Journal of Southeast University (English Edition) Vol. 25, No. 4, pp. 549 – 551 Dec. 2009 ISSN 1003—7985

Infinitely many periodic solutions for second-order Hamiltonian systems

Yin Cuicui Zhang Fubao Huang Chengshan

(Department of Mathematics, Southeast University, Nanjing 211189, China)

Abstract: The existence of high energy periodic solutions for the second-order Hamiltonian system $-\ddot{\boldsymbol{u}}(t)+\boldsymbol{A}(t)\boldsymbol{u}(t)=\nabla F(t,\boldsymbol{u}(t))$ with convex and concave nonlinearities is studied, where $F(t,\boldsymbol{u})=F_1(t,\boldsymbol{u})+F_2(t,\boldsymbol{u})$. Under the condition that F is an even functional, infinitely many solutions for it are obtained by the variant fountain theorem. The result is a complement for some known ones in the critical point theory.

Key words: variant fountain theorem; second-order Hamiltonian system; infinitely periodic solutions; even functional

Consider the following second-order Hamiltonian systems:

$$\left.\begin{array}{l}-\ddot{\boldsymbol{u}}(t)+\boldsymbol{A}(t)\boldsymbol{u}(t)=\nabla F(t,\boldsymbol{u}(t))\\ \boldsymbol{u}(0)-\boldsymbol{u}(T)=\dot{\boldsymbol{u}}(0)-\dot{\boldsymbol{u}}(T)=\boldsymbol{0}\end{array}\right\}\qquad(1)$$

where $\boldsymbol{A}(\cdot)$ is a continuous, symmetric, positive definite matrix. We denote by(,) and $\|\cdot\|_p$ the usual L^2 inner product and L^p norm, respectively.

In previous years, the existence and multiplicity of T-periodic solutions for system (1) have been extensively studied by means of the critical point theory. And many results have been obtained based on various hypotheses on $\boldsymbol{A}$ and F, which we refer to below.

When $\boldsymbol{A}=\boldsymbol{0}$ and F is convex, the existence problem is completely solved(see Ref. [1]). In the case that $\boldsymbol{A}=\boldsymbol{0}$ and F is sublinear, sub-quadratic or super-quadratic, some results have been obtained in Refs. [2 – 7]. In the case that $\boldsymbol{A}(\cdot)$ is a continuous symmetric matrix and F satisfies different conditions, many results have been obtained in Refs. [8 – 9].

In this paper, we deal with the existence problem of infinitely many T-periodic solutions of system (1) under the assumption that $F(t,\boldsymbol{x})$ is an even functional of $\boldsymbol{x}$, i. e. $F(t,-\boldsymbol{x})=F(t,\boldsymbol{x})$, $\boldsymbol{x}\in\mathbf{R}^N$.

The proof of the existence of high energy solutions(i. e. the symmetric mountain pass theorem) is based on the (P. S.) condition and the fountain theorem(see Ref. [10]).

We denote C as the various positive constants whose exact values are irrelevant.

1 Main Theorem

Assume that $F=F_1+F_2$ and F satisfies the following conditions:

1) F_1: $[0,T]\times\mathbf{R}^N\to\mathbf{R}$ is measurable in t for every $\boldsymbol{x}\in\mathbf{R}^N$ and continuously differentiable in x for a. e. $t\in[0,T]$; $\lim\limits_{|\boldsymbol{x}|\to 0}\dfrac{\nabla F_1(t,\boldsymbol{x})}{|\boldsymbol{x}|}=0$ uniformly for a. e. $t\in[0,T]$; $\nabla F_1(t,\boldsymbol{x})\boldsymbol{x}\geqslant\boldsymbol{0}$ for all $\boldsymbol{x}\in\mathbf{R}^N$.

2) $|\nabla F_1(t,\boldsymbol{x})|\leqslant c(1+|\boldsymbol{x}|^{p-1})$ for a. e. $t\in[0,T]$ and all $\boldsymbol{x}\in\mathbf{R}^N$, where $p>2$.

3) $\liminf\limits_{|\boldsymbol{x}|\to\infty}\dfrac{\nabla F_1(t,\boldsymbol{x})\boldsymbol{x}}{|\boldsymbol{x}|^{\mu}}\geqslant C>0$ uniformly for $\boldsymbol{x}\in\mathbf{R}^N$, where $\mu>2$ is a constant.

4) $F_2\in C([0,T]\times\mathbf{R}^N,\mathbf{R})$, and there exist $\sigma,\delta\in(1,2)$, $c_1>0$, $c_2>0$, $c_3>0$ such that $c_1|\boldsymbol{x}|^{\sigma}\leqslant\nabla F_2(t,\boldsymbol{x})\boldsymbol{x}\leqslant c_2|\boldsymbol{x}|^{\sigma}+c_3|\boldsymbol{x}|^{\delta}$ for a. e. $t\in[0,T]$ and all $\boldsymbol{x}\in\mathbf{R}^N$.

5) $\dfrac{1}{2}\nabla F(t,\boldsymbol{x})\boldsymbol{x}-F(t,\boldsymbol{x})$ is a nondecreasing function of $\boldsymbol{x}$ for $x_1>0$ and a. e. $t\in[0,T]$, where $\boldsymbol{x}=\{x_1,x_2,\dots,x_N\}$.

6) $F(t,\boldsymbol{x})$ is even of $\boldsymbol{x}$ for $\boldsymbol{x}\in\mathbf{R}^N$ and a. e. $t\in[0,T]$.

Remark 1 In view of condition 3), $F(t,\boldsymbol{x})$ is of super-quadratic growth as $|\boldsymbol{x}|\to\infty$, which is weaker than the global A-R condition. There are many results on the existence of T-periodic solutions for system (1) under super quadratic and other stronger conditions(see Ref. [10]). The global A-R condition ensures the boundedness of (P. S.) sequences of the corresponding functional. But in this paper, we use the variant fountain theorem in Ref. [7], and can be free from this stronger condition when studying the existence of infinitely many periodic solutions.

Theorem 1 Suppose that $\boldsymbol{A}(t)\in C([0,T],\mathbf{R}^{N^2})$ is a symmetric positive definite matrix. And assume that conditions 1) to 6) hold. Then system (1) has infinitely many solutions $\{\boldsymbol{u}_k\}$ satisfying $\dfrac{1}{2}\displaystyle\int_0^T(|\dot{\boldsymbol{u}}_k|^2+\boldsymbol{A}(t)|\boldsymbol{u}_k|^2)\mathrm{d}t-\int_0^T F(t,\boldsymbol{x})\mathrm{d}t\to\infty$ as $k\to\infty$.

Remark 2 The following example satisfies the conditions of theorem 1. $F(t,\boldsymbol{u})=F_1(t,\boldsymbol{u})+F_2(t,\boldsymbol{u})$ with $\nabla F_1(t,\boldsymbol{u})=\mu\boldsymbol{u}\ln(1+|\boldsymbol{u}|)+c|\boldsymbol{u}|^{\mu-2}\boldsymbol{u}$ and $\nabla F_2(t,\boldsymbol{u})=\boldsymbol{u}|\boldsymbol{u}|^{\sigma-2}\ln(2+|\boldsymbol{u}|)$, where $\mu>2$ and $\sigma\in(1,2)$ are constants.

2 Proof of Theorem

Let $E=\left\{\boldsymbol{u}\in H_T^1:\ \displaystyle\int_0^T(|\dot{\boldsymbol{u}}(t)|^2+\boldsymbol{A}(t)|\boldsymbol{u}(t)|^2)\mathrm{d}t<\infty\right\}$. Then E is a Hilbert space with the inner product $\langle\boldsymbol{u},\boldsymbol{v}\rangle=\displaystyle\int_0^T(\dot{\boldsymbol{u}}(t),\dot{\boldsymbol{v}}(t))+(\boldsymbol{A}(t)\boldsymbol{u}(t),\boldsymbol{v}(t))\mathrm{d}t$ and the norm $\|\boldsymbol{u}\|=\langle\boldsymbol{u},\boldsymbol{u}\rangle^{1/2}$. Obviously, $0\notin\sigma\left(-\dfrac{\mathrm{d}^2}{\mathrm{d}t^2}+\boldsymbol{A}(t)\right)$. Hence, $\|\cdot\|$ is equivalent to $\|\cdot\|_{H_T^1}$. Then $\left(-\dfrac{\mathrm{d}^2}{\mathrm{d}t^2}+\boldsymbol{A}(t)\right)$ has a sequence of positive eigenvalues λ_m with $\lambda_m\to\infty$, as $m\to\infty$.

Received 2009-06-19.

Biographies: Yin Cuicui(1985—), female, graduate; Zhang Fubao(corresponding author), male, doctor, professor, 101009933@seu.edu.cn.

Citation: Yin Cuicui, Zhang Fubao, Huang Chengshan. Infinitely many periodic solutions for second-order Hamiltonian systems[J]. Journal of Southeast University (English Edition), 2009, 25(4): 549 – 551.

Let X_j be the sub-space corresponding to λ_j, $j=1,2,\ldots$, and $X_j<\infty$, then $E=\bigoplus_{j=1}^{\infty}X_j$.

Set $W_k=\bigoplus_{j=1}^{k}X_j$, $Z_k=\bigoplus_{j=k}^{\infty}X_j$, $B_k=\{u\in W_k:\ \|u\|\leqslant\rho_k\}$ and $S_k=\{u\in Z_k:\ \|u\|=r_k\}$, for $\rho_k>r_k>0$.

Since $A(t)\in C([0,T],\ \mathbf{R}^{N^2})$ is a symmetric, positive definite matrix, the embedding $E\to L^s[0,T]$ for $1\leqslant s\leqslant\infty$ is compact by virtue of the T-periodic condition.

Consider a family of functions $\Phi_\lambda: E\to\mathbf{R}$ of the form

$$\Phi_\lambda(u)=\frac{1}{2}\|u\|^2-\lambda\int_0^T F(t,u(t))\,\mathrm{d}t:=I(u)-\lambda J(u)$$

where $F(t,u)=F_1(t,u)+F_2(t,u)$, $\lambda\in[1,2]$, $I(u)=\frac{1}{2}\|u\|^2$, $J(u)=\int_0^T(F_1(t,u)+F_2(t,u))\,\mathrm{d}t$. Then $J(u)\geqslant 0$, $I(u)\to\infty$ as $\|u\|\to\infty$. $\Phi_\lambda(-u)=\Phi_\lambda(u)$ for all $\lambda\in[1,2]$ and $u\in E$.

Let $a_k(\lambda)=\max_{u\in W_k,\ \|u\|=\rho_k}\Phi_\lambda(u)$ and $b_k(\lambda)=\inf_{u\in Z_k,\ \|u\|=r_k}\Phi_\lambda(u)$. Define $c_k(\lambda)=\inf_{\gamma\in\Gamma_k}\max_{u\in B_k}\Phi_\lambda(\gamma(u))$, where $\Gamma_k:=\{\gamma\in C(B_k,E):\gamma$ is odd, $\gamma|_{\partial B_k}=\mathrm{Id}\}$, $k\geqslant 2$.

Lemma 1 Assuming that conditions 1), 2) and 4) hold, then $\Phi\in C(B_k,E)$ and J' is compact. We make the following assumptions:

(A_1) Φ_λ maps bounded sets into bounded sets uniformly for $\lambda\in[1,2]$. Moreover, for all $(\lambda,u)\in[1,2]\times E$, $\Phi_\lambda(-u)=\Phi_\lambda(u)$.

(A_2) $J(u)\geqslant 0$ for all $u\in E$, $I(u)\to\infty$ or $J(u)\to\infty$ as $\|u\|\to\infty$, or

(A_3) $J(u)\leqslant 0$ for all $u\in E$, $J(u)\to\infty$ as $\|u\|\to\infty$.

The proof of lemma 1 is based on lemma 1.20 and lemma 1.22 in Ref. [10].

Lemma 2[10] Assume that (A_1) and either (A_2) or (A_3) hold. If $b_k>a_k$ for all $\lambda\in[1,2]$, then $c_k\geqslant b_k$ for all $\lambda\in[1,2]$. Moreover, for almost every $\lambda\in[1,2]$, there exists a sequence $\{u_n^k(\lambda)\}_{n=1}^{\infty}$ such that $\sup_n\|u_n^k(\lambda)\|<\infty$, $\Phi'_\lambda(u_n^k(\lambda))\to 0$, $\Phi_\lambda(u_n^k(\lambda))\to c_k(\lambda)$, as $n\to\infty$.

Lemma 3 Under the assumptions of theorem 1, for each $k\geqslant 2$, there exist

$$r_k>0,\ \tau_n\to 1\qquad\text{as } n\to\infty$$
$$\bar{b}_k\to\infty\qquad\text{as } k\to\infty$$
$$\bar{c}_k>\bar{b}_k>0$$
$$\{z_n\}_{n=1}^{\infty}\subset E$$

such that

$$\Phi'_{\tau_n}(z_n)=0,\quad \Phi'_{\tau_n}(z_n)\in[\bar{b}_k,\ \bar{c}_k]$$

Proof By conditions 1), 2) and 4), for any $\varepsilon>0$, there exists a C_ε such that $\nabla F_1(t,u)u+\nabla F_2(t,u)u\geqslant C_\varepsilon|u|^\mu-\varepsilon|u|^2+c_1|u|^\sigma$ for any $u\in E$. Therefore,

$$\Phi_\lambda(u)=\frac{1}{2}\|u\|^2-\lambda\int_0^T(F_1(t,u)+F_2(t,u))\,\mathrm{d}t=$$
$$\frac{1}{2}\|u\|^2-\lambda\int_0^T\int_0^1(\nabla F_1(t,su)+\nabla F_2(t,su))u\,\mathrm{d}s\mathrm{d}t+$$
$$\lambda\int_0^T(F_1(t,0)+F_2(t,0))\,\mathrm{d}t\leqslant\frac{1}{2}\|u\|^2-$$
$$\lambda C(C_\varepsilon\|u\|_\mu^\mu-\varepsilon\|u\|_2^2+c_1\|u\|_\sigma^\sigma)+C$$

Since $\mu>2$, $\sigma\in(1,2)$, for some $\rho_k>0$ large enough, we have $a_k(\lambda)=\max_{u\in W_k,\ \|u\|=\rho_k}\Phi_\lambda(u)\leqslant 0$ uniformly for $\lambda\in[1,2]$. On the other hand, by conditions 1), 2) and 4) for any $\varepsilon>0$, there exists a C_ε such that $|\nabla F_1(t,u)+\nabla F_2(t,u)|\leqslant C_\varepsilon|u|^{p-1}+\varepsilon|u|+c_2|u|^{\sigma-1}+c_3|u|^{\delta-1}$ for any $u\in E$ and a. e. $t\in[0,T]$. Let $\alpha_k(p)=\sup_{u\in Z_k,\ \|u\|=1}\|u\|_p$, $\alpha_k(\sigma)=\sup_{u\in Z_k,\ \|u\|=1}\|u\|_\sigma$, $\alpha_k(\delta)=\sup_{u\in Z_k,\ \|u\|=1}\|u\|_\delta$. Then $\alpha_k(p)\to 0$, $\alpha_k(\sigma)\to 0$, $\alpha_k(\delta)\to 0$, as $k\to\infty$. We only prove $\alpha_k(p)\to 0$, as $k\to\infty$. Indeed, if not, then there exist an ε_0 and $\{u_j\}\subset E$ with $u_j\perp W_{k_j-1}$, $\|u_j\|=1$, $\|u_j\|_p\geqslant\varepsilon_0$, where $k_j\to\infty$ as $j\to\infty$. For any $v\in E$, we may find a $w_j\in W_{k_j-1}$ such that $w_j\to v$ as $j\to\infty$. Therefore, $|\langle u_j,v\rangle|=|\langle u_j,w_j-v\rangle|\leqslant\|w_j-v\|\to 0$, as $j\to\infty$, i. e. $u_j\to 0$ weakly in E. Hence, $u_j\to 0$ in $L^q[0,T]$, where $q\geqslant 1$, a contradiction. In the same way, we can prove other cases. Therefore, for $u\in Z_k$ and ε small enough, we have the following estimates:

$$\Phi_\lambda(u)=\frac{1}{2}\|u\|^2-\lambda\int_0^T(F_1(t,u)+F_2(t,u))\,\mathrm{d}t\geqslant$$
$$\frac{1}{2}\|u\|^2-\frac{\lambda\varepsilon}{2}\|u\|_2^2-\frac{\lambda C_\varepsilon}{p}\|u\|_p^p-\frac{\lambda C_\varepsilon}{\sigma}\|u\|_\sigma^\sigma-$$
$$\frac{\lambda C_\varepsilon}{\delta}\|u\|_\delta^\delta\geqslant\frac{1}{4}\|u\|^2-C(\|u\|_p^p+\|u\|_\sigma^\sigma+\|u\|_\delta^\delta)\geqslant$$
$$\frac{1}{4}\|u\|^2-C(\alpha_k^p(p)\|u\|^p+\alpha_k^\sigma(\sigma)\|u\|^\sigma+\alpha_k^\delta(\delta)\|u\|^\delta)$$

If we choose $r_k=[16c(p\alpha_k^p(p)+\sigma\alpha_k^\sigma(\sigma)+\delta\alpha_k^\delta(\delta))]^{\frac{1}{2-p}}$, then for $u\in Z_k$ with $\|u\|=r_k$, we obtain

$$\Phi_\lambda(u)\geqslant[16c(p\alpha_k^p(p)+\sigma\alpha_k^\sigma(\sigma)+\delta\alpha_k^\delta(\delta))]^{\frac{2}{2-p}}\cdot$$
$$\left(\frac{1}{4}-\frac{1}{16p}-C(\alpha_k^\sigma(\sigma)r_k^{\sigma-2}+\alpha_k^\delta(\delta)r_k^{\delta-2})\right):=\bar{b}_k$$

It follows that $b_k(\lambda)=\inf_{u\in Z_k,\ \|u\|=r_k}\Phi_\lambda(u)\geqslant\bar{b}_k\to\infty$ as $k\to\infty$ uniformly for λ. Therefore, $b_k(\lambda)\geqslant a_k(\lambda)$. By lemma 2, for a. e. $\lambda\in[1,2]$, there exists a sequence $\{u_n^k(\lambda)\}_{n=1}^{\infty}$ such that $\sup_n\|u_n^k(\lambda)\|<\infty$, $\Phi'_\lambda(u_n^k(\lambda))\to 0$, and $\Phi_\lambda(u_n^k(\lambda))\to c_k(\lambda)\geqslant b_k(\lambda)\geqslant\bar{b}_k$, as $n\to\infty$. Furthermore, since $c_k(\lambda)\leqslant\sup_{u\in B_k}\Phi_\lambda(u):=\bar{c}_k$, by lemma 1, J' is compact, and then $\{u_n^k(\lambda)\}_{n=1}^{\infty}$ has a convergent subsequence. Hence, there exists a $z_k(\lambda)$ such that $\Phi'_\lambda(z_k(\lambda))=0$ and $\Phi_\lambda(z_k(\lambda))\in[\bar{b}_k,\bar{c}_k]$. Evidently, we may find $\tau_n\to 1$ and z_n desired by lemma 3.

Lemma 4 The sequence $\{z_n\}_{n=1}^{\infty}$ obtained in lemma 3 is bounded.

Proof If not, up to a subsequence, $\|z_n\|\to\infty$. We consider $w_n=z_n/\|z_n\|$, and then $\|w_n\|=1$. Up to a subsequence, we obtain $w_n\to w$ weakly in E. Then $w_n\to w$ in $L^q[0,T]$ for $q\geqslant 1$ and $w_n(t)\to w(t)$ a. e. $t\in[0,T]$. We shall show that $\{w_n\}_{n=1}^{\infty}$ is neither vanishing nor nonvanishing, thereby obtaining a contradiction.

Case 1 Nonvanishing of $\{w_n\}_{n=1}^{\infty}$ is impossible. If $w\neq 0$ in E, by lemma 3, we know $\Phi'_{\tau_n}(z_n)=0$, i. e. $\|z_n\|^2-\tau_n\int_0^T\nabla F(t,z_n)z_n\,\mathrm{d}t=0$. By $\tau_n\to 1$, we have

$\int_0^T \frac{\nabla F(t,z_n)z_n}{\|z_n\|^2}\mathrm{d}t \leqslant C$. Furthermore, by Fatou's lemma and conditions 3) to 5), we obtain

$$\int_0^T \frac{\nabla F(t,z_n)z_n}{\|z_n\|^2}\mathrm{d}t = \int_{\{t \mid w_n \neq 0\}} |w_n(t)|^2 \frac{\nabla F_1(t,z_n)z_n + \nabla F_2(t,z_n)z_n}{\|z_n\|^2}\mathrm{d}t \geqslant \int_{\{t \mid w_n \neq 0\}} |w_n(t)|^2 (C|z_n(t)|^{\mu-2} + c_1 |z_n(t)|^{\sigma-2})\mathrm{d}t \to \infty$$

A contradiction.

Case 2 Vanishing of $\{w_n\}_{n=1}^{\infty}$ is impossible. If $w = \mathbf{0}$ in E, we define $\Phi_{\tau_n}(s_n z_n) := \max_{s\in[0,1]} \Phi_{\tau_n}(s z_n)$, where s_n is irrelevant to subscript n. For any $c > 0$ and $\bar{w}_n := (4c)^{1/2} w_n$, by conditions 1) and 4), $J(u) \in C^1(\mathbf{R}^N, \mathbf{R})$. Then for an n large enough, we have

$$\Phi_{\tau_n}(s_n z_n) = \frac{1}{2}\|s_n z_n\|^2 - \tau_n \int_0^T (F_1(t, s_n z_n) + F_2(t, s_n z_n))\mathrm{d}t \geqslant \Phi_{\tau_n}(\bar{w}_n) = \frac{1}{2}\|\bar{w}_n\|^2 - \tau_n \int_0^T (F_1(t,\bar{w}_n) + F_2(t,\bar{w}_n))\mathrm{d}t = 2c - \tau_n \int_0^T (F_1(t,\bar{w}_n) + F_2(t,\bar{w}_n))\mathrm{d}t \geqslant c$$

It follows that $\lim_{n\to\infty} \Phi_{\tau_n}(s_n z_n) = \infty$. Obviously, $s_n \in [0, 1]$. Hence,

$$(\Phi'_{\tau_n}(s_n z_n), s_n z_n) = 0$$

$$\Phi_{\tau_n}(s_n z_n) - \frac{1}{2}(\Phi'_{\tau_n}(s_n z_n), s_n z_n) = \tau_n \int_0^T \left(\frac{1}{2}\nabla F(t, s_n z_n) s_n z_n - F(t, s_n z_n)\right)\mathrm{d}t \to \infty$$

By condition 5), $\frac{1}{2}\nabla F(t,u)u - F(t,u)$ increases in u for $u_1 > \mathbf{0}$. Combining these with the evenness of $F(t, \cdot)$ for a. e. $t \in [0, T]$, and noting that $\tau_n \int_0^T \left(\frac{1}{2}\nabla F(t,z_n)z_n - F(t,z_n)\right)\mathrm{d}t = \Phi_{\tau_n}(z_n) \in [\bar{b}_k, \bar{c}_k]$. We have

$$\Phi(z_n) - \Phi'(z_n)z_n = \int_0^T \left(\frac{1}{2}\nabla F(t,z_n)z_n - F(t,z_n)\right)\mathrm{d}t \geqslant \int_0^T \left(\frac{1}{2}\nabla F(t,s_n z_n)s_n z_n - F(t,s_n z_n)\right)\mathrm{d}t \to \infty$$

This provides a contradiction. Hence $\{z_n\}_{n=1}^{\infty}$ is bounded. The proof of lemma 4 is completed.

Proof of theorem 1 By lemma 3 and lemma 4, we have obtained infinitely many critical points of Φ. A standard argument shows that $u \in E$ which is a critical point of Φ is a solution of second-order system (1). Therefore, the proof of theorem 1 is completed.

References

[1] Mawhin J, Williem M. *Critical point theory and Hamiltonian systems* [M]. New York: Springer Verlag, 1989.

[2] Ding Y H. Existence and multiplicity results for homocilinic solutions to a class of Hamiltonian systems [J]. *Nonlinear Analysis*, 1995, **25**(11): 1095 - 1113.

[3] Bartsch T, Willem M. On an elliptic equation with concave and convex nonlinearities [J]. *Proc Amer Math Soc*, 1995, **123**(11): 3555 - 3561.

[4] Tang C L. Existence and multiplicity of periodic solutions for nonautonomous second-order systems [J]. *Nonlinear Anal TMA*, 1998, **32**(3): 299 - 304.

[5] Wang Z Q. On a superlinear elliptic equation [J]. *Ann Inst H Poincare Anal Nonlineaire*, 1991, **8**: 43 - 57.

[6] Tang C L, Wu X P. Notes on periodic solutions of subquadratic second-order Hamiltonian systems [J]. *J Math Anal Appl*, 2003, **285**(1): 8 - 16.

[7] Zou W M. Variant fountain theorems and their applications [J]. *Manuscripta Math*, 2001, **104**(3): 343 - 358.

[8] Zou W M. Infinitely many solutions for Hamiltonian systems [J]. *J Differential Equations*, 2002, **186**(1): 141 - 164.

[9] Tang C L, Wu X P. Periodic solutions for a class of nonautonomous subquadratic second-order Hamiltonian systems [J]. *J Math Anal Appl*, 2002, **275**(2): 870 - 882.

[10] Zou W M, Schechter M. *Critical point theory and its applications* [M]. New York: Springer Verlag, 2006.

二阶哈密顿系统的无限多周期解

尹翠翠　张福保　黄成山

（东南大学数学系，南京 211189）

摘要：研究了二阶哈密顿系统 $-\ddot{u}(t) + A(t)u(t) = \nabla F(t,u(t))$ 的高能量周期解的存在性问题，其中 $F(t,u) = F_1(t,u) + F_2(t,u)$，而 $F_1(t,u)$ 和 $F_2(t,u)$ 分别满足某种凸性及凹性条件. 利用喷泉定理及其推广获得了上述哈密顿系统在 F 为偶泛函的条件下存在无穷多个解的结果，在一定程度上本质地推广和补充了已有的临界点理论中的某些结论.

关键词：喷泉定理；二阶哈密顿系统；无限多周期解；偶泛函

中图分类号：O175