a major problem in asphalt concrete (AC) pavements. Previous studies predicting fracture properties were based on extensive physical experiments such as beam bending tests and indirect tensile tests. However, fracture properties predictions from tests come at the expense of conducting time-consuming and costly test procedures. Micromechanical modeling has tremendous potential benefits in the field of asphalt technology to reduce or eliminate costly tests to characterize asphalt-aggregate mixtures for design and control of these materials.

Over the past 10 years, the use of micromechanics to predict properties of asphalt mixtures and mastics has drawn increasing attention, and a number of approaches have been investigated. Birgisson et al. [1] used the boundary element method (BEM) to predict viscoelastic response and crack growth in asphalt mixtures. Sadd et al. [2] used the finite element method (FEM) to simulate the indirect tensile test. Finite element modeling of the asphalt concrete microstructure allows accurate modeling of aggregates and mastic microstructure geometry. However, the current limitations of this approach are the convergence difficulties in modeling the irregular shapes of aggregate and the contact geometry between the aggregate and the binder. Furthermore, the modeling of the aggregate or the mastic fracture during strength test simulations is very cumbersome based upon current finite element capabilities, unless a continuum approach is adopted, in which case the microstructural features are homogenized into an equivalent material.

The DEM is a promising technique for micromechanical modeling of the asphalt concrete microstructure. The discrete element method analyzes particulate systems by modeling the translational and rotational behavior of each particle using Newton's second law with appropriate interparticle contact forces. Rothenburg et al. [3] developed a micromechanical discrete element model for asphalt concrete to investigate pavement rutting. Chang and Meegoda[4] proposed a micromechanical model based upon the discrete element program ASBAL, which was developed by modifying the TRUBAL program to simulate hot mix asphalt (HMA). The modified program allowed aggregate-binder-aggregate contact and aggregate-aggregate contact. An innovative feature in the model is the consideration of both aggregate-binder-aggregate contact and aggregate-aggregate contact. Zhong and Chang[5] adopted a micromechanics approach to consider a contact law for the interparticle behavior of two particles connected by a binder. The model is based upon the premises that the interparticle binder initially contains microcracks, and the discrete element method is used to simulate the cracking behavior under uniaxial and biaxial conditions. Besides the researches mentioned above, some DEM studies were conducted on granular materials or asphalt mixtures such as the works by Trent and Margolin[6], Sadd et al. [2], and Buttlar et al. [7-10]. However, the gradation and irregular shapes of the aggregate in asphalt mixtures have not been considered in previous studies.

1 Numerical Beam Sample Generation

1.1 2D quantity gradation of aggregate

Assuming that the shape of an aggregate is spherical, the probability that a circular plane with a diameter of d_j cut from one sphere with a diameter of d_i can be written as

$$P(D_c = d_j \mid D_s = d_i) \tag{1}$$

According to the statistical distribution theory, for large numbers of spherical aggregates with diameters from d_1 to d_i in asphalt mixtures, the probability that the circular plane with a diameter of d_j in the section of asphalt mixtures can be represented by a set of linear equations,

$$P(D_c = d_j) = \sum P(D_s = d_i) P(D_c = d_j \mid D_s = d_i) \tag{2}$$

where $P(D_c = d_j)$ is the probability that the circular plane

Received 2009-06-18.

Biographies: Chen Jun (1981—), male, graduate; Huang Xiaoming (corresponding author), male, doctor, professor, huangxm@seu.edu.cn.

Foundation items: The National High Technology Research and Development Program of China (863 Program) (No. 2006AA11Z110), the Research and Innovation Foundation for Graduate Students in Jiangsu Province (No. CX07B_156Z), the Excellent Doctoral Dissertation Foundation of Southeast University.

Citation: Chen Jun, Huang Xiaoming. Virtual fracture test of asphalt mixture based on discrete element method[J]. Journal of Southeast University (English Edition), 2009, 25(4): 518 – 522.

with a diameter of d_j occurs in the section of asphalt mixtures and $P(D_s = d_i)$ is the percentage of aggregate with d_i of all aggregates. If the difference in density between the coarse aggregate and the fine aggregate is ignored, $P(D_s = d_i)$ can be considered as the weight proportion of the aggregate with a diameter of d_j to the total aggregates in asphalt mixtures. So the probability $P(D_c = d_j \mid D_s = d_i)$ is crucial to obtain $P(D_c = d_j)$ in the section of asphalt mixtures.

DeHoff and Rhines[11] derived the relationship of statistical distributions between $N(D_c = d_j)$ and $N(D_s = d_i)$,

$$N(D_c = d_j) = 2\Delta \sum_{i=j}^{n} k_{ji} N(D_s = d_i) \tag{3}$$

where $N(D_s = d_i)$ is the statistical distribution of an aggregate group whose diameter ranges from d_i to d_{i+1} in all the aggregates; $N(D_c = d_j)$ is the statistical distribution of the circular plane whose diameter ranges from d_j to d_{j+1} in the section of asphalt mixtures; n is the number of aggregate groups; m is the number of circular plane groups; Δ represents the diameter ranges of aggregate groups, and k_{ji} is the distribution of the aggregate group whose diameter ranges from d_i to d_{i+1} to the circular plane whose diameter ranges from d_j to d_{j+1} in the section.

$$k_{ji} = 0 \qquad j \neq i,\ j > i \tag{4}$$

$$k_{ji} = \left[\left(i - \frac{1}{2} \right)^2 - (j-1)^2 \right]^{1/2} = \left(i - \frac{3}{4} \right)^{1/2} \qquad i = j \tag{5}$$

$$k_{ji} = \left[\left(i - \frac{1}{2} \right)^2 - (j-1) \right]^{1/2} - \left[\left(i - \frac{1}{2} \right)^2 - j \right]^{1/2} \qquad j \neq i,\ j < i \tag{6}$$

According to Eqs. (3) to (6), the number of the circular plane whose diameter ranges from d_j to d_{j+1} in a certain section of asphalt mixtures can be obtained by the following expression,

$$M(D_c = d_j) = 2\Delta \sum_{i=j}^{n} k_{ji} M(D_s = d_i) \tag{7}$$

where $M(D_c = d_j)$ is the number of the circular plane with the diameter ranging from d_j to d_{j+1} and $M(D_s = d_i)$ is the number of aggregates with the diameter ranging from d_i to d_{i+1} in a given volume of asphalt mixtures. $M(D_s = d_i)$ can be obtained by the following equation,

$$M(D_s = d_i) = \frac{48}{\pi(d_i + d_{i+1})^3} V(D_s = d_i) \tag{8}$$

where $M(D_s = d_i)$ is the volume of aggregates whose diameters range from d_i to d_{i+1}. $V(D_s = d_i)$ can be easily obtained from the gradation of asphalt mixtures when the density of the coarse aggregates is similar to that of the fine aggregates.

Accordingly, the 2D quantity gradation of the virtual structure of two-dimensional digital specimens can be obtained by Eqs. (4) to (8). The gradation of the coarse aggregates in a mixture called AC16 in China and the 2D quantity gradation of AC16 are listed in Tab. 1.

Tab. 1 Gradation of asphalt mixture and 2D quantity gradation of the section

Sieve size/mm	Percent passing/%	
	Asphalt concrete	2D numerical asphalt concrete
19.0	100	100
16.0	95	98
13.2	84	95
9.5	70	90
4.75	48	70
2.36	34	0

1.2 Generation of numerical beam sample

As mentioned in the previous section, an inverse-stereology based approach has been successfully developed to convert the volumetric aggregate gradation into the 2D quantity gradation. In this section, the particle generation method is combined with the particle arrangement algorithm to form an automated virtual sample fabrication procedure.

The 2D projective shape of the aggregate in asphalt mixtures is always polygonal with an arbitrary shape. By the digital image processing of asphalt mixtures, the 2D shape of aggregates can be described as polygonal and the number of polygonal edges varies from 4 to 10. In this section, the polygon with n sides (shown in Fig. 1) in polar coordinates is generated by

$$r_i = R_0 + (2\lambda - 1)R \tag{9}$$

$$\theta_i = \frac{2\pi}{n} + (2\eta - 1)\frac{2\pi}{n}\delta \tag{10}$$

where λ and η are random numbers $(0 < \lambda < 1,\ 0 < \eta < 1)$; R_0 is the average sieve size of two serial sieves; R is the range of an aggregate radius between two serial sieves; δ is a parameter to reflect the extent of angle fluctuation in polar coordinates. The angle fluctuation $\theta_i \in (2\pi(1-\delta)/n,\ 2\pi(1+\delta)/n)$ is combined with the radius fluctuation $r_i \in (R_0 - R,\ R_0 + R)$ to reflect the arbitrary shape of the aggregate.

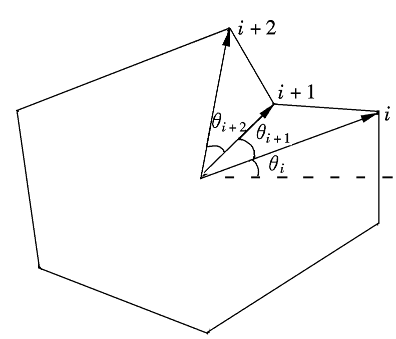

Fig. 1 Particle generation procedure

The resulting aggregate particle is then randomly placed into the area of the specimen. A check for possible overlaps of the particles is made, and the generated particle is accepted only if there are no overlaps with other previously generated particles inside the specimen. Starting with the largest aggregate, the above procedure recursively proceeds until the 2D quantity percentage of aggregates is reached[12]. The procedure and algorithm presented above are implemented into a program written in Microvisual C, namely AMVF(as-

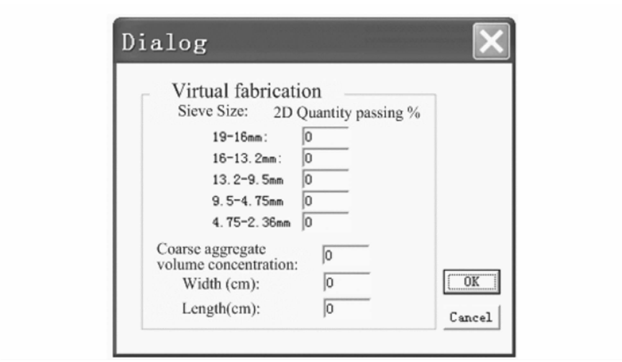

Fig. 2 Interface of AMVF

phalt mixture virtual fabrication). Fig. 2 shows the interface of this program. By inputting the area percentage of coarse aggregates whose size are larger than 2. 36 mm in the specimen and the 2D quantity gradation obtained from inverse stereology and virtual structure size, AMVF can generate the 2D virtual structure.

A study of quantitative stereology shows that the apparent area fraction determined on cut surfaces can be statistically represented by the actual volume fraction. Thus, in the current study, the percentages of the aggregate and the binder in 2D virtual microstructure are assumed to be the same as those in the volumetric mixture design. Based on the above analysis, the area percentage of coarse aggregates in the specimen can be 45% in this paper. Fig. 3 shows the 2D virtual structure with a nominal sieve size of 16 mm specimen (30 mm in length and 5 mm in width). Thus, the virtual structure is composed of coarse aggregates and sand mastic which is a mixture of fine aggregates in asphalt mixtures passing the 2. 36 mm sieve.

Fig. 3 2D virtual structure with nominal sieve size of 16 mm

2 Virtual Fracture Test of Asphalt Mixture

The virtual fracture test of asphalt mixtures using the DEM is based on defining three aspects, namely discrete element modeling, the material properties and the contact model between discrete elements. Each aspect is covered separately in the following subsections.

2.1 Discrete element modeling with particle-flow code of virtual fracture test

Particle-flow code in two dimensions (PFC 2D) is a commercially available distinct element code and it is used in this paper. The bending beam of center-point loading is adopted. Fig. 4 shows the discrete model of the virtual beam with nominal sieve size of 16 mm, in which the coarse aggregate portion is made up of 1 725 particles, and the mastic consists of 2 025 particles.

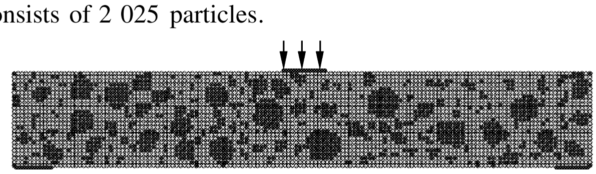

Fig. 4 Discrete model of virtual beam

2.2 Contact model of discrete element

In virtual fracture tests, the parallel bonds are combined with the linear contact stiffness model to simulate the contact behavior of discrete particles in each coarse aggregate. Considering the viscoelastics characteristics of sand mastic, the contact behavior of discrete particles in the sand mastic can be simulated by the Burgers model (see Fig. 5) and the parallel bonds. For convenience of virtual tests, the contacts between the sand mastic and the adjacent particles in coarse aggregates are assumed to be the same as the contact behavior of discrete particles in the sand mastic.

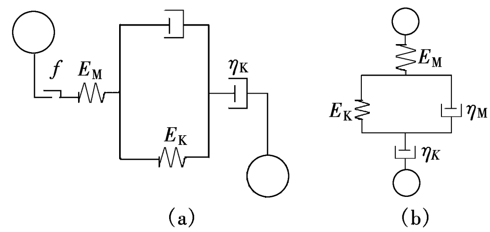

Fig. 5 Burgers model in PFC 2D. (a) Normal direction; (b) Shear direction

2.3 Material properties

A typical value of 55. 5 GPa is assumed for the aggregates modulus[10]. Properties of the sand mastic must be tested for mechanical parameters. The macroscopical mechanical parameters in the Burgers model and the strength with normal direction and shear direction can be obtained respectively by the direct tensile test and the shear test of the sand mastic in laboratory. Then, the micro-parameters are obtained by the relationship between the micro-parameter and macroscopical performance of the sand mastic. Tab. 2 and Tab. 3 list some parameters with normal and shear directions in the Burgers model and parallel bonds of the sand mastic in asphalt mixtures with nominal sieve size of 16 mm.

Tab. 2 Parameters in the Burgers model

Direction	Maxwell		Kelvin	
	Stiffness/ ($MN\cdot m^{-1}$)	Viscosity/ ($MN\cdot s\cdot m^{-1}$)	Stiffness/ ($MN\cdot m^{-1}$)	Viscosity/ ($MN\cdot s\cdot m^{-1}$)
Normal	101. 8	4 103. 3	304. 7	5 400. 0
Shear	34. 2	536. 7	52. 7	410. 2

Tab. 3 Parameters of sand mastic at 15 ℃

Direction	Strength/Pa	Radius/mm
Normal	1. 0	0. 5
Shear	1. 0	0. 5

2.4 Virtual fracture test

Based on the model set up above, the virtual fracture test of AC16 has been conducted under the vertical loading velocity of 1 mm/s. Fig. 6(a) shows the beam of asphalt mixtures at the initial stage from the PFC 2D plotting interface, illustrating the tensile force, compression force magnitudes and directions with a vector-style plot. From Fig. 6(a), the compression stress appears on the top of the beam and the tensile stress appears on the underside of the beam.

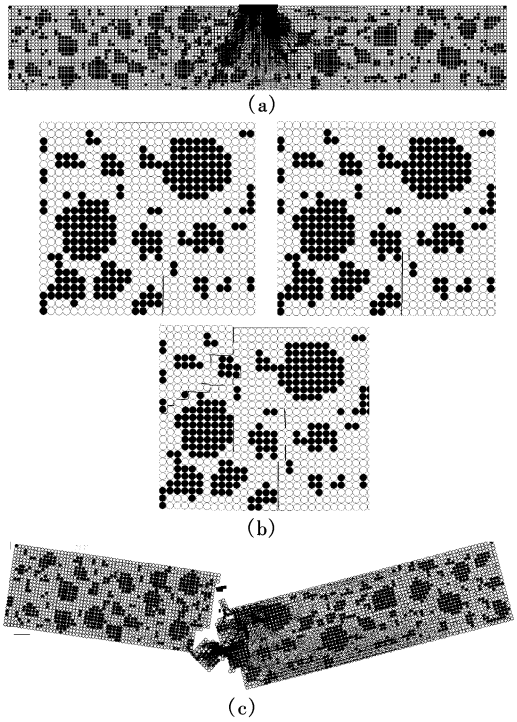

Fig. 6 Fracture patterns of the model presented in Fig. 4. (a) Tensile and compressive force chains after loading; (b) Cracks of mid-span under bending deflection of 0. 2, 0. 3 and 0. 8 mm; (c) Tensile and compressive force chains after failure

Cracks initiation propagation are presented in Fig. 6(b). From Fig. 6(b), a great deal of cracks appear in the middle area of the beam and the spread path of marcocracks is visible for analysis. It can be noticed that cracking is concentrated within the middle portion of the specimen. It can also be observed that the cracks have the tendency to occur at the interface between the aggregate and the binder due to the high stress concentration. Fig. 6(c) shows the failure of the specimen after virtual bending tests.

The relationship between the vertical force and the bending deflection of the virtual beam observed by PFC 2D is displayed in Fig. 7. The fracture intensity of 1. 6 MPa and the fracture strain of 8.94×10^{-6} are obtained from the curve in Fig. 7.

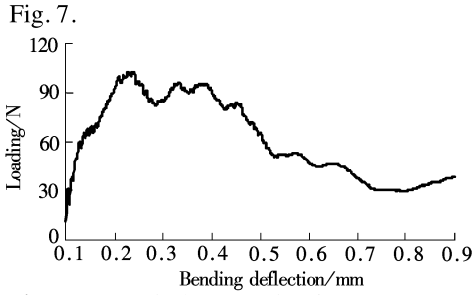

Fig. 7 Numerical progressive fracture process

3 Laboratory Testing on Similar Beam Samples

Asphalt mixtures called AC13, AC16 and SMA13 in China are selected and tested to validate virtual fracture tests. Limestone aggregate and Limestone filler are used in the all specimens. The aggregate gradation is presented in Tab. 4 and meets the requirements for asphalt mixture gradation in China. 70# asphalt and the asphalt content of 5. 1%, 4. 7% and 6. 4% by weight of mixture are used respectively in the mixture design of AC13, AC16 and SMA13. Beam tests of center-point loading are conducted under the load control mode at 15 ℃.

Tab. 4 Aggregate gradation

Sieve size/mm	Percent passing/%		
	AC13	AC16	SMA13
19. 00	100	100	100
16. 00	100	95	100
13. 20	90	84	95
9. 50	60	70	62. 5
4. 75	30	48	27
2. 36	20	34	20. 5
1. 18	15	25	19
0. 60	10	17	16
0. 30	7	12	13
0. 15	5	9	12
0. 075	4	6	10

Tab. 5 lists the fracture stress obtained by the test in laboratory of three asphalt mixtures at 15 ℃. For comparison, virtual tests of AC13 and SMA13 are conducted under the same test conditions. The fracture intensity of AC13 and SMA13 and that of AC16 obtained above by virtual tests are supplemented in Tab. 5. As shown in Tab. 5, there is no obvious difference in intensity between prediction and laboratory test data.

Tab. 5 Fracture intensity of asphalt mixtures

Type size	Fracture intensity/MPa	
	Laboratory test	Virtual test
AC13	1. 9	1. 7
AC16	1. 6	1. 7
SMA13	2. 1	1. 9

4 Conclusion

In this paper, the microstructure-based DEM model is developed and used to analyze the fracture performance of asphalt mixtures. The theory of probability is used to convert the volumetric aggregate gradation into a 2D quantity gradation. The 2D microstructure of asphalt mixtures is obtained by the virtual structure generation procedure, in which the 2D quantity gradation and the irregular shape of the aggregates are considered. The method of the virtual fracture test is established, which comprises discrete element modeling of virtual tests, the material properties and the contact model between discrete elements. Center-point beam bending tests are accomplished in laboratory to evaluate the DEM model and validate the method of virtual tests. By the comparison of fracture intensity between laboratory tests and virtual tests, the simulation results on 2D models have satisfactory prediction. It is concluded that the micro-structural numerical tests can be used as a supplemental tool to evaluate fracture properties of asphalt mixtures. The significance of the research is that with the laboratory validated micro-structural model, field asphalt pavement fracture performance can also be predicted precisely. In the near future, the DEM technique will be extended to consider viscoplasticity and three-dimensional microstructure reconstructions.

References

[1] Birgisson B, Sangpetngam B, Roque R. Predicting viscoelastic response and crack growth in asphalt mixtures with the boundary element method [J]. *Transportation Research Board*, 2002(1789): 129 − 135.

[2] Sadd M H, Dai Q, Parameswaran V, et al. Simulation of asphalt materials using a finite element micromechanical model with damage mechanics[J]. *Transportation Research Board*, 2003(1832): 86 − 95.

[3] Rothenburg L, Bogobowicz A, Hass R, et al. Micromechanical modeling of asphalt concrete in connection with pavement rutting problems[C]//*Proceedings of the 7th International Conference on Asphalt Pavements*. Ann Arbor, Michigan, USA, 1992: 230 − 245.

[4] Chang K G, Meegoda J N. Micromechanical simulation of hot mix asphalt[J]. *J Eng Mech*, 1997, **123**(5): 495 − 503.

[5] Zhong X, Chang C S. Micromechanical modeling for behavior of cementitious granular materials [J]. *J Eng Mech*, 1999, **125**(11): 1280 − 1285.

[6] Trent B C, Margolin L G. Modeling fracture in cemented granular materials[C]//*Proceedings of the ASCE National Convention*. Atlanta, GA, USA, 1994: 54 − 69.

[7] Buttlar W G, You Z P. Discrete-element modeling of asphalt concrete: a micro-fabric approach[J]. *Transportation Research Board*, 2001(1757): 111 − 118.

[8] Ullidtz P. A study of failure in cohesive particulate media using the discrete element method[C]//*Proceedings of the 80th Transportation Research Board Meeting*. Washington, DC, USA, 2001: 7 − 11.

[9] You Z P, Buttlar W G. Discrete element modeling to predict the modulus of asphalt concrete mixtures[J]. *Journal of Materials in Civil Engineering*, 2004, **16**(2): 140 − 144.

[10] You Z P. Development of a micromechanical modeling approach to predict asphalt mixture stiffness using the discrete element method[D]. Illinois, USA: Department of Civil and Environmental Engineering of the University of Illinois at Urbana-Champaign, 2003.

[11] DeHoff R T, Rhines F N. *Quantitative microscopy*[M]. New York: McGraw Hill, 1968.

[12] Zhang P. Microstructure generation of asphalt concrete and lattice modeling of its cracking behavior under low temperature[D]. North Carolina, USA: Department of Civil, Construction, and Environmental Engineering of North Carolina State University, 2003.

基于离散元方法的沥青混合料虚拟断裂试验

陈　俊　　黄晓明

(东南大学交通学院, 南京 210096)

摘要: 为了深入研究沥青混合料开裂特性, 设计了基于沥青混合料二维数字试件的断裂虚拟试验. 在考虑混合料二维截面的集料数量级配和集料不规则形状的基础上, 通过生成程序建立了沥青混合料的二维虚拟试件; 运用离散元方法, 建立了混合料二维离散元模型, 并进行了虚拟的中点弯曲小梁断裂试验; 为了验证虚拟试验方法的正确性, 在实验室内进行了沥青混合料小梁的中点弯曲断裂试验. 室内实体试验与虚拟试验结果的比较发现, 虚拟断裂试验获得的混合料断裂强度与室内试验结果一致, 表明虚拟断裂试验可以作为混合料断裂性能分析的辅助手段.

关键词: 沥青混合料; 断裂; 离散元方法; 虚拟试验

中图分类号: U414