

# Outage performance analysis of multiple users in cognitive networks

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**Abstract:** A cooperative model of multiple primary and secondary users coexisting cognitive network is presented. In this model, the control center is aware of all the users' locations in order to allocate the nearest secondary user to the primary user. The control center is aware of the information of the unused spectral resources in terms of the feedback of the sensing results from the secondary users. It allocates idle frequency bands among the secondary users. The primary user accesses the base station (BS) in orthogonal subchannels, and it cooperatively transmits packets with the secondary user and exploits the free band assigned by the control center to amplify-and-forward what it receives immediately. Under this scenario, the outage probability of the cooperative transmission pair of the primary and secondary transmitters is derived. The numerical simulation of the outage probabilities as a function of primary transmission probability  $p_s$ , power allocation ratio  $\xi$  between the primary and secondary users, and the numbers of the primary and secondary users are given respectively. The results show that the optimal system performance is achieved under the conditions of  $\xi = 0.5$  and the numbers of the primary and the secondary users being equal.

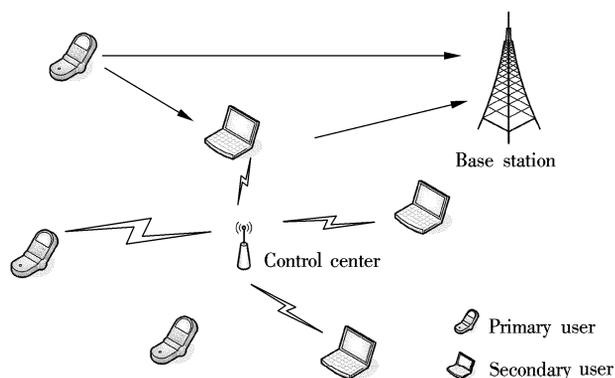
**Key words:** cooperative transmission; outage probability; amplify-and-forward; cognitive network

As user demands for data services and rates steadily increase, the efficient usage of existing spectrum resources is becoming a critical issue<sup>[1]</sup>. The cognitive radio enables a secondary network to dynamically “borrow” and reuse the licensed spectrum allocated to the primary system under the condition that no harmful interference is caused to the primary services<sup>[2-4]</sup>. This motivates the research on the sharing spectrum between the primary and the secondary users which is called a coexisting cognitive network.

A basic cognitive network including two source-destination links and sharing the same spectrum resources (i. e., time slots and frequency bands) was investigated in Refs. [5 – 6] from an information theoretical standpoint. For MAC capacity analyses of cognitive networks, Simeone et al.<sup>[7-8]</sup> derived the maximum throughput of the secondary user in the case that the primary arrival rate is given. And the maximum throughput is investigated under a simple scenario with two links, one is the primary and the other one is the secondary, in which the primary user transmits packets with and without secondary user relay.

Most of the recent cognitive models being studied are the common models which means that primary terminals are oblivious to the presence of secondary users. Secondary users sense the radio environment in search of spectrum holes and exploit the detected transmission opportunities.

In this paper, we focus on the model of the primary user and the secondary user cooperatively transmitting packets to the destination. In Fig. 1, the primary users are aware of the presence of the secondary user. All the secondary users cooperatively sense the idle spectral resources, and then the sensing results are reported to a spectrum sharing pool at the control center. The secondary user chooses one band from the spectrum sharing pool (i. e., the band used by the secondary user is assigned by the control center) to amplify-and-forward what it receives immediately with the arrival of primary packets. As shown in Fig. 1, the system model consists of  $N$  primary users connecting to the base station (BS) in orthogonal subchannels, i. e.,  $f_1, f_2, \dots, f_N$ . The secondary users are considered to be relays for the primary user when they sense idle spectrum resources. Furthermore, the control center is aware of the locations of every user and allocates one secondary user which is the nearest to the primary user to cooperatively transmit with the primary user (if there exists enough secondary users). Assuming that all the channel state information (CSI) (e. g., the channels from the primary user and the secondary users to the BS) is available in the BS, the outage performance of this system model is analyzed.



**Fig. 1** Wireless cooperative cognitive network of the system model

## 1 System Model

### 1.1 A note on the model

In this system model, we only consider the secondary users to be relay nodes for the primary users. The control center has the ability to allocate a secondary user which is the nearest to the primary user for transmitting packets cooperatively with the primary user based on the information of user

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locations, and it is responsible for spectrum allocation among different secondary users. In addition, the BS must be aware of the CSI of all the channels in order to compute the outage probability. In practice, this entails the need for a control channel to be used, e. g., for exchanging the information of locations, allocating a relay for the primary user and the communication of CSI parameters from all the users to the BS.

## 1.2 Physical layer

The channels between different users are modeled as zero-mean independent circularly-symmetric complex Gaussian random variables. We define  $a_{i,j}$  as the channel gain between nodes  $i$  and  $j$  with variance  $\sigma_{i,j}^2$ , so that the magnitude  $|a_{i,j}|$  is Rayleigh distributed ( $\beta_{i,j} = |a_{i,j}|^2$  is exponentially distributed with mean  $\sigma_{i,j}^2$ ) and the phase is uniformly distributed on  $[0, 2\pi]$ . Furthermore, we model  $z_j$  as zero-mean mutually independent, circularly-symmetric complex Gaussian random variables with variance  $N_0$ .

Based on the above definition, the received signal at the BS is modeled as

$$y_d = a_{p_i,d}x_{p_i} + a_{s_i,d}x_{s_i} + z_d \quad (1)$$

where  $x_{p_i}$  is the  $i$ -th primary source transmitting signal, and  $x_{s_i}$  is the  $i$ -th secondary relaying signal based on the amplify-and-forward scheme<sup>[9]</sup>.  $a_{p_i,d}$  and  $a_{s_i,d}$  denote the channel gains of the links among the  $i$ -th primary and secondary user to the BS respectively.

If we assume that every primary-secondary cooperative transmitting pair has an average power constraint  $P_{\text{tot}}$ , the primary user transmitting power can be regarded as  $P_p = \xi P_{\text{tot}}$  and the secondary user transmitting power is  $P_s = (1 - \xi)P_{\text{tot}}$ , where  $\xi \in (0, 1]$  and  $(1 - \xi) \in [0, 1)$  denote the fractions of the total end-to-end power  $P_{\text{tot}}$  allocated to the primary source and the assigned secondary relay, respectively. As a result, the channel model is parameterized by the SNR random variables  $\xi \text{SNR} \beta_{p_i,j}$  or  $(1 - \xi) \text{SNR} \beta_{s_i,j}$ , where  $\text{SNR} = P_{\text{tot}}/N_0$ <sup>[9]</sup>. Furthermore, due to the burst and random characteristics of data transmission, we define the transmission probability of the primary user as  $p_s$ . Under the above definition, the mutual information between the  $i$ -th primary user and the BS is given by

$$I_{p_i} = \log(1 + \xi \text{SNR} \beta_{p_i,d} + f(\xi \text{SNR} \beta_{p_i,s_i}, (1 - \xi) \text{SNR} \beta_{s_i,d})) \\ i = 1, 2, \dots, N; j = 1, 2, \dots, S_r \quad (2)$$

where  $S_r$  is the number of the secondary users, and  $f(x, y) \triangleq xy/(x + y + 1)$ .

Due to the primary and secondary relay transmitting on the orthogonal frequency bands, the nodes achieve the total number of the available degrees of freedom in the channel. In contrast, the available degrees of freedom of the channels in Ref. [9] can only be achieved by one half of the total degrees of freedom. The outage probability in this paper is defined as the probability that the mutual information between the primary user and the BS falls below the required rate. We give the outage performance and numerical simulations of the model in the following section.

## 2 Outage Performance Analysis

In this section, we analyze the outage performance of a cooperative transmission between the primary and the secondary users using a free band to the BS. The outage event occurs when satisfying the following conditions:

$$\mathbf{1}[O_{\text{out}}] = [\mathbf{1}\{O_{p_i,s_i,d} \cap O_f \cap O_s\} \cup \mathbf{1}\{O'_{p_i,s_i,d} \cap O_f^c \cap O_s\} \cup \\ \mathbf{1}\{O_{p_i,d} \cap O_s^c\}] \cap \mathbf{1}\{O_t\} \quad (3)$$

where  $\mathbf{1}\{\cdot\}$  is the indicator function;  $O_{\text{out}}$  is the overall outage event;  $O_{p_i,s_i,d}$  denotes the outage event of the cooperative transmission to the BS;  $O_f$  describes the event that there is at least one free band in the spectrum sharing pool;  $O_s$  denotes the event that there exists at least one secondary user relaying for the primary;  $O'_{p_i,s_i,d}$  means the outage event on the condition of there being no free band in the spectrum sharing pool;  $O_{p_i,d}$  denotes the outage event of direct transmission to the BS;  $O_t$  means the event that the primary user has packets to transmit;  $O_f^c$  and  $O_s^c$  are the complements of  $O_f$  and  $O_s$ , respectively.

Based on the system model of the physical layer, we can give the probabilities of these events. The outage probability of cooperative transmission is<sup>[9]</sup>

$$P[O_{p_i,s_i,d}] = P[I_{p_i,s_i,d} < R] = P\{\log(1 + \xi \text{SNR} \beta_{p_i,d} + \\ f(\xi \text{SNR} \beta_{p_i,s_i}, (1 - \xi) \text{SNR} \beta_{s_i,d})) < R\} \sim \\ \frac{(1 - \xi) \sigma_{s_i,d}^2 + \xi \sigma_{p_i,s_i}^2}{2\xi^2 (1 - \xi) \sigma_{p_i,d}^2 \sigma_{p_i,s_i}^2 \sigma_{s_i,d}^2} \left(\frac{2^R - 1}{\text{SNR}}\right)^2 \quad (4)$$

The last row of (4) is tenable when SNR is great.

The probability of the existing free band in the spectrum sharing pool is given by

$$P[O_f] = 1 - P[O_f^c] \quad (5)$$

where  $O_f^c$  means that all the bands except for the licensed band of the  $i$ -th primary user in the system are occupied by the primary or secondary users. If the control center allocates one secondary user for the primary user and the cooperative transmission is adopted, the working bands are occupied in pairs (One band is for the primary user transmission to the BS, and the other band is for the secondary user to amplify-and-forward what it receives simultaneously to the BS). Assuming that all primary users have the same transmitting probability  $p_s$ , the probability of the event  $O_f^c$  is given as

$$P[O_f^c] = \begin{cases} p_s^{(N-1)/2} + p_s^{(N-1)/2+1} + \dots + p_s^{N-1} & N \text{ is odd} \\ p_s^{N/2} + p_s^{N/2+1} + \dots + p_s^{N-1} & N \text{ is even} \end{cases} \quad (6)$$

It reveals that no free band in the spectrum sharing pool is used for the cooperative transmission only when at least half the number of the primary users access the channel via secondary relaying which is due to the orthogonal access protocol. If more than one half the number of primary users access the channel, they must exploit a direct transmission or time diversity access.

When the free band does not exist, the primary user makes use of its license band to transmit packets in two slots

(named as time diversity access). One slot is for primary transmission, and the other one is for the secondary retransmission to the BS. As a result, the probability of  $O'_{p_i, s_i, d}$  is written as

$$P[O'_{p_i, s_i, d}] = P\left[\frac{1}{2}I_{p_i, s_i, d} < R\right] = P\left\{\frac{1}{2}\log(1 + \xi\text{SNR}\beta_{p_i, d} + f(\xi\text{SNR}\beta_{p_i, s_j}, (1 - \xi)\text{SNR}\beta_{s_j, d})) < R\right\} \sim \frac{(1 - \xi)\sigma_{s_j, d}^2 + \xi\sigma_{p_i, s_j}^2}{2\xi^2(1 - \xi)\sigma_{p_i, d}^2\sigma_{p_i, s_j}^2\sigma_{s_j, d}^2} \left(\frac{2^{2R} - 1}{\text{SNR}}\right)^2 \quad (7)$$

The analysis is similar to Ref. [9].

If the number of secondary users is less than that of primary users, there exists no secondary relaying for at least one primary user. Under this condition, the relaying probability for one primary user is considered to be  $S_r/N$ . Hence, the probability of  $O_s$  is

$$P[O_s] = \min\left\{\frac{S_r}{N}, 1\right\} \quad (8)$$

The primary user adopts a direct transmission only when no secondary user is assigned for it. Besides, the outage probability of direct transmission is shown as follows:

$$P[O_{p_i, d}] = P(I_{p_i, d} < R) = P\{\log(1 + \xi\text{SNR}\beta_{p_i, d}) < R\} = 1 - \exp\left(-\frac{2^R - 1}{\xi\text{SNR}\sigma_{p_i, d}^2}\right) \quad (9)$$

The overall outage probability is

$$P_{\text{out}} = p_s \{P[O_{p_i, s_i, d}]P[O_f]P[O_s] + P[O'_{p_i, s_i, d}]P[O_f^c] \cdot P[O_s] + P[O_{p_i, d}]P[O_s^c]\} \quad (10)$$

### 3 Simulations

In this section, we give numerical simulations of the outage probability as a function of SNR, the power allocation ratio  $\xi$ , and the numbers of the primary and secondary users based on different primary user transmission probabilities  $p_s$ .

#### 3.1 Simulation parameters

We consider a symmetric channel model which has the same channel gain among different primary links (including the channel from the primary user to the BS  $p_i \rightarrow d$  and the channel from the primary user to the secondary relay user  $p_i \rightarrow s_j$ ) and different secondary links (the channel from the secondary user to the BS  $s_i \rightarrow d$ ). We define the channel variances as  $\sigma_{p_i, s_j}^2 = 2$ ,  $\sigma_{s_j, d}^2 = 2$  and  $\sigma_{p_i, d}^2 = 1$  ( $i = 1, 2, \dots, N$ ;  $j = 1, 2, \dots, S_r$ ) in our simulation. Furthermore, the required data rate  $R = 1$  bit/(s · Hz).

#### 3.2 Simulation and discussions

Fig. 2 shows the analytical and simulated outage probability of the proposed scheme as a function of SNR based on different  $p_s$  ( $p_s = 1, 0.6, 0.3, 0.1$ ) and the system parameters of  $\xi = 0.5$ ,  $N = 5$ ,  $S_r = 5$ ,  $R = 1$  bit/(s · Hz). According to Fig. 2, the simulated results are very similar to the analytical performances. In addition, the outage probabilities decline with the transmission probabilities of primary users. It means that the secondary relay user can make good use of

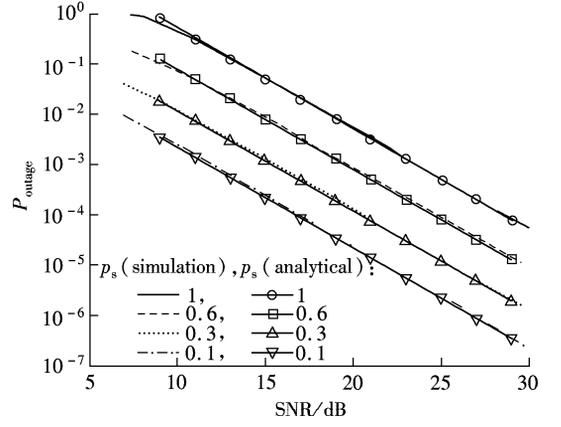


Fig. 2 Outage probability of simulation and analysis

the free channel resource and reduce the probabilities of outage events.

Fig. 3 shows the outage probabilities as a function of  $\xi$  with  $p_s = 0.5$ ,  $\text{SNR} = 15, 20, 25$  and  $30$  dB. From Fig. 3, it can be seen that the optimal power allocation ratio  $\xi$  is equal to  $0.5$  which can be computed by taking the derivate of Eq. (10) to  $\xi$ , i. e., by solving the equation  $\partial P_{\text{out}}/\partial \xi = 0$ . It is a natural choice of  $\xi$  when implementing in the practical networks without the knowledge of the CSI of the system.

Fig. 4 shows the outage probability vs. the parameter  $N/S_r$  with  $N = 1, 2, \dots, 10$ ;  $S_r = 5$ ;  $\xi = 0.5$  and  $p_s = 0.5, 0.3, 0.1$ , respectively. It can be seen from Fig. 4 that the outage probability decreases when  $N$  is less than the number of secondary relay users  $S_r$  and increases when  $N$  is greater than

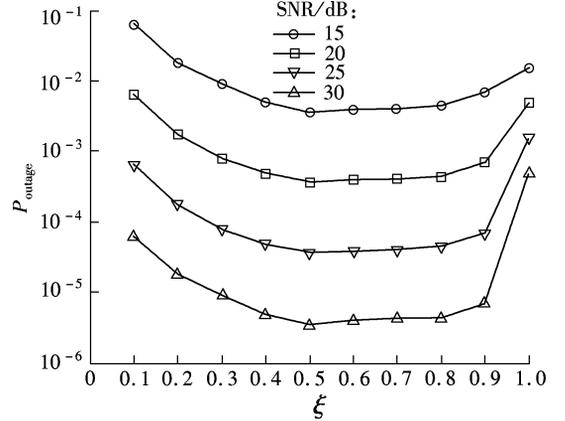


Fig. 3 Outage probability as a function of power allocation  $\xi$  at the end-end spectral efficiency  $R = 1$  bit/(s · Hz)

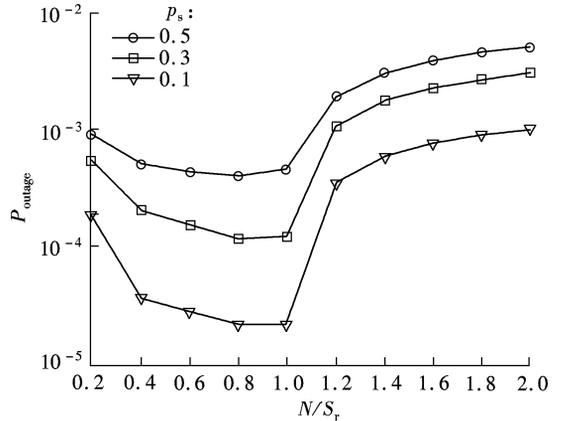


Fig. 4 Outage probability as a function of  $N/S_r$

$S_r$ . It is due to the fact that every primary user is allocated one secondary relay user and cooperative transmission is adopted when  $N < S_r$ . Under this scenario, the secondary relay user can obtain more opportunities to access a free channel to retransmit the primary packet when  $N$  increases. As a result, the mutual information increases so as to decrease the outage probability. However, when  $N > S_r$ , the relayed probability (i. e.,  $S_r/N$ ) of the primary user is reduced accordingly. The primary user must transmit directly to the BS with greater probability. Obviously, the overall outage probability increases under this condition.

Fig. 5 gives the simulation results of the outage probabilities vs. the system parameters  $S_r/N$  with  $S_r = 1, 2, \dots, 10$ ;  $N = 5$ ;  $\xi = 0.5$  and  $p_s = 0.5, 0.3, 0.1$ , respectively. According to Fig. 5, more primary users can be relayed through the increasing number of secondary users. It can reduce the outage probability when  $S_r < N$ . However, it is not necessary to keep more secondary relay users than the primary users because every primary node transmits with only one secondary user cooperatively.

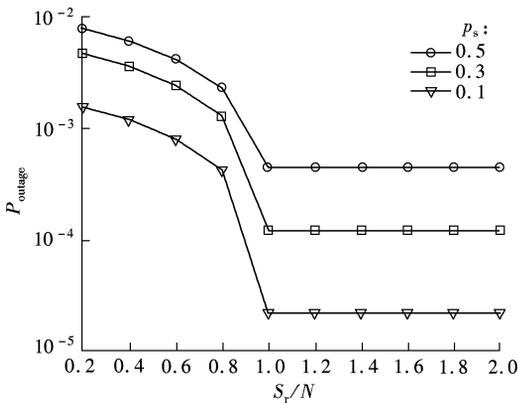


Fig. 5 Outage probability vs. system parameters

## 4 Conclusion

The model of  $N$  primary users transmitting in orthogonal subchannels with  $S_r$  secondary relaying is studied in a stationary

fading environment. The primary user transmits packets cooperatively with the secondary user which is assigned by the control center using the free band of the spectrum sharing pool. We analyze the outage probabilities as a function of primary transmission probability  $p_s$ , power allocation ratio  $\xi$ , and the numbers of the primary and secondary users, respectively. Numerical simulations verify the results of the theoretical analysis.

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# 基于多用户认知网络的中断概率性能研究

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**摘要:**提出了一种多主从用户相互协作的认知共存网络系统模型. 在该模型中, 控制中心已知所有用户的位置并为每个主用户分配最近的从用户进行协作通信, 控制中心通过从用户反馈的感知结果来获取频谱空洞信息并为从用户分配工作频段. 主用户利用正交子信道接入基站. 从用户利用网络中未用的频谱空洞采用放大前传机制为主用户转发数据. 推导了在该模型下主用户的中断概率, 并对主用户发送概率  $p_s$ 、主从用户间发射功率分配系数  $\xi$  和主从用户数对系统的影响进行了仿真. 仿真结果表明, 主从用户间平均分配发射功率在  $\xi = 0.5$  且网络中主从用户数目相同时系统性能最优.

**关键词:**协作通信; 中断概率; 放大前传; 认知网络

**中图分类号:** TN915.02