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Viscoelastic model for asphalt mixture under repeated haversine load

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Abstract: The series-wound dashpot of the Burgers model is modified by introducing the strain hardening parameter, and the new model is considered as a combination of the modified dashpot and the Van Der Poel model. The cyclical pulse load consisting of a haversine load time and a rest period is adopted to simulate the actual vehicle load, and the permanent strain model under the repeated load is derived from the rheological and viscoelastic theories. Subsequently, the model is validated by the results of uniaxial repeated load permanent deformation tests of three asphalt mixtures. It is indicated that the proportion of residual viscoelastic strain to permanent strain decreases gradually with the load cycles, and only accounts for 2% to 3% during most of the loading period. If the rest period is long, the residual viscoelastic strain is little. The rest period of the actual vehicle load may be long enough, so the residual viscoelasticity can be ignored and the simplified model can be obtained. The proposed model can well describe the permanent deformation of asphalt mixtures under repeated load.

Key words: asphalt mixture; permanent deformation; mechanics model; repeated haversine load

Permanent deformation is one of the most critical distress types that affect the serviceability of asphalt pavement. Permanent deformation is an accumulation of small amounts of unrecoverable deformation which occurs in asphalt mixture vehicle loads[1]. Based on the stiffness modulus of asphalt mixtures, the traditional method considers the vertical permanent strain as a function of tire pressures and traffic volumes, and the rut depth can be predicted by the layer strain approach[2]. But there are some disadvantages in the traditional method, and more and more researchers prefer applying the elastic-viscoplastic theory on rut investigation[3]. A complex model, in which elastic strains, plastic strains and viscoplastic strains are respectively evaluated, has been suggested[4]. The total strain can be represented as

$$\varepsilon_{\mathrm{total}} = \varepsilon_{\mathrm{e}} + \varepsilon_{\mathrm{p}} + \varepsilon_{\mathrm{ve}} + \varepsilon_{\mathrm{vp}} \tag{1}$$

where ε_{e} is the elastic strain; ε_{p} is the plastic strain; $\varepsilon_{\mathrm{ve}}$ is the viscoelastic strain; and $\varepsilon_{\mathrm{vp}}$ is the viscoplastic strain.

Usually, the viscosity and the plastic strain are considered together as a non-linear deformation, and the cumulative permanent strain is the accumulation of the viscous and residual viscoelastic strains.

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Now, it is indicated that the repeated load permanent deformation test can simulate the dynamic effects of actual vehicle loads well and can better reflect the permanent deformation properties of asphalt mixtures[5]. Most mechanistic models are only descriptions of the creep behavior of asphalt mixtures under the static load and cannot reflect the dynamic effect of the actual vehicle load[6-7]. So, it is required to modify the existing model and deduct a new one to describe the permanent deformation of asphalt mixtures under repeated loads. If developable, the new model should be able to describe the permanent deformation of asphalt mixtures under repeated loads and can also provide necessary parameters for pavement mechanical analysis and rut prediction.

1 Mechanics Model of Permanent Strain for Asphalt Mixtures under Repeated Loads

1.1 Modification of Burgers model

The Burgers model is selected for modification due to its good applicability and accurateness[8-9]. Introducing the strain hardening parameter, the viscosity of modified dashpot can be considered as a function of strain[3], and the viscous strain rate can be represented as

$$\dot{\varepsilon}_{\mathrm{vp}} = \frac{B\sigma^n}{\eta_{\mathrm{vp}}} = \frac{B\sigma^n}{A\varepsilon_{\mathrm{vp}}^p} \tag{2}$$

where $\varepsilon_{\mathrm{vp}}$ and $\dot{\varepsilon}_{\mathrm{vp}}$ are the viscous strain and the viscous strain rate, respectively; η_{vp} is the viscosity of modified dashpot; σ is the load stress; and A, B, n, p are the material parameters.

The model is considered as a combination of the modified dashpot and the Van Der Poel model in tandem, as shown in Fig. 1. So the viscous strain and the elastic strain (instantaneous elastic strain and viscoelastic strain) can be separated, and the permanent strain can be represented as the summation of the viscous strain and the residual viscoelastic strain,

$$\varepsilon_{\mathrm{PD}}^N = \varepsilon_{\mathrm{Rve}}^N + \varepsilon_{\mathrm{vp}}^N \tag{3}$$

where $\varepsilon_{\mathrm{PD}}^N$ is the permanent strain after N load cycles; $\varepsilon_{\mathrm{Rve}}^N$ is the residual viscoelastic strain after N load cycles; and $\varepsilon_{\mathrm{vp}}^N$ is the viscous strain after N load cycles.

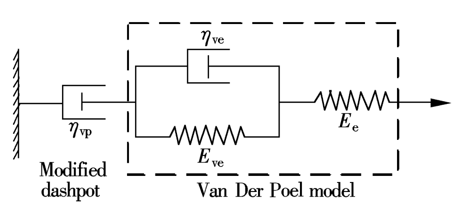

Fig. 1 Modification of Burgers model

1.2 Repeated load form

The cyclical pulse load consisting of a haversine load time and a rest period is adopted. The function of the load pulse is represented as

$$\sigma_t = \begin{cases} \dfrac{\sigma_0}{2}\left(1 - \cos\dfrac{2\pi}{t_0}t\right) & 0 \leqslant t \leqslant t_0 \\ 0 & t_0 < t \leqslant T \end{cases} \tag{4}$$

where σ_0 is the peak value of the haversine load; t_0 is the haversine load time; T is the duration of a load cycle, and $T = t_0 + t_d$, t_d is the rest period.

1.3 Permanent strain under repeated load

1.3.1 Residual viscoelastic strain under repeated load

Residual elastic strain can be derived from the Van Der Poel model. Creep compliance of the Van Der Poel model, $J(t)$, is represented as[1]

$$J(t) = \frac{1}{E_e} + \frac{1}{E_{ve}}\left(1 - \exp\left(-\frac{E_{ve}t}{\eta_{ve}}\right)\right) \tag{5}$$

The Van Der Poel model is a linear viscoelastic model, and it obeys the Boltzmann principle. During the period of the i-th haversine load, the viscoelastic strain $\varepsilon_{ve,\,i}$ is

$$\varepsilon_{ve,\,i} = \int_0^{t_0} J(t_0 - \tau)\frac{d\sigma(\tau)}{d\tau}d\tau = -\frac{\pi f \sigma_0}{E_{ve}}\int_0^{t_0}\exp\left(-\frac{E_{ve}}{\eta_{ve}}(t_0 - \tau)\right)\sin(2\pi f t)\,d\tau$$

Then we obtain

$$\varepsilon_{ve,\,i} = \frac{\sigma_0}{2E_{ve}}\left(\exp\left(\frac{E_{ve}}{\eta_{ve}t_0} - 1\right)\right)\cdot \left(\frac{t_0^2E_{ve}^2}{4\pi^2\eta_{ve}^2} + 1\right)^{-1}\exp\left(-\frac{E_{ve}}{\eta_{ve}}t_0\right) \tag{6}$$

And, at the moment of the end of N load pulse cycles, the residual viscoelastic strain $\varepsilon_{Rve,\,i}$ of the i-th haversine load is

$$\varepsilon_{Rve,\,i} = \int_0^{t_0} J[NT - (i-1)T - \tau]\frac{d\sigma(\tau)}{d\tau}d\tau = -\frac{\pi f\sigma_0}{E_{ve}}\int_0^{t_0}\exp\left(-\frac{E_{ve}}{\eta_{ve}}(NT - (i-1)T - \tau)\right)\sin(2\pi f\tau)\,d\tau$$

Then we obtain

$$\varepsilon_{Rve,\,i} = \frac{\sigma_0}{2E_{ve}}\left(\exp\left(\frac{E_{ve}}{\eta_{ve}}t_0\right) - 1\right)\left(\frac{t_0^{\,2}E_{ve}^{\,2}}{4\pi^2\eta_{ve}^{\,2}} + 1\right)^{-1}\cdot \exp\left(-\frac{E_{ve}}{\eta_{ve}}(N + 1 - i)T\right) \tag{7}$$

So, after N load cycles the residual viscoelastic strain ε_{Rve}^N is

$$\varepsilon_{Rve}^N = \sum_{i=1}^N \varepsilon_{Rve,\,i} = \frac{\sigma_0\left(\exp\left(\frac{E_{ve}}{\eta_{ve}}t_0\right) - 1\right)\exp\left(-\frac{E_{ve}}{\eta_{ve}}T\right)}{2E_{ve}\left(\frac{t_0^2E_{ve}^2}{4\pi^2\eta_{ve}^2} + 1\right)\left(1 - \exp\left(-\frac{E_{ve}}{\eta_{ve}}T\right)\right)}\cdot \left(1 - \exp\left(-\frac{E_{ve}}{\eta_{ve}}NT\right)\right) \tag{8}$$

From Eq. (8), it can be concluded that if the rest period is long, the residual viscoelastic strain is little.

1.3.2 Viscous strain under repeated load

Since the rest period does not affect the nonlinear viscous strain, the nonlinear viscous strain is only correlative to the haversine load. The Boltzmann principle does not fit for the nonlinear viscoelastic model, so it is assumed that during the $d\tau$ period of the i-th haversine load, the viscosity of modified dashpot holds the value at the τ moment. Then,

$$\frac{d\varepsilon_{vp,\,i}}{d\tau} = \frac{B\sigma^n}{A(\varepsilon_{vp}^{i-1} + \varepsilon_{vp,\,i})^p} \tag{9}$$

where $\varepsilon_{vp,\,i}$ is the viscous strain produced during the period of the i-th haversine load; ε_{vp}^{i-1} is the accumulated viscous strain of the $(i-1)$-th haversine load cycle.

Substituting the initial condition $\varepsilon_{vp,\,i}\big|_{t=0} = 0$, we can obtain

$$(\varepsilon_{vp}^{i-1} + \varepsilon_{vp,\,i})^{p+1} = \frac{B(p+1)}{A}\int_0^t\left(\frac{\sigma_0}{2}(1 - \cos 2\pi f\tau)\right)^n d\tau + (\varepsilon_{vp}^{i-1})^{p+1} \tag{10}$$

At the end of the i-th haversine load cycle, the accumulated viscous strain of the i-th haversine load cycle can be represented as

$$(\varepsilon_{vp}^{i})^{p+1} = (\varepsilon_{vp}^{i-1} + \varepsilon_{vp,\,i})^{p+1} = \frac{B(p+1)}{A}\cdot \int_0^{t_0}\left(\frac{\sigma_0}{2}(1 - \cos 2\pi f\tau)\right)^n d\tau + (\varepsilon_{vp}^{i-1})^{p+1} \tag{11}$$

Eq. (11) can be changed into the arithmetical progression,

$$(\varepsilon_{vp}^{i})^{p+1} - (\varepsilon_{vp}^{i-1})^{p+1} = \frac{B(p+1)}{A}\int_0^{t_0}\left(\frac{\sigma_0}{2}(1 - \cos 2\pi f\tau)\right)^n d\tau \tag{12}$$

The viscous strain after N load cycles ε_{vp}^N can be represented as a sum of arithmetical progressions,

$$\varepsilon_{vp}^N = \left(N\frac{B(p+1)}{A}\int_0^{t_0}\left(\frac{\sigma_0}{2}(1 - \cos 2\pi f\tau)\right)^n d\tau\right)^{1/(p+1)} \tag{13}$$

Then, substituting Eqs. (8) and (13) into Eq. (3), the permanent strain after N load cycles can be represented as

$$\varepsilon_{PD}^N = \sigma_0P_2(1 - \exp^{-P_1N}) + P_3\sigma_0^{P_4}N^{P_5} \tag{14}$$

where

$$P_1 = \frac{E_{ve}}{\eta_{ve}},\quad P_2 = \frac{\left(\exp\left(\frac{E_{ve}t_0}{\eta_{ve}}\right) - 1\right)\exp\left(-\frac{E_{ve}T}{\eta_{ve}}\right)}{2E_{ve}\left(\frac{t_0^2E_{ve}^2}{4\pi^2\eta_{ve}^2} + 1\right)\left(1 - \exp\left(-\frac{E_{ve}T}{\eta_{ve}}\right)\right)}$$

$$P_3 = \left(\frac{B(p+1)}{A}\int_0^{t_0}\left(\frac{1}{2}(1 - \cos 2\pi f\tau)\right)^n d\tau\right)^{1/(p+1)}$$

$$P_4 = \frac{n}{p+1},\quad P_5 = \frac{1}{p+1}$$

2 Verification of the Modified Model

2.1 Preparation for permanent deformation test under uniaxial repeated load

Three asphalt mixtures (SMA13, AC20 and AC25) are selected, and basalt aggregate and limestone filler are used in all the specimens with SBS modified asphalt. The mixture design is shown in Tab. 1.

First, we prepare 165 mm high gyratory specimens to the 4% air void content, and then core the nominal 100 mm diameter test specimens from the center of the gyratory specimens. Then we prepare the ends of the specimen by sawing with a double-bladed saw to obtain the cylindrical specimens, 100 mm in diameter and 150 mm in height. The ends of all the test specimens shall be smooth and perpendicular to the axes of the specimens. Then permanent deformation tests under the uniaxial repeated loads are carried out on the cylindrical specimens to validate the proposed model. The tests are performed at 40, 50 and 60 ℃, and the procedure uses a loading cycle of 1.0 s in duration, consisting of applying a 0.1 s haversine load followed by a 0.9 s rest period. The peak values of the haversine load are 0.2, 0.4 and 0.6 MPa, respectively.

Tab. 1 Design of the three asphalt mixtures %

Asphalt mixtures	Passing percent of the below sieves (size in mm)													Asphalt content	Fiber content
	31.5	26.5	19	16	13.2	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075		
SMA13				100	95	65	30	21	19	16	13	12	10	5.8	0.3
AC20		100	95	85	75	64	43	29	18	12	10	7	5	4.7	
AC25	100	96	87	78	68	53	35	21	14	9	7	6	5	4.4	

2.2 Model verification

Actual permanent strains and the fitting curves of the three mixtures under different temperatures and stresses are shown in Fig. 2 to Fig. 4.

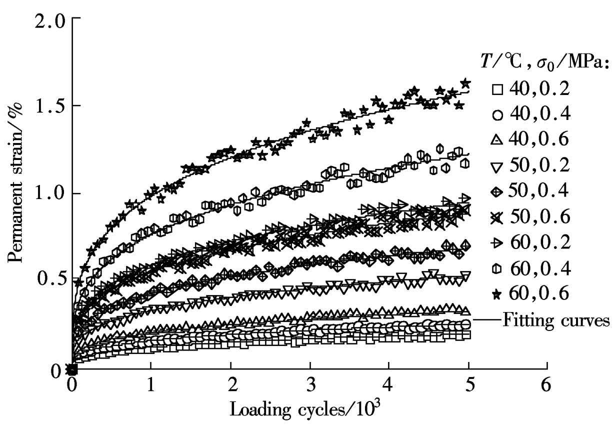

Fig. 2 Actual permanent strains and the fitting curves of SMA13

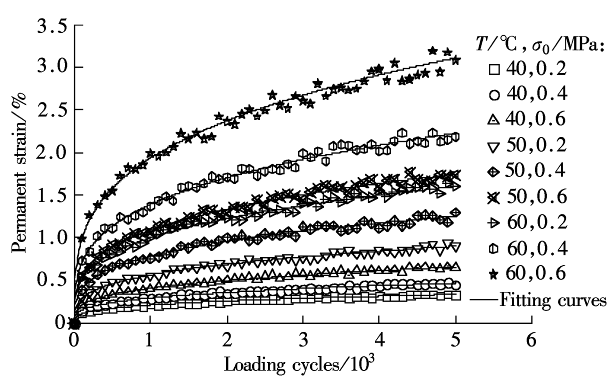

Fig. 3 Actual permanent strains and the fitting curves of AC20

The regression model can be expressed as

$$\varepsilon_{\text{PD-SMA13}} = 0.027e^{0.011T}\sigma_0(1 - e^{-0.170e^{0.004T}N}) + 0.063e^{0.025T}\sigma_0^{1.412}N^{0.284} \qquad R^2 = 92.3\% \tag{15}$$

$$\varepsilon_{\text{PD-AC20}} = 0.034e^{0.014T}\sigma_0(1 - e^{-0.224e^{0.006T}N}) + 0.076e^{0.033T}\sigma_0^{1.416}N^{0.287} \qquad R^2 = 90.5\% \tag{16}$$

$$\varepsilon_{\text{PD-AC25}} = 0.034e^{0.012T}\sigma_0(1 - e^{-0.182e^{0.005T}N}) +$$

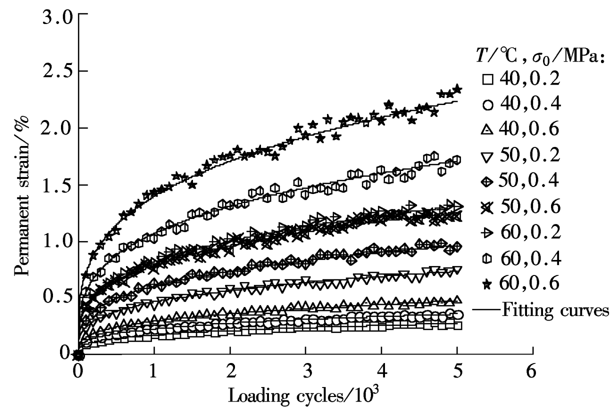

Fig. 4 Actual permanent strains and the fitting curves of AC25

$$0.082e^{0.026T}\sigma_0^{1.415}N^{0.286} \qquad R^2 = 91.5\% \tag{17}$$

where σ_0 is the peak value of the haversine load; T is the test temperature; and N is the number of the load cycles.

After substituting the relevant parameters into Eqs. (15) to (17), the proportion of residual viscoelastic strain to permanent strain can be calculated. Take mixture AC25 for example, as shown in Fig. 5.

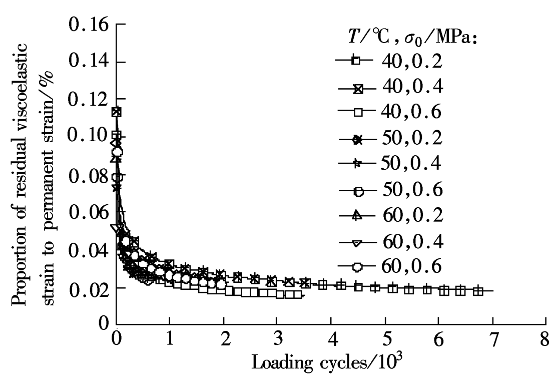

Fig. 5 Proportion of residual viscoelastic strain to permanent strain of AC25

The residual viscoelastic strain increases with the increase of loading cycles and tends to maintain a fixed value, and it is greater when stress and temperature are greater. The proportion of the residual viscoelastic strain to the permanent strain decreases gradually with the load cycles, and only accounts for 2% to 3% during most of the loading period. If the rest period is long, the residual viscoelastic strain is little. In actual pavement, the rest period of the vehicle load may be long enough and the residual viscoelasticity can be ignored, so the proposed model can be simplified as

$$\varepsilon_{\mathrm{PD}} = \alpha e^{\beta T} \sigma_0^{\gamma} N^{\lambda} \tag{18}$$

where α, β, γ and λ are regression parameters.

3 Conclusion

The existing mechanistic models cannot describe the permanent deformation of asphalt mixtures under repeated loads. So the Burgers model is selected for its good applicability and accurateness and the series-wound dashpot is modified by introducing the strain hardening parameter. The new model is considered as the combination of the modified dashpot and the Van Der Poel model, and the permanent strain can be represented as the summation of the viscous strain and the residual viscoelastic strain. Then, the permanent strain model is derived from the rheological and viscoelastic theories. Subsequently, permanent deformation tests under the uniaxial repeated loads for three asphalt mixtures are conducted at 40, 50 and 60 ℃ to validate the proposed model, and the test procedure uses a loading cycle of 1.0 s in duration, consisting of applying a 0.1 s haversine load followed by a 0.9 s rest period. It is indicated that the proportion of the residual viscoelastic strain to the permanent strain decreases gradually with the load cycles, and only accounts for 2% to 3% during most of the loading period. If the rest period is long, the residual viscoelastic strain is little. The rest period of the actual vehicle load may be long enough, so the residual viscoelasticity can be ignored. The proposed model can well describe the permanent deformation of asphalt mixtures under repeated loads.

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周期性半正矢荷载下沥青混合料的粘弹性本构模型

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摘要： 通过引入应变硬化变量，对 Burgers 模型中串联粘壶进行了改进，并将改进模型看成是由 Van Der Poel 模型与改进粘壶串联组成. 采用半正矢波间歇荷载模拟实际轮载作用，综合运用流变学和粘弹性力学理论推导了周期性荷载作用下沥青混合料永久变形的力学模型. 然后根据 3 种沥青混合料的室内重复荷载永久变形试验结果，对该模型进行了验证. 结果表明：随着荷载作用次数的增加，残余粘弹性应变占永久应变的比例先迅速衰减，而后逐渐趋向稳定，一般仅占 2% ～3%. 荷载间歇时间越长，残余粘弹性应变越小. 实际行车荷载的间歇时间足够长，因此残余粘弹性应变可以忽略，从而得到了简化模型. 该模型可以很好地描述重复荷载下沥青混合料的永久变形特性.

关键词： 沥青混合料；永久变形；本构模型；周期性半正矢荷载

中图分类号： U416.2