

Dynamic vaccine distribution model based on epidemic diffusion rule and clustering approach

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Abstract: Due to the fact that the emergency medicine distribution is vital to the quick response to urgent demand when an epidemic occurs, the optimal vaccine distribution approach is explored according to the epidemic diffusion rule and different urgency degrees of affected areas with the background of the epidemic outbreak in a given region. First, the SIQR (susceptible, infected, quarantined, recovered) epidemic model with pulse vaccination is introduced to describe the epidemic diffusion rule and obtain the demanded vaccine in each pulse. Based on the SIQR model, the affected areas are clustered by using the self-organizing map (SOM) neural network to qualify the results. Then, a dynamic vaccine distribution model is formulated, incorporating the results of clustering the affected areas with the goals of both reducing the transportation cost and decreasing the unsatisfied demand for the emergency logistics network. Numerical study with twenty affected areas and four distribution centers is carried out. The corresponding numerical results indicate that the proposed approach can make an outstanding contribution to controlling the affected areas with a relatively high degree of urgency, and the comparison results prove that the performance of the clustering method is superior to that of the non-clustering method on controlling epidemic diffusion.

Key words: epidemic diffusion rule; clustering approach; SIQR model; self-organizing map (SOM) neural network; vaccine distribution model

The breakout of severe acute respiratory syndrome (SARS) in 2003 profoundly shocked us and the advent of influenza A (H1N1) has now sharply alarmed us that epidemics have always been one of the strongest enemies threatening our lives and property. When an epidemic breaks out, the key challenge for us is whether the medicine can be efficiently distributed to the affected areas in order to control the epidemic diffusion.

A majority of pioneering studies have made great contributions in this area. On the one hand, some scholars have put forward fresh views on how to effectively control the diseases^[1-4]. For example, Shulgin et al.^[1] drew the important conclusion that the system converges to a stable point where the number of infected individuals equals zero under a planned pulse vaccination using the SIR epidemic model. It is usually difficult to control the spread by a one-time vaccination, and thus a planned pulse vaccination is an effective

way to control the epidemic. On the other hand, the research on relief distribution has been the core in the scope of emergency logistics^[5-8]. A very recent study by Wang et al.^[9] developed a multi-objective stochastic programming model with time-varying demand and conducts sensitivity analysis of the latent period based on the SEIR model.

Nevertheless, it is not difficult to find that few papers are dedicated to the research on the effective distribution with respect to different degrees of urgency of the affected areas, which is the objective of this paper. Unlike those pioneering studies, this paper is unique in quantitatively incorporating the clustering results into the vaccine distribution model and concentrating on establishing the optimal vaccine distribution approach according to different degrees of urgency of the affected areas. We suppose that an epidemic breaks out in m areas, and there are n distribution centers around these affected areas. Based on the epidemic diffusion rule, we propose a feasible plan to dynamically distribute the vaccine under the condition of inadequate supplies of vaccine before each pulse vaccination.

1 SIQR Epidemic Diffusion Model

Due to the great difficulty in eradicating the epidemic by a one-time vaccination, we use the pulse vaccination method with a low vaccination rate to control the epidemic. Thus, the SIQR epidemic model with pulse vaccination close to the actual situation is employed to obtain the amount of vaccine demanded. The SIQR epidemic model with pulse vaccination is adopted^[10] as

$$\left. \begin{aligned} \frac{dS}{dt} &= d - (\beta I(t) + d)S(t) \\ \frac{dI}{dt} &= \beta S(t)I(t) - (\delta + \gamma + d + \alpha_1)I(t) \\ \frac{dQ}{dt} &= \delta I(t) - (d + \alpha_2 + \theta)Q(t) \\ \frac{dR}{dt} &= \gamma I(t) + \theta Q(t) - dR(t) \end{aligned} \right\} \quad (1)$$

$$t \neq k\tau; \quad k = 0, 1, \dots$$

$$S(t^+) = (1-p)S(t), \quad R(t^+) = R(t) + pS(t) \quad t = k\tau; \quad k = 1, 2, \dots$$

where $S(k\tau^+) = \lim_{\lambda \rightarrow 0^+} S(k\tau + \lambda)$, $R(k\tau^+) = \lim_{\lambda \rightarrow 0^+} R(k\tau + \lambda)$.

In this epidemic diffusion model, $S(t)$, $I(t)$, $Q(t)$ and $R(t)$ are the time-based parameters, denoting the number of susceptible people, the number of infected people, the number of the quarantined people and the number of recovered people, respectively. d is the natural birth rate and mortality rate of the population; β is the propagation coefficient of disease; γ and θ are the removal rate constants from groups I and Q , respectively; δ is the rate constant for individuals leaving the infectious compartment and I for the quarantine compartment Q ; α_1 and α_2 are the extra disease-related death

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rate constants in classes I and Q , respectively. The pulse vaccination is applied every τ , and p denotes the successful proportion of those vaccinations. Therein, parameters d , β , γ , θ , δ , p and τ are positive constants, and α_1 and α_2 are non-negative constants. Dai and Zhao^[10] studied the relationship between the pulse vaccination period and the vaccination rate in the SIQR model, and determined the minimal vaccination rate p , which is given by

$$p = \min \left\{ \frac{dT(e^{dT} - 1)(\beta - \delta - \gamma - d - \alpha_1)}{\beta(e^{dT} - 1) - dT(\beta - \delta - \gamma - d - \alpha_1)}, \frac{(e^{dT} - 1)(\beta - \delta - \gamma - \alpha_1)}{\delta + \gamma + \alpha_1} \right\} \quad (2)$$

2 Dynamic Vaccine Distribution Model

To facilitate the progress of model formulation, we make the following three assumptions: 1) The locations of the distribution centers are previously known; 2) Affected areas are isolated from each other without population mobility when an epidemic strikes; 3) There are enough vehicles to deliver the vaccine without loss.

2.1 Clustering affected areas

The SOM neural network is carried out to cluster the affected areas in this section. The merit of the SOM neural network is that it can automatically classify the affected areas with similar degrees of urgency according to the given evaluation indices instead of artificially establishing the evaluation function. Before each pulse, three attributes are selected as the severity determinants of the clustering affected areas, which are specified:

1) $w_j^1(t)$ represents the percentage of the number of susceptible people relative to the total number of the population in a given affected area j ;

2) $w_j^2(t)$ represents the percentage of the number of infectious people relative to the total number of the population in a given affected area j ;

3) $w_j^3(t)$ represents the percentage of low-immunity people relative to the total number of the population in a given affected area j .

The concrete operation procedures of the SOM algorithm are summarized in the following three steps:

Step 1 Input the primary parameters of the SOM, involving the number of input neurons, the number of output neurons, the number of input data, the learning iteration, and the learning coefficient. In addition, the initial weight matrix M is stochastically generated.

Step 2 Conduct the following three sub-steps for each input vector $\mathbf{v}_i = \{v_{i1}, v_{i2}, \dots, v_{ik}\}$ sequentially, where $i = 1, 2, \dots, s$; s is the total number of input data points and k is the number of neurons in the input layer. Note that k is also the number of features for an input point.

1) Calculate the Euclidean distance l_j between the input vector \mathbf{v}_i and the output neuron \mathbf{u}_j . That is, $l_j = \|\mathbf{v}_i - \mathbf{u}_j\|$ where $\mathbf{u}_j = \{u_{j1}, u_{j2}, \dots, u_{jr}\}$; $j = 1, 2, \dots, r$ and r is the number of the output neurons.

2) Find the output neuron \mathbf{u}_{j^*} with the minimum Euclidean distance between the output neurons and the input vector. Mathematically, it is represented as $l_{\min} = \min_{j=1,2,\dots,r} l_j$.

3) Update the weights of the output neurons \mathbf{u}_j using $\mathbf{u}_j^{\text{new}} = \mathbf{u}_j^{\text{old}} + \sigma(\mathbf{v}_i - \mathbf{u}_j^{\text{old}})$, where σ is the learning rate.

Step 3 Decrease the learning rate σ and repeat step 2 until the stopping criteria of the learning process are reached.

Suppose that all the affected areas are classified into three groups using the SOM neural network. The priority of each affected area is given by

$$\gamma_j^k(t) = \sum_{s=1}^3 \tilde{\omega}_s^k w_s^k(t) \quad (3)$$

where $w_s^k(t)$ represents the mean value of the attribute s associated with the group that has affected area j and $\tilde{\omega}_s^k$ represents the weight associated with a given attribute s , which is specified to determine the significance of the corresponding attribute.

2.2 Cluster-based vaccine distribution model

Based on the results of the epidemic diffusion rule and clustering the affected areas, two objective functions are proposed to establish the optimal plan to distribute the vaccine: one is to minimize the distribution costs while penalty should be given if the transportation time exceeds the time threshold; the other is to minimize the unsatisfied demand in each pulse vaccination. Therein, $F_j^{1k}(t)$ and $F_j^{2k}(t)$ are given by

$$\min F_j^{1k}(t) = \sum_{\forall k} \sum_{\forall j} \sum_{i=1}^n (c_{ij} x_{ij}^k(t) + \beta_j \max\{0, t_{ij} - T^k\}) \quad \forall (k, j); t = k\tau \quad (4)$$

$$\min F_j^{2k}(t) = \sum_{\forall k} \sum_{\forall j} DM_j^k(t) - \sum_{\forall k} \sum_{\forall j} \sum_{i=1}^n x_{ij}^k(t) \quad \forall (k, j); t = k\tau \quad (5)$$

where c_{ij} is the unit transportation cost from distribution center i to affected area j ; t_{ij} is the time needed from distribution center i to affected area j ; T^k is the time threshold (Penalty should be given if exceeding T); β_j is the penalty coefficient (The cost is increased if punished); $DM_j^k(t)$ is the amount of demanded vaccine distributed to the affected area; $x_{ij}^k(t)$ is the decision variable denoting planned amount from distribution center i to affected area j .

Then $F_j^{1k}(t)$ and $F_j^{2k}(t)$ are normalized as

$$\min \overline{F_j^{1k}(t)} = \frac{F_j^{1k}(t) - F_j^{1k}(t)_{\min}}{F_j^{1k}(t)_{\max} - F_j^{1k}(t)_{\min}} \quad \forall (k, j); t = k\tau \quad (6)$$

$$\min \overline{F_j^{2k}(t)} = \frac{F_j^{2k}(t) - F_j^{2k}(t)_{\min}}{F_j^{2k}(t)_{\max} - F_j^{2k}(t)_{\min}} \quad \forall (k, j); t = k\tau \quad (7)$$

where $F_j^{1k}(t)_{\max}$ and $F_j^{1k}(t)_{\min}$ represent the expected maximum and minimum values associated with $F_j^{1k}(t)$, while $F_j^{2k}(t)_{\max}$ and $F_j^{2k}(t)_{\min}$ represent the expected maximum and minimum values associated with $F_j^{2k}(t)$.

Considering different priorities and effects of the two objectives mentioned above, the weight is introduced to in-

tegrate the two objective functions into a composite optimization problem. The composite objective function $F_j^k(t)$ is given by

$$\min F_j^k(t) = \mu_1 \overline{F_j^{2k}(t)} + \mu_2 \overline{F_j^{1k}(t)} \quad \forall (k, j); t = k\tau \quad (\mu_1 + \mu_2 = 1) \quad (8)$$

Extended from Eq. (8), the aggregate function with multiple affected areas is formulated below, which involves the distribution priority associated with each affected area determined in the previous phase.

$$\min F^k(t) = \sum_{j=1}^m (1 - \omega_j^k(t)) F_j^k(t) \quad \forall k; t = k\tau \quad (9)$$

where $\omega_j^k(t)$ is the weight associated with each affected area, and it is defined as

$$\omega_j^k(t) = \frac{\gamma_j^k(t)}{\sum_{j=1}^m \gamma_j^k(t)} \quad (10)$$

In addition, three constraints are given below:

$$D_j^k(t) = p^k S_j(t^-) \quad (11)$$

$$\sum_{j=1}^m x_{ij}^k(t) \leq \text{CAP}_i \leq \text{DM}_j^k(t) \quad (12)$$

$$x_{ij}^k(t) \geq 0 \quad (13)$$

where $D_j^k(t)$ is the planned amount distributed to the affected area j ; and CAP_i is the capacity of the distribution center i .

Eq. (11) represents the total amount of demand vaccine for the affected area j and it is obtained from the SIQR epidemic model. Eq. (12) ensures that the capacity of each distribution center is large enough to supply the vaccine. Eq. (13) indicates a feasible domain of the decision variable $x_{ij}^k(t)$.

If the amount of vaccine distributed to the affected area j cannot be satisfied, $\text{DM}_j^{k+1}(t)$ will be regulated.

Accordingly, $\text{DM}_j^{k+1}(t)$ is given by

$$\left. \begin{aligned} \Delta D_j^k(t) &= D_j^k(t) - \sum_{i=1}^n x_{ij}^k(t) \\ \text{DM}_j^{k+1}(t) &= \text{DM}_j^{k+1}(t) - \Delta D_j^k(t) \end{aligned} \right\} \quad k = 1, 2, \dots; t = k\tau \quad (14)$$

where $\text{DM}_j^1(t) = D_j^1(t)$.

3 Numerical Example

In this section, we take an example to demonstrate the availability of the proposed method when measles strikes. Suppose that $m = 20$, $n = 4$ and $p = 0.15$. The initial data of the twenty affected areas are shown in Tab. 1. Therein, the initial values of susceptible people and infected people are generated stochastically, following a normal distribution. The value of the total population is the sum of that of susceptible people and infectious people. Besides, the young

people at the age of 15 to 25 are considered as the low-immunity people since they are more vulnerable to the measles, and their number is also stochastically obtained following a normal distribution. The estimated cost and time needed from each distribution center to each affected area are listed in Tab. 2. Tab. 3 summarizes the primary parameters in this paper, which is composed of two parts: the parameters of the SIQR epidemic diffusion model and the parameters of the vaccine distribution model. In the vaccine distribution model, there four distribution centers and the corresponding capacities are 380, 350, 420 and 400, respectively.

Tab. 1 Initial data of the affected areas

Affected area	Total population	Susceptible people	Infected people	Low-immunity
1	1 442	1 327	115	326
2	867	834	33	194
3	1 686	1 550	136	442
4	1 796	1 615	181	219
5	1 107	1 041	65	353
6	2 119	1 976	143	322
7	2 138	1 976	163	208
8	1 505	1 485	20	83
9	1 659	1 631	28	294
10	1 698	1 570	129	199
11	1 505	1 425	80	361
12	1 925	1 790	134	351
13	1 406	1 265	141	469
14	2 509	2 373	136	359
15	1 610	1 445	165	236
16	1 679	1 546	133	338
17	2 086	1 927	160	199
18	1 564	1 524	40	298
19	1 561	1 462	99	295
20	1 259	1 167	92	140

Tab. 2 Cost and time

Affected area	Cost				Time			
	I	II	III	IV	I	II	III	IV
1	3	4	7	8	3	2	5	4
2	4	5	6	8	2	1	4	6
3	7	6	6	8	5	4	4	6
4	5	4	6	9	2	2	4	7
5	6	7	5	7	4	5	3	5
6	4	5	4	5	2	1	2	1
7	8	6	5	4	6	4	1	2
8	3	4	7	5	1	1	5	2
9	8	9	5	6	6	7	2	4
10	3	5	3	2	1	3	2	2
11	4	8	4	5	2	6	2	1
12	4	5	6	4	1	3	4	2
13	5	8	6	3	3	6	4	2
14	9	5	6	7	7	3	4	5
15	4	5	7	6	2	2	5	4
16	3	2	5	3	1	2	3	2
17	4	4	4	4	3	3	2	1
18	5	3	6	8	3	5	3	5
19	5	7	5	4	2	5	3	3
20	5	3	8	4	3	2	6	4

Tab. 3 Primary parameters in this study

SIQR model							Dynamic vaccine distribution model						
d	β	δ	γ	α_1	α_2	θ	$\tilde{\omega}_1$	$\tilde{\omega}_2$	$\tilde{\omega}_3$	μ_1	μ_2	β_j	T
10^{-5}	5×10^{-4}	0.3	0.002	10^{-4}	5×10^{-5}	0.3	0.5	0.25	0.25	0.3	0.7	10	3

Based on the related data and parameters, the proposed method is carried out to establish the distribution plan. The time interval between two pulse vaccinations is set to be 5 d and the vaccination rate is incremented 0.15 in each pulse. Conveniently, we study the two previous pulses, and the amount of vaccine demanded can be obtained using Matlab 7.1 (see Tab. 4). The SOM is employed to classify the affected areas into three groups. The input vectors are the attributes of the twenty affected areas. The output vector is the result of the cluster. The epoch is set to be 10^3 and the initial learning rate is 0.5. Tab. 5 shows the results of clustering the affected areas, indicating the urgency degree of each group. Using Lingo as an optimal tool, the multi-objective model can be solved and Tab. 6 shows the amount of vaccine distributed to a given affected area in the two previous pulses.

Tab. 4 Amount of vaccine demanded

Affected area	First pulse	Second pulse
1	109	196
2	112	165
3	99	62
4	81	48
5	40	50
6	77	34
7	70	31
8	58	53
9	177	126
10	98	61
11	210	327
12	98	53
13	98	81
14	65	21
15	91	62
16	99	88
17	76	35
18	166	200
19	118	83
20	252	364

Tab. 5 Results of clustering the affected areas

Pulse	Group	Number of affected area	γ_j^k
First	1	2, 8, 9, 18	0.625 145
	2	5, 6, 11, 12, 14, 19	0.610 967
	3	1, 3, 4, 7, 10, 13, 15, 16, 17, 20	0.602 046
Second	1	2, 4, 5, 13, 20	0.239 272
	2	1, 3, 8, 9, 11, 16, 18	0.144 021
	3	6, 7, 10, 12, 14, 15, 17, 19	0.125 467

Comparing Tab. 4 with Tab. 6, in the first pulse, it is not difficult to determine that the amounts of vaccine demanded associated with areas 1, 11, 18, and 20 are not satisfied, while it can be learned from Tab. 5 that areas 1, 11, and 20 are all listed in group 2 and group 3, the urgency degrees of which are both lower than those of group 1, and area 18 is satisfied in part. In the second pulse, the unsatisfied areas 1, 6, 11, 16 are all included in group 2 and

group 3. Due to the large demand associated with area 20, a fraction of vaccine is carried to area 20. In the two previous pulses, the areas with high priority are basically satisfied, and the unsatisfied areas are mostly included in the low priority group. As a consequence, we can draw the conclusion that the proposed approach meets the objective to effectively control the affected areas with a relatively high degree of urgency when the amount of medicine is deficient.

Tab. 6 Amount of vaccine distributed

Affected area	First pulse	Second pulse
1	0	94
2	112	165
3	99	62
4	81	48
5	40	50
6	77	0
7	70	31
8	58	53
9	177	126
10	98	61
11	0	195
12	98	53
13	98	81
14	65	21
15	91	62
16	99	0
17	76	35
18	93	200
19	118	83
20	0	130

In addition, to evaluate the availability of the clustering method, we compare the numerical results with those obtained without employing the clustering approach. Correspondingly, we set $\omega_j^k(t) = 0.05$; that is to say, the twenty affected areas enjoy the same priority. The results show that areas 9, 11, 18, 20 cannot be satisfied in the first pulse, two of which are part of group 1. In the second pulse, it is definitely unreasonable that the amount of vaccine distributed to area 20 is zero given that there is no vaccine distributed to area 20 in the first pulse. Accordingly, the effect of the clustering method on controlling the epidemic diffusion is superior to that of the non-clustering method, since the areas with relatively high degrees of urgency are a priority for obtaining the vaccine using the clustering method.

4 Conclusion

This paper presents a clustering method to establish a series of dynamic vaccine distribution operations for quickly responding to urgent medicinal needs. Based on the SIQR epidemic model with pulse vaccination, the amount of vaccine demanded is forecasted. By the SOM neural network, the affected areas are clustered and the priority is determined. A cluster-based vaccine distribution model is for-

mulated based on the two aforementioned mechanisms with the goal of reducing the transportation cost and decreasing the unsatisfied demand. The numerical results indicate that the proposed approach helps to control the affected areas with a relatively high degree of urgency and the comparison results demonstrate the good performance of the clustering approach applied to vaccine distribution. Nonetheless, there is still great potential for improving the performance of the distribution plan. One example is considering more rescue materials, such as medicine, antibiotics, and so on. Furthermore, more influential factors can be integrated into the attributes for advancing the accuracy of the priorities associated with a given affected area. Overall, the clustering approach can be employed by the decision-makers to efficiently classify the affected areas based on different degrees of urgency and make corresponding decisions scientifically.

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基于传染病扩散规律和聚类分析的动态疫苗配送模型

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摘要: 针对当传染病发生时医药物资的配送对于应急需求响应至关重要这一特点, 以在一定区域内发生传染病疫情为研究背景, 根据传染病扩散规律和疫区紧急救助程度的不同, 研究了具有多个疫区和多个应急配送中心的疫苗配送最优方案. 引入具有脉冲接种的 SIQR 模型来描述传染病的扩散规律和获得各区域疫苗需求量, 在 SIQR 模型的基础上, 利用 SOM 神经网络将各疫区进行聚类并量化结果. 以实现运输成本和缺货量最小为目标, 在聚类的基础上建立了动态的疫苗配送模型. 以 20 个受感染区域和 4 个分布中心为例, 给出了算例和仿真分析, 结果表明所提出的方法有助于控制具有较高紧急等级区域的疫情, 且与不使用聚类方法进行疫苗配送相比, 利用聚类方法能更好地控制疫情扩散.

关键词: 传染病扩散规律; 聚类分析; SIQR 模型; SOM 神经网络; 疫苗配送模型

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