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T-S-fuzzy-model-based quantized control for nonlinear networked control systems

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Abstract: In order to overcome data-quantization, networked-induced delay, network packet dropouts and wrong sequences in the nonlinear networked control system, a novel nonlinear networked control system model is built by the T-S fuzzy method. Two time-varying quantizers are added in the model. The key analysis steps in the method are to construct an improved interval-delay-dependent Lyapunov functional and to introduce the free-weighting matrix. By making use of the parallel distributed compensation technology and the convexity of the matrix function, the improved criteria of the stabilization and stability are obtained. Simulation experiments show that the parameters of the controllers and quantizers satisfying a certain performance can be obtained by solving a set of LMIs. The application of the nonlinear mass-spring system is provided to show that the proposed method is effective.
Key words: T-S fuzzy model; linear matrix inequalities (LMIs); quantizers

Feedback control systems in which the control loops are closed through a real-time network are called networked control systems (NCSs). Such NCSs have received increasing attention in recent years[1-6], while nonlinear NCSs (NNCSs) are challenging. The fuzzy control is a useful approach to solve the control problems of nonlinear systems. A novel model of NNCSs based on the fuzzy model is established[7], in which the PDC (parallel distributed compensation) technique is extended to the controller design in the field of NNCSs. However, it is worth mentioning that little progress has been reported for NNCSs when considering the effects of quantization, which motivates the present study. The quantization problems have been paid considerable attention in recent years. However, when considering the effects of network conditions, such as network-induced delays, packet dropouts and wrong sequences, only a little attention has been paid. Very recently, Peng and Tian[8] proposed some new results about the guaranteed cost control and H_∞ control of continuous systems over networks with quantization, where the effects of both network conditions and data quantization were taken into considerations. However, only linear NCSs were considered in their works. According to the best of the author's knowledge, quantization control for the NNCSs has not been investigated and still remains challenging. In this paper, the NNCSs with quantizers is studied.

1 System Description and Preliminaries

1.1 System description

Considering the limited capacity of the communication channels and also for reducing the data transmission rate in the network, the sensor and the controlled output signals are quantized individually by two quantizers. The NNCS with quantizers is described in Fig. 1. In this paper, the quantizers are chosen as the following time-varying form:

$$\mu q\left(\frac{z}{\mu}\right)=\begin{cases}\mu M\Delta & \dfrac{z}{\mu}>(M+0.5)\Delta\\ -\mu M\Delta & \dfrac{z}{\mu}<-(M+0.5)\Delta\\ \mu\Delta\left[\dfrac{z}{\mu}\Delta\right] & -(M+0.5)\Delta\leqslant\dfrac{z}{\mu}\leqslant(M+0.5)\Delta\end{cases}\tag{1}$$

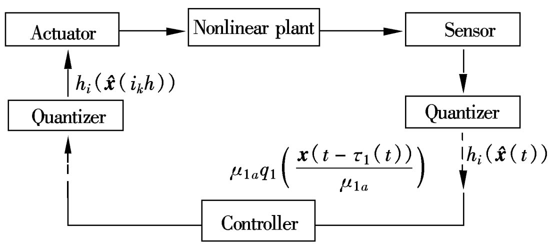

Fig. 1 Structure of the NNCS with two time-varying quantizers

where $M>0$ and $\Delta>0$ are the quantization range and the quantization error, respectively. The variable μ which is positive is varied in a discrete fashion according to the variations of z.

The interval time-varying delay $\tau_m\leqslant\tau(t)\leqslant\tau_M$ is employed in the analysis.

The T-S fuzzy model is described as follows.

Plant rule i: If $\boldsymbol{\theta}_1(t)$ is F_1^i, …, and $\boldsymbol{\theta}_s(t)$ is F_s^i, then $\dot{\boldsymbol{x}}(t)=\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}_i\boldsymbol{u}(t)$

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where $\boldsymbol{x}(t) \in \mathbf{R}^n$, $\boldsymbol{u}(t) \in \mathbf{R}^m$ are the state vector and the input vector, respectively; $\boldsymbol{A}_i$, $\boldsymbol{B}_i$ are the constant matrices with appropriate dimensions; $\boldsymbol{\theta}_1(t)$, $\boldsymbol{\theta}_2(t)$, …, $\boldsymbol{\theta}_s(t)$ are the premise variables. The inferred system can be expressed by[9-10]

$$\left.\begin{aligned} \dot{\boldsymbol{x}}(t) &= \sum_{i=1}^{r} h_i(\boldsymbol{\theta}(t))[\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t)] \qquad i = 1,2,\ldots, r \\ \boldsymbol{x}(t) &= \boldsymbol{\varphi}(t) \qquad t \in [-\tau_M, 0] \end{aligned}\right\} \tag{2}$$

where $h_i(\boldsymbol{\theta}(t)) \geqslant 0$ for $i=1, 2, \ldots, r$ and $\sum_{i=1}^{r} h_i(\boldsymbol{\theta}(t)) = 1$.

Now in order to stabilize the closed-loop system, we design the controller as follows.

Control rule i: If $\boldsymbol{\theta}_1(i_k h)$ is F_1^i, …, and $\boldsymbol{\theta}_s(i_k h)$ is F_s^i, then $\boldsymbol{u}(t^+) = \boldsymbol{K}_j\hat{\boldsymbol{x}}(t-\tau(t))$

where $i_k(k=1, 2, \ldots)$ are some integers such that $\{i_1, i_2, i_3\} \subset \{0, 1, 2, \ldots\}$; $t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}]$.

Hence, the input to the system is

$$\boldsymbol{u}(t^+) = \sum_{j=1}^{r} h_j(\theta(t-\tau(t)))\boldsymbol{K}_j\hat{\boldsymbol{x}}(t-\tau(t)) = \sum_{j=1}^{r} h_j^{\tau}[\boldsymbol{K}_j\boldsymbol{x}(t-\tau(t)) - \mu_{2k}\delta(\mu_{1k}, \mu_{2k})] \qquad t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}] \tag{3}$$

where $h_j^{\tau} = h_j(\theta(t-\tau(t)))$, $\delta(\mu_{1k}, \mu_{2k}) = \mu_{2k}^{-1}\boldsymbol{K}_j\boldsymbol{x}(t-\tau(t)) - q_2(\mu_{2k}^{-1}\boldsymbol{K}_j\mu_{1k}q_1(\mu_{1k}^{-1}\boldsymbol{x}(t-\tau(t))))$.

Substituting Eq. (3) into Eq. (2), the closed-loop fuzzy quantized system is

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j^{\tau}[\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i(\boldsymbol{K}_j\boldsymbol{x}(t-\tau(t)) - \mu_{2k}\delta(\mu_{1k}, \mu_{2k}))] \qquad \boldsymbol{x}(t) = \boldsymbol{\varphi}(t);\ t \in [-\tau_M, 0] \tag{4}$$

1.2 Preliminary lemmas

Lemma 1 For any matrices $\boldsymbol{X}_i$, $\boldsymbol{Y}_i(1 \leqslant i \leqslant r)$ and $\boldsymbol{S} > 0$ with appropriate dimensions, we obtain

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{p=1}^{r}\sum_{l=1}^{r} h_i h_j h_p h_l \boldsymbol{X}_{ij}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{Y}_{pl} \leqslant \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j(\boldsymbol{X}_{ij}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{X}_{ij} + \boldsymbol{Y}_{ij}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{Y}_{ij})$$

Lemma 2 $\boldsymbol{\Xi}_1$, $\boldsymbol{\Xi}_2$ and $\boldsymbol{\Omega}$ are constant matrices of appropriate dimensions, $0 \leqslant \tau_m \leqslant \tau(t) \leqslant \tau_M$, and then $(\tau(t) - \tau_m)\boldsymbol{\Xi}_1 + (\tau_M - \tau(t))\boldsymbol{\Xi}_2 + \boldsymbol{\Omega} < 0$ if and only if the following inequations hold:

$$(\tau_M - \tau_m)\boldsymbol{\Xi}_1 + \boldsymbol{\Omega} < 0, \qquad (\tau_M - \tau_m)\boldsymbol{\Xi}_2 + \boldsymbol{\Omega} < 0$$

2 Stability Analysis and State Feedback Controller Design

First, we suppose that the feedback gain $\boldsymbol{K}_j$ is known, and then we have the following results on asymptotical stability.

Theorem 1 For scalars τ_m, τ_M, constant $d > 0$ and matrices $\boldsymbol{K}_j(j \in S)$, system (4) is asymptotically stable if there exist matrices $\boldsymbol{P} > \boldsymbol{0}$, $\boldsymbol{Q}_i(i=1, 2, 3) > \boldsymbol{0}$, $\boldsymbol{N}_{lij}$, $\boldsymbol{M}_{lij}$, $\boldsymbol{V}_{lij}(i, j \in s;\ l=1, 2, \ldots, 5)$, $\boldsymbol{R}_l > \boldsymbol{0}(l=1, 2)$ and $\boldsymbol{S}_i(i=1, 2, 3)$ of appropriate dimensions such that the following LMIs hold$(n=1, 2)$:

$$\begin{bmatrix} \boldsymbol{\Gamma}_{11}^{ii} + \boldsymbol{Q} & & \\ \boldsymbol{\Gamma}_{21}^{ii} & -\tau_m\boldsymbol{R}_1 & \\ \boldsymbol{\Gamma}_{31}^{ii}(n) & \boldsymbol{0} & -(\tau_M - \tau_m)\boldsymbol{R}_2 \end{bmatrix} < \boldsymbol{0},\ \begin{bmatrix} \boldsymbol{\Gamma}_{11}^{ij} + \boldsymbol{\Gamma}_{11}^{ji} + \boldsymbol{Q} & & \\ \boldsymbol{\Gamma}_{21}^{ij} + \boldsymbol{\Gamma}_{21}^{ji} & -2\tau_m\boldsymbol{R}_1 & \\ \boldsymbol{\Gamma}_{31}^{ij}(n) + \boldsymbol{\Gamma}_{31}^{ji}(n) & \boldsymbol{0} & -2(\tau_M - \tau_m)\boldsymbol{R}_2 \end{bmatrix} < \boldsymbol{0} \tag{5}$$

where

$$\boldsymbol{\Gamma}_{11}^{ij} = \begin{bmatrix} \boldsymbol{\Omega}_1 & & & & & \\ \boldsymbol{N}_{2ij} - \boldsymbol{N}_{1ij}^{\mathrm{T}} + \boldsymbol{M}_{1ij}^{\mathrm{T}} & \boldsymbol{\Omega}_3 & & & & \\ \boldsymbol{\Omega}_2 & -\boldsymbol{N}_{3ij} + \boldsymbol{M}_{3ij} - \boldsymbol{M}_{2ij}^{\mathrm{T}} + \boldsymbol{V}_{2ij}^{\mathrm{T}} & \boldsymbol{\Omega}_4 & & & \\ \boldsymbol{N}_{4ij} - \boldsymbol{V}_{1ij}^{\mathrm{T}} & -\boldsymbol{N}_{4ij} + \boldsymbol{M}_{4ij} - \boldsymbol{V}_{2ij}^{\mathrm{T}} & -\boldsymbol{M}_{4ij} + \boldsymbol{V}_{4ij} - \boldsymbol{V}_{3ij}^{\mathrm{T}} & -\boldsymbol{Q}_3 - \boldsymbol{V}_{4ij} - \boldsymbol{V}_{4ij}^{\mathrm{T}} & & \\ \boldsymbol{P} + \boldsymbol{N}_{5ij} - \boldsymbol{S}_3\boldsymbol{A}_i + \boldsymbol{S}_1^{\mathrm{T}} & -\boldsymbol{N}_{5ij} + \boldsymbol{M}_{5ij} & \boldsymbol{\Omega}_5 & -\boldsymbol{V}_{5ij} & \boldsymbol{\Omega}_6 \end{bmatrix}$$

$$\boldsymbol{\Gamma}_{21}^{ij} = [\tau_m\boldsymbol{N}_{1ij}^{\mathrm{T}} \quad \tau_m\boldsymbol{N}_{2ij}^{\mathrm{T}} \quad \tau_m\boldsymbol{N}_{3ij}^{\mathrm{T}} \quad \tau_m\boldsymbol{N}_{4ij}^{\mathrm{T}} \quad \tau_m\boldsymbol{N}_{5ij}^{\mathrm{T}}]$$

$$\boldsymbol{\Gamma}_{31}^{ij}(1) = [(\tau_M - \tau_m)\boldsymbol{M}_{1ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\boldsymbol{M}_{2ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\boldsymbol{M}_{3ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\boldsymbol{M}_{4ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\boldsymbol{M}_{5ij}^{\mathrm{T}}]$$

$$\boldsymbol{\Gamma}_{31}^{ij}(2) = [(\tau_M - \tau_m)\boldsymbol{V}_{1ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\boldsymbol{V}_{2ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\boldsymbol{V}_{3ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\boldsymbol{V}_{4ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\boldsymbol{V}_{5ij}^{\mathrm{T}}]$$

$$\boldsymbol{\Omega}_1 = \boldsymbol{Q}_1 + \boldsymbol{Q}_2 + \boldsymbol{Q}_3 + \boldsymbol{N}_{1ij} + \boldsymbol{N}_{1ij}^{\mathrm{T}} - \boldsymbol{S}_1\boldsymbol{A}_i - \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{S}_1^{\mathrm{T}}, \quad \boldsymbol{\Omega}_2 = \boldsymbol{N}_{3ij} - \boldsymbol{M}_{1ij}^{\mathrm{T}} + \boldsymbol{V}_{1ij}^{\mathrm{T}} - \boldsymbol{S}_2\boldsymbol{A}_i - \boldsymbol{K}_j^{\mathrm{T}}B_i^{\mathrm{T}}\boldsymbol{S}_1^{\mathrm{T}}$$

$$\boldsymbol{\Omega}_3 = -\boldsymbol{Q}_1 - \boldsymbol{N}_{2ij} - \boldsymbol{N}_{2ij}^{\mathrm{T}} + \boldsymbol{M}_{2ij} + \boldsymbol{M}_{2ij}^{\mathrm{T}}, \quad \boldsymbol{\Omega}_4 = -(1-d)\boldsymbol{Q}_2 - \boldsymbol{M}_{3ij} - \boldsymbol{M}_{3ij}^{\mathrm{T}} + \boldsymbol{V}_{3ij} + \boldsymbol{V}_{3ij}^{\mathrm{T}} - \boldsymbol{S}_2\boldsymbol{B}_i\boldsymbol{K}_j - \boldsymbol{K}_j^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{S}_2^{\mathrm{T}}$$

$$\boldsymbol{\Omega}_5 = -\boldsymbol{M}_{5ij} + \boldsymbol{V}_{5ij} - \boldsymbol{S}_3\boldsymbol{B}_i\boldsymbol{K}_j + \boldsymbol{S}_2^{\mathrm{T}}, \quad \boldsymbol{\Omega}_6 = \tau_m\boldsymbol{R}_1 + (\tau_M - \tau_m)\boldsymbol{R}_2 + \boldsymbol{S}_3 + \boldsymbol{S}_3^{\mathrm{T}}$$

The chosen quantizers' parameters satisfy the following criterion for $i = 1, 2, \ldots, r$; $j = 1, 2, \ldots, r$.

1) $M_1 > \dfrac{2\Delta(1+\delta)\ \|\boldsymbol{SB}_i + \boldsymbol{SB}_j\|\ \|\boldsymbol{Q}^{-1}\|}{1-\delta}$ and $M_2 \geqslant \|\boldsymbol{K}_j\|(\Delta_1 + M_1)$, where $\Delta = \|\boldsymbol{K}_j\|\Delta_1 + \Delta_2$;

2) μ_{1k} is chosen on the plant side such that $2\Delta\ \|\boldsymbol{SB}_i + \boldsymbol{SB}_j\|\ \|\boldsymbol{Q}^{-1}\| \leqslant \mu_{1k}^{-1}\ \|\boldsymbol{x}(t-\tau(t))\| \leqslant M_1$;

3) $\mu_{2k} = \mu_{1k}$, for $k = 1, 2, \ldots, r$.

Proof Choose the Lyapunov-Krasovskii functional candidate as

$$V(\boldsymbol{x}_t) = V_1(\boldsymbol{x}_t) + V_2(\boldsymbol{x}_t) + V_3(\boldsymbol{x}_t)$$

where

$$V_1(\boldsymbol{x}_t) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{x}(t)$$

$$V_2(\boldsymbol{x}_t) = \int_{t-\tau_m}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_1 x(s)\,\mathrm{d}s + \int_{t-\tau(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_2\boldsymbol{x}(s)\,\mathrm{d}s + \int_{t-\tau_M}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_3\boldsymbol{x}(s)\,\mathrm{d}s$$

$$V_3(\boldsymbol{x}_t) = \int_{t-\tau_m}^{t}\int_{s}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(v)\boldsymbol{R}_1\dot{\boldsymbol{x}}(v)\,\mathrm{d}v\mathrm{d}s + \int_{t-\tau_M}^{t-\tau_m}\int_{s}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(v)\boldsymbol{R}_2\dot{\boldsymbol{x}}(v)\,\mathrm{d}v\mathrm{d}s$$

Taking the derivation of $V(\boldsymbol{x}_t)$, employing the free-weighting method, then according to the Schur complements, lemma 2 and theorem 1, we can conclude that

$$\dot{V}(\boldsymbol{t}) \leqslant \sum_{i=1}^{r} h_i h_i^{\tau}[-\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{\xi}(t) + 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{SB}_i\mu_{2k}\delta(\mu_{1k}, \mu_{2k})] + \sum_{i,j=1}^{r}\sum_{i<j} h_i h_j^{\tau}[-\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{\xi}(t) + 2\boldsymbol{\xi}^{\mathrm{T}}(t)(\boldsymbol{SB}_i + \boldsymbol{SB}_j)\mu_{2k}\delta(\mu_{1k}, \mu_{2k})]$$

Based on theorem 1 and the properties of the quantizers q_1 and q_2, we know that $\dot{V}(t) < 0$; thus the proof is completed. In the following, we give a result on the design of the feedback gain $\boldsymbol{K}_j$.

Theorem 2 For scalars τ_m, τ_M, constant $d > 0$, system (4) is asymptotically stable if there exist matrices $\tilde{\boldsymbol{P}} > 0$, $\tilde{\boldsymbol{Q}}_i (i = 1, 2, 3) > 0$, $\tilde{\boldsymbol{R}}_l > 0$, $\tilde{\boldsymbol{N}}_{lij}$, $\tilde{\boldsymbol{M}}_{lij}$, $\tilde{\boldsymbol{V}}_{lij} (l = 1, 2, \ldots, 5;\ i, j \in s)$, $\boldsymbol{X}$ and $\boldsymbol{Y}_j$ of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} \tilde{\boldsymbol{\Gamma}}_{11}^{ii} + \tilde{\boldsymbol{Q}} & & \\ \tilde{\boldsymbol{\Gamma}}_{21}^{ii} & -\tau_m\tilde{\boldsymbol{R}}_1 & \\ \tilde{\boldsymbol{\Gamma}}_{31}^{ii}(l) & \boldsymbol{0} & -(\tau_M - \tau_m)\tilde{\boldsymbol{R}}_2 \end{bmatrix} < \boldsymbol{0}, \quad \begin{bmatrix} \tilde{\boldsymbol{\Gamma}}_{11}^{ij} + \tilde{\boldsymbol{\Gamma}}_{11}^{ii} + \tilde{\boldsymbol{Q}} & & \\ \tilde{\boldsymbol{\Gamma}}_{21}^{ij} + \tilde{\boldsymbol{\Gamma}}_{21}^{ii} & -2\tau_m\tilde{\boldsymbol{R}}_1 & \\ \tilde{\boldsymbol{\Gamma}}_{31}^{ij}(l) + \tilde{\boldsymbol{\Gamma}}_{31}^{ii}(l) & \boldsymbol{0} & -2(\tau_M - \tau_m)\tilde{\boldsymbol{R}}_2 \end{bmatrix} < \boldsymbol{0} \tag{6}$$

where

$$\tilde{\boldsymbol{\Gamma}}_{11}^{ij} = \begin{bmatrix} \tilde{\boldsymbol{\Omega}}_1 & & & & \\ \tilde{\boldsymbol{N}}_{2ij} - \tilde{\boldsymbol{N}}_{1ij}^{\mathrm{T}} + \tilde{\boldsymbol{M}}_{1ij}^{\mathrm{T}} & \tilde{\boldsymbol{\Omega}}_3 & & & \\ \tilde{\boldsymbol{\Omega}}_2 & -\tilde{\boldsymbol{N}}_{3ij} + \tilde{\boldsymbol{M}}_{3ij} - \tilde{\boldsymbol{M}}_{2ij}^{\mathrm{T}} + \tilde{\boldsymbol{V}}_{2ij}^{\mathrm{T}} & \tilde{\boldsymbol{\Omega}}_4 & & \\ \tilde{\boldsymbol{N}}_{4ij} - \tilde{\boldsymbol{V}}_{1ij}^{\mathrm{T}} & -\tilde{\boldsymbol{N}}_{4ij} + \tilde{\boldsymbol{M}}_{4ij} - \tilde{\boldsymbol{V}}_{2ij}^{\mathrm{T}} & -\tilde{\boldsymbol{M}}_{4ij} + \tilde{\boldsymbol{V}}_{4ij} - \tilde{\boldsymbol{V}}_{3ij}^{\mathrm{T}} & -\tilde{\boldsymbol{Q}}_3 - \tilde{\boldsymbol{V}}_{4ij} - \tilde{\boldsymbol{V}}_{4ij}^{\mathrm{T}} & \\ \tilde{\boldsymbol{P}} + \tilde{\boldsymbol{N}}_{5ij} - \rho_3\boldsymbol{A}_i\boldsymbol{X}^{\mathrm{T}} + \rho_1\boldsymbol{X} & -\tilde{\boldsymbol{N}}_{5ij} + \tilde{\boldsymbol{M}}_{5ij} & \tilde{\boldsymbol{\Omega}}_5 & -\tilde{\boldsymbol{V}}_{5ij} & \tilde{\boldsymbol{\Omega}}_6 \end{bmatrix}$$

$$\tilde{\boldsymbol{\Gamma}}_{21}^{ij} = [\tau_m\tilde{\boldsymbol{N}}_{1ij}^{\mathrm{T}} \quad \tau_m\tilde{\boldsymbol{N}}_{2ij}^{\mathrm{T}} \quad \tau_m\boldsymbol{N}_{3ij}^{\mathrm{T}} \quad \tau_m\tilde{\boldsymbol{N}}_{4ij}^{\mathrm{T}} \quad \tau_m\tilde{\boldsymbol{N}}_{5ij}^{\mathrm{T}}]$$

$$\tilde{\boldsymbol{\Gamma}}_{31}^{ij}(1) = [(\tau_M - \tau_m)\tilde{\boldsymbol{M}}_{1ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\tilde{\boldsymbol{M}}_{2ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\tilde{\boldsymbol{M}}_{3ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\tilde{\boldsymbol{M}}_{4ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\tilde{\boldsymbol{M}}_{5ij}^{\mathrm{T}}]$$

$$\tilde{\boldsymbol{\Gamma}}_{31}^{ij}(2) = [(\tau_M - \tau_m)\tilde{\boldsymbol{V}}_{1ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\tilde{\boldsymbol{V}}_{2ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\tilde{\boldsymbol{V}}_{3ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\tilde{\boldsymbol{V}}_{4ij}^{\mathrm{T}} \quad (\tau_M - \tau_m)\tilde{\boldsymbol{V}}_{5ij}^{\mathrm{T}}]$$

$$\tilde{\boldsymbol{\Omega}}_1 = \tilde{\boldsymbol{Q}}_1 + \tilde{\boldsymbol{Q}}_2 + \tilde{\boldsymbol{Q}}_3 + \tilde{\boldsymbol{N}}_{1ij} + \tilde{\boldsymbol{N}}_{1ij}^{\mathrm{T}} - \rho_1\boldsymbol{A}_i\boldsymbol{X}^{\mathrm{T}} - \rho_1\boldsymbol{X}\boldsymbol{A}_i^{\mathrm{T}}, \quad \tilde{\boldsymbol{\Omega}}_2 = \tilde{\boldsymbol{N}}_{3ij} - \tilde{\boldsymbol{M}}_{1ij}^{\mathrm{T}} + \tilde{\boldsymbol{V}}_{1ij}^{\mathrm{T}} - \rho_2\boldsymbol{A}_i\boldsymbol{X}^{\mathrm{T}} - \rho_1\boldsymbol{Y}_j^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}}$$

$$\tilde{\boldsymbol{\Omega}}_3 = -\tilde{\boldsymbol{Q}}_1 - \tilde{\boldsymbol{N}}_{2ij} - \tilde{\boldsymbol{N}}_{2ij}^{\mathrm{T}} + \tilde{\boldsymbol{M}}_{2ij} + \tilde{\boldsymbol{M}}_{2ij}^{\mathrm{T}}, \quad \tilde{\boldsymbol{\Omega}}_4 = -(1-d)\tilde{\boldsymbol{Q}}_2 - \tilde{\boldsymbol{M}}_{3ij} - \tilde{\boldsymbol{M}}_{3ij}^{\mathrm{T}} + \tilde{\boldsymbol{V}}_{3ij} + \tilde{\boldsymbol{V}}_{3ij}^{\mathrm{T}} - \rho_2\boldsymbol{B}_i\boldsymbol{Y}_j - \rho_2\boldsymbol{Y}_j^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}}$$

$$\tilde{\boldsymbol{\Omega}}_5 = -\tilde{\boldsymbol{M}}_{5ij} + \tilde{\boldsymbol{V}}_{5ij} - \rho_3\boldsymbol{B}_i\boldsymbol{Y}_j + \rho_2\boldsymbol{X}, \quad \tilde{\boldsymbol{\Omega}}_6 = \tau_m\tilde{\boldsymbol{R}}_1 + (\tau_M - \tau_m)\tilde{\boldsymbol{R}}_2 + 2\rho_3\boldsymbol{X}$$

The chosen quantizers' parameters satisfy the following for $i = 1, 2, \ldots, r$; $j = 1, 2, \ldots, r$.

1) $M_1 > \dfrac{2\Delta(1+\delta)\left\|[\rho_1\boldsymbol{X}^{-\mathrm{T}} \quad 0 \quad \rho_2\boldsymbol{X}^{-\mathrm{T}} \quad 0 \quad \rho_3\boldsymbol{X}^{-\mathrm{T}}](\boldsymbol{B}_i + \boldsymbol{B}_j)\right\|\ \|\boldsymbol{Q}^{-1}\|}{1-\delta}$, $M_2 \geqslant \max\limits_j \|\boldsymbol{Y}_j\boldsymbol{X}^{-\mathrm{T}}\|(\Delta_1 + M_1)$, where $\Delta = \max\limits_j(\|\boldsymbol{Y}_j\boldsymbol{X}^{-\mathrm{T}}\|\Delta_1 + \Delta_2)$;

2) μ_{1k} is chosen on the plant side such that

$$2\Delta\left\|\left[\rho_1 \boldsymbol{X}^{-\mathrm{T}} \quad \mathbf{0} \quad \rho_2 \boldsymbol{X}^{-\mathrm{T}} \quad \mathbf{0} \quad \rho_3 \boldsymbol{X}^{-\mathrm{T}}\right](\boldsymbol{B}_i+\boldsymbol{B}_j)\right\|\left\|\boldsymbol{Q}^{-1}\right\| \leqslant \mu_{1k}^{-1}\left\|\boldsymbol{x}(t-\tau(t))\right\| \leqslant \boldsymbol{M}_1$$

3) $\mu_{2k}=\mu_{1k}$, for $k=1, 2, \cdots, r$ and the controller feedback gain $\boldsymbol{K}_j=\boldsymbol{Y}_j\boldsymbol{X}^{-\mathrm{T}}$.

3 Numerical Example

Consider the following nonlinear mass-spring system:

$$\dot{\boldsymbol{x}}_1=\boldsymbol{x}_2, \quad \dot{\boldsymbol{x}}_2=-0.01\boldsymbol{x}_1-0.67\boldsymbol{x}_1^3+\boldsymbol{u} \tag{7}$$

where $x_1 \in[-1, 1]$. Choose fuzzy membership functions as $\mu_1(\boldsymbol{x}_1)=1-\boldsymbol{x}_1^2$ and $\mu_2(\boldsymbol{x}_1)=1-\mu_1(\boldsymbol{x}_1)$. The following fuzzy model is used to model the NNCS: 1) If $\boldsymbol{x}_1$ is μ_1, then $\dot{\boldsymbol{x}}=\boldsymbol{A}_1\boldsymbol{x}+\boldsymbol{B}_1\boldsymbol{u}$; 2) If $\boldsymbol{x}_1$ is μ_2, then $\dot{\boldsymbol{x}}=\boldsymbol{A}_2\boldsymbol{x}+\boldsymbol{B}_2\boldsymbol{u}$, where

$$\boldsymbol{A}_1=\begin{bmatrix}0 & 1\\ -0.01 & 0\end{bmatrix}, \quad \boldsymbol{A}_2=\begin{bmatrix}0 & 1\\ -0.68 & 0\end{bmatrix}, \quad \boldsymbol{B}_1=\boldsymbol{B}_2=\begin{bmatrix}0\\ 1\end{bmatrix}$$

For this example, we use theorem 2 to find feedback gains $\boldsymbol{K}_j$ such that the controller with a set of suitable parameters can guarantee that the closed-loop system has a good performance. We set $\rho_1=1$, $\rho_2=0.3$, $\rho_3=20$ and $\widetilde{\boldsymbol{Q}}=\boldsymbol{I}$. By solving the LMIs (6) for $i=1, 2$ and $j=1, 2$ simultaneously, the upper bound of τ_M can be obtained when τ_m is given, which is shown in Tab. 1. The corresponding matrices $\boldsymbol{K}_1$, $\boldsymbol{K}_2$, $\boldsymbol{Y}_1$, $\boldsymbol{Y}_2$ can be solved. For example, when τ_m = 0.5 s, the corresponding $\boldsymbol{Y}_1=[-9.8803 \quad 8.3238]$, $\boldsymbol{Y}_2=[-2.6077 \quad 0.4183]$; and the corresponding feedback gains $\boldsymbol{K}_1=[-0.1683 \quad -0.6705]$, $\boldsymbol{K}_2=[0.4895 \quad -0.7142]$. Choosing $\delta=1/9$, $\|\boldsymbol{K}_1\|=0.6913$ and $\|\boldsymbol{K}_2\|=0.8658$, the quantization range of q_1 is $M_1>7.9345\times10^{-5}\Delta$ and that of q_2 is $M_2>6.8697\times10^{-5}\Delta+0.8658\Delta_1$, where $\Delta=0.8658\Delta_1+\Delta_2$. Using the obtained feedback gain $\boldsymbol{K}_j$ given above, the responses of system (7) are shown in Fig. 2, where the initial condition is $\boldsymbol{x}(0)=[0.3, 0]^{\mathrm{T}}$. From Fig. 2, we can see the effect of signal quantization on system (7).

Tab. 1 Upper bound of τ_M for given τ_m

τ_m	τ_M
0.05	1.2060
0.10	1.2246
0.20	1.2300
0.30	1.2318
0.50	1.2330

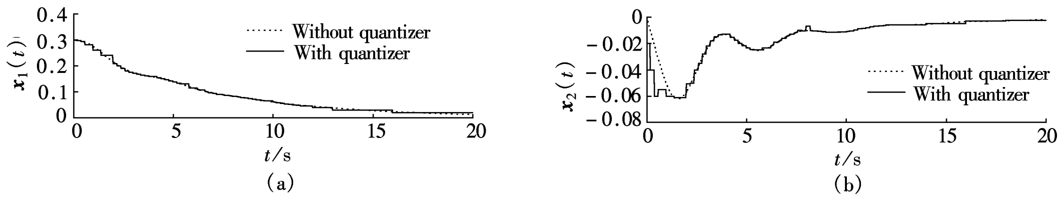

Fig. 2 State responses of system (7) with and without quantizers. (a) State response of x_1; (b) State response of x_2

4 Conclusion

This paper investigates the NNCSs with interval time-varying delay when considering the effects of quantization. Based on the Lyapunov-Krasovskii functional method, a less conservative condition is formulated. A numerical example is given to show the effectiveness of the proposed method. Some interesting problems on this topic to be considered in the future include the design of an output-based quantized feedback and the H_∞ controller design, and the guaranteed-cost controller and the filtering design for the NNCSs.

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基于 T-S 模糊模型的非线性网络控制系统的量化控制

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摘要： 为了克服数据量化、网络诱导时滞、网络丢包及错序对非线性网络控制系统造成的影响，采用 T-S 模糊建模方法建立了一个新的非线性网络控制系统模型,并在系统模型中加入 2 个时变量化器. 主要分析方法在于通过构造一个改进的区间时滞依赖的李雅普诺夫函数，并引入自由权矩阵. 利用并行分布式补偿技术和矩阵函数的凸性，得出了改进系统的稳定和镇定的条件. 仿真实验表明，通过求解一组线性矩阵不等式,可得保证系统渐近稳定并满足一定性能的控制器参数和量化器参数. 在具有非线性的弹簧系统中的应用验证了所提方法的有效性.

关键词： T-S 模糊模型；线性矩阵不等式；量化器
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