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Application of stiffness matrix of a beam element considering section distortion effect

Li Haifeng Luo Yongfeng

(Department of Building Engineering, Tongji University, Shanghai 200092, China)

Abstract: According to the stationary principle of potential energy and the generalized coordinate method, a stiffness matrix of a beam element considering distortion effects is derived. Using the stiffness matrix of the beam element, a finite element program for computing thin-walled box steel beams is developed. And the program can take the section distortion and warping effects into account. The influences of diaphragm spacing on the mechanical behavior of thin-walled box beams are analyzed by the program. The numerical analysis shows that setting diaphragms have the greatest influence on the distortion normal stress, while there is very little influence on the bending normal stress. Only when the distance of adjacent diaphragms decreases to a certain value, will the distortion normal stress in the thin-walled box beam obviously reduce under the distortion load. Finally, a distortion-warping coefficient γ is introduced for simplifying the calculation of the longitudinal normal stress of thin-walled box beams. When the ratio of diaphragms adjacent space L to the maximum section dimension H is less than 2, the distortion-warping coefficient γ tends to one, which means that the distortion normal stress of the thin-walled box beam tends to zero, and the effect of the section distortion can be ignored.

Key words: thin-walled box beam; stationary principle of potential energy; generalized coordinate method; distortion; warping; distortion-warping coefficient

A large quantity of thin-walled box steel beams has been used in bridges, marine engineering, high-rise buildings and heavy plant buildings in recent years, especially in large-span steel structures. As an actual project, seven different sections of thin-walled box beams have been adopted in the Nanjing Olympic Sports Center Stadium[1].

Much research has been done on the mechanical performance of thin-walled box beams till now. However, there are still many puzzles in engineering, such as distortion and warping of thin-walled members[2-4]. St. Venant first studied the problem of free distortion in thin-walled members in 1850. Vlasov put forward two assumptions about distortion and warping of thin-walled members in 1903. It is assumed that the shear strain in the middle plane of a plate is zero and the box-beam has a rigid periphery. But the calculation precision has not been satisfied in actual projects. In 1939, Ymanhck proposed a new theory of distortion about closed-section thin-walled members without the assumption of zero shear strain in the middle plane of a plate. However, the assumption that the box-beam had a rigid periphery was retained in his theory. The distortion has obvious influences on the mechanical performance of thin-walled members. Therefore, the theory of Ymanhck also has large errors in applications. In recent years, numerical calculation[5-6] is popular in actual projects. However, the element stiffness matrix[7-9] is the most important of those methods. The finite stripe method proposed by Cheung[10] can reduce a three-dimensional problem to a two-dimensional problem, but the problem of warping is still not perfectly solved.

With the application in actual projects, the special mechanical performance of thin-walled box beams[11], especially the effects of section distortion and warping have been recognized and caught designers' attention. In actual projects, if diaphragms are set up in a thin-walled box beam, the effects of the section distortion can be ignored and the restrained torsion can be analyzed by the theory of Ymanhck. This method is not convincing without considering the effects of the section distortion[12-13].

With the requirements of the higher calculation precision of modern structures, the internal forces of box-beams used in large-scale complex structures cannot be calculated exactly with the traditional assumption that a box-beam has a rigid periphery when loading. Compared with the traditional beams, computation of a thin-walled box steel beam must take the influences of the section distortion and warping into account[14], while the mechanical performance of the box beams is directly influenced by the diaphragms setting in the box beams.

Based on the generalized coordinate method and the stationary principle of potential energy, a stiffness matrix of the beam element considering distortion effect is deduced. Using the stiffness matrix of the beam element, a finite element program for computing thin-walled box steel beams is developed in this paper by Matlab. The program can take the section warping and the beam distortion into account, and it can be used to analyze the influences of diaphragms on the effects and the distribution of stresses in the thin-walled box beams.

1 Element Model and Flow Chart of Program

Based on the generalized coordinate method and the stationary principle of potential energy[11, 15], an element stiffness matrix for computing thin-walled box steel beams under any load is developed. Parameters such as axial tension, lateral shear, two axial-bending, torsion, section warping and distortion are taken into account. The end forces and displacements of a thin-walled beam element are shown in Fig. 1.

The positive directions of the end forces and the end displacements in generalized coordinates are in accord with clockwise direction of torsion displacement θ_i. Where $\boldsymbol{u}_i$ is the axial linear displacement of node i; $\boldsymbol{v}_i$ is the linear dis-

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Biographies: Li Haifeng (1983—), male, graduate; Luo Yongfeng (corresponding author), male, doctor, professor, shyfluo93@126.com.

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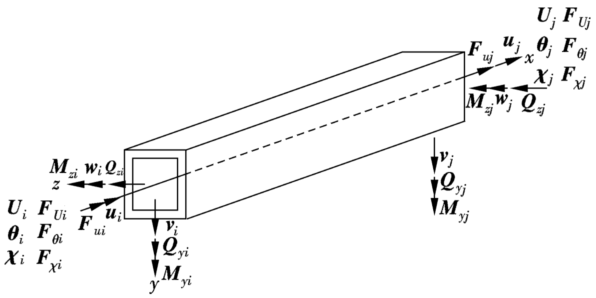

Fig. 1 End forces and displacements of a thin-walled box beam element

placement along axis y of node i; v'_i is the rotation angle around axis z of node i; w_i is the linear displacement along axis z of node i; w'_i is the rotation angle around axis y of node i; U_i is the generalized warping displacement of node i; θ_i is the torsion angle of node i; χ_i is the generalized distortion displacement of node i; F_{ui} is the axial force of node i; Q_{yi} is the shear force along axis y of node i; M_{zi} is the bending moment around axis z of node i; Q_{zi} is the shear force along axis z of node i; $-M_{yi}$ is the bending moment around axis y of node i; F_{Ui} is the longitudinal bimoment of node i; $F_{\theta i}$ is the torque of node i; and $F_{\chi i}$ is the transverse bimoment of node i. The symbols of node j are similar to node i.

Joint displacements of end i and end j of the beam element are as follows:

$$\boldsymbol{u}_e = \begin{bmatrix} \boldsymbol{u}_i \\ \boldsymbol{u}_j \end{bmatrix} =$$

$$[\, u_i \;\; v_i \;\; v'_i \;\; w_i \;\; w'_i \;\; U_i \;\; \theta_i \;\; \chi_i \;\; u_j \;\; v_j \;\; v'_j \;\; w_j \;\; w'_j \;\; U_j \;\; \theta_j \;\; \chi_j \,]^{\mathrm{T}}$$

Joint forces of end i and end j of the beam element are

$$\boldsymbol{F}_e = \begin{bmatrix} \boldsymbol{F}_i \\ \boldsymbol{F}_j \end{bmatrix} =$$

$$[\, F_{ui} \; Q_{yi} \; M_{zi} \; Q_{zi} \; -M_{yi} \; F_{Ui} \; F_{\theta i} \; F_{\chi i} \; F_{uj} \; Q_{yj} \; M_{zj} \; Q_{zj} \; -M_{yj} \; F_{Uj} \; F_{\theta j} \; F_{\chi j} \,]^{\mathrm{T}}$$

The equilibrium equation of the element is

$$\boldsymbol{K}^{(e)} \boldsymbol{u}_e = \boldsymbol{F}_e \tag{1}$$

where $\boldsymbol{K}^{(e)}$ is the element stiffness matrix, and the theoretical deduction and its detailed expression can be seen in Ref. [16]. Based on equilibrium equation (1), a finite element program for computing thin-walled box steel beams is developed using Matlab[17].

2 Checking Program

Both the program in this paper and ANSYS are used for numerical calculation and comparison. The example considered is a thin-walled box cantilever beam under torsion. The section of the beam is shown in Fig. 2, which is 1 400 mm × 500 mm × 12 mm × 25 mm. The beam length $L = 10$ m. The beam material is steel Q235. The torsion T acted on free end is 7.5×10^5 N·m. When the restrained torsion of a thin-walled box steel beam is calculated by means of the generalized coordinate method, the equilibrium shear flow[11] S is usually used to simulate the action of torsion T. The loading state of the thin-walled box cantilever beam is shown in Fig. 2.

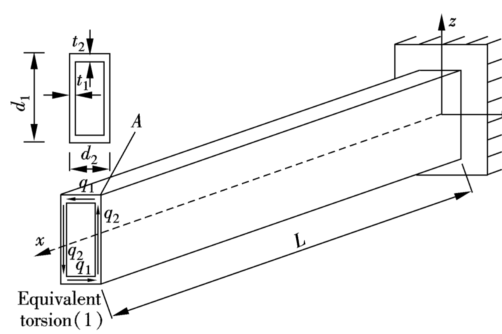

Fig. 2 Force diagram of the thin-walled box cantilever beam

Numerical results calculated by ANSYS and the program in this paper are shown in Tab. 1. The differences of warping stress, shear force in beam web and shear force in flange are 0.442%, 2.02% and 12.6%, respectively.

Tab. 1 Comparisons of numerical analysis

Numerical analysis and difference	Warping stress σ_ω	Shear force in web τ_b	Shear force in flange τ_f
Maximal stress by ANSYS/MPa	18.11	45.12	21.86
Maximal stress by program in this paper/MPa	18.19	46.03	24.61
Difference*/%	0.442	2.02	12.6

Note: * $\text{Difference} = \dfrac{\text{Results by the program} - \text{Results by ANSYS}}{\text{Results by ANSYS}} \times 100\%$.

For further checking the precision of the program in this paper, the variation of warping normal stress along the beam axial direction is shown in Fig. 3. The section position 0 denotes the fixed end, and the section position 10 m denotes the free end. The warping normal stress shown in Fig. 3 represents the stresses at point A in Fig. 2.

As shown in Fig. 3, there is a little difference in the results calculated by the program and ANSYS. The maximal difference of the warping normal stress is only 8.4%.

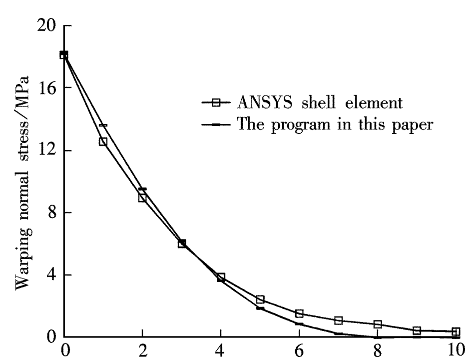

Fig. 3 Variation curve of warping normal stresses along beam axis

3 Influences of Diaphragms on Beam Stresses

A numerical example is analyzed for the investigation of the influences of the diaphragms on the loading behavior of the thin-walled box beams. The section and the material of the beam are 1 200 mm × 600 mm × 12 mm × 30 mm and steel Q235. The beam length $L = 10$ m. The uniformly distributed eccentric load acted on the box beam is $q = 8 \times 10^4$ N/m. For comparison, the adjacent space of diaphragms in the cantilever beam is taken as L, $L/2$, $L/4$, $L/5$, $L/10$, respectively. While the adjacent space of the diaphragms setting in the fixed supported beam or the simply supported beam is taken as $L/2$, $L/4$, $L/5$, $L/8$, $L/10$, respectively. In the following section-stress curves, the horizontal axis denotes the position of the beam sections along beam axis x. The longitudinal axis denotes the normal stresses. For example, the section position 0 m denotes the fixed end section, and the section position 10 m denotes the free section of the cantilever beam. The positive normal stress denotes the tension stress.

3.1 Influences on longitudinal normal stresses of cantilever box beam

The loading state of the thin-walled box cantilever beam is shown in Fig. 4.

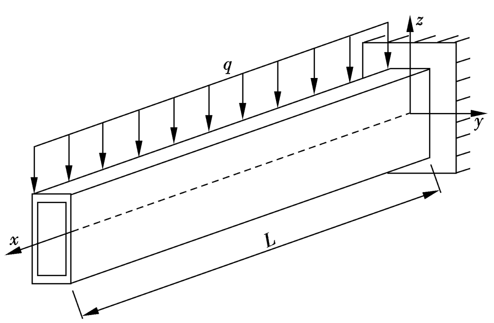

Fig. 4 Thin-walled box cantilever beam under a uniformly distributed eccentric line load

For analyzing the influences of diaphragms on the warping stresses of the thin-walled box beam under bending, torsion and distortion, respectively, a concentrated eccentric load can be decomposed from a bending load, a torsion load and a distortion load. The decomposition is shown in Fig. 5.

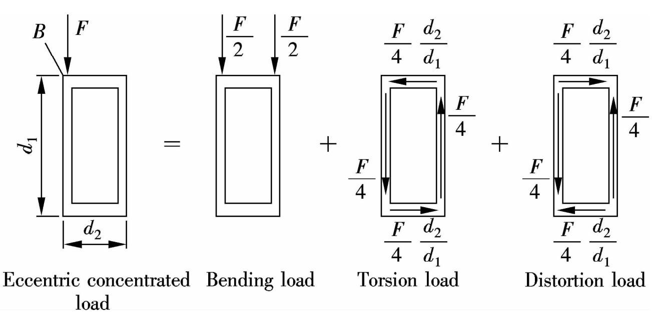

Fig. 5 Decomposition of a concentrated eccentric load

The longitudinal normal stresses of point B are selected for comparison. The influences of the diaphragms on longitudinal normal stresses are shown in Figs. 6 to 9.

Some conclusions can be drawn from Fig. 6 to Fig. 9.

1) Diaphragms setting in beams have a great influence on distortion normal stresses, while there is very little influence on bending normal stresses.

2) Only when the diaphragm space decreases to a certain value, does the distortion normal stress in the thin-walled box beam obviously reduce. When the adjacent space of diaphragms is less than one-fifth of the beam length, the distortion normal stress in the thin-walled box beam tends to zero, and the influence of section distortion can be ignored.

3) With the decrease in the diaphragm space, the warping

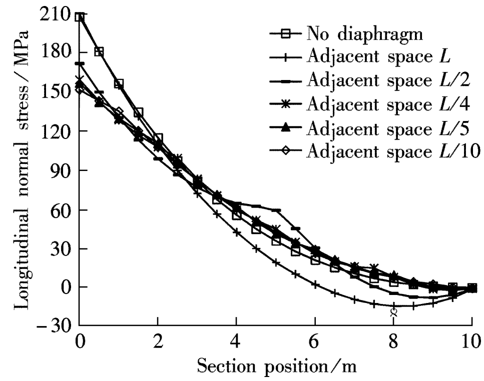

Fig. 6 Longitudinal normal stress distribution under eccentric load

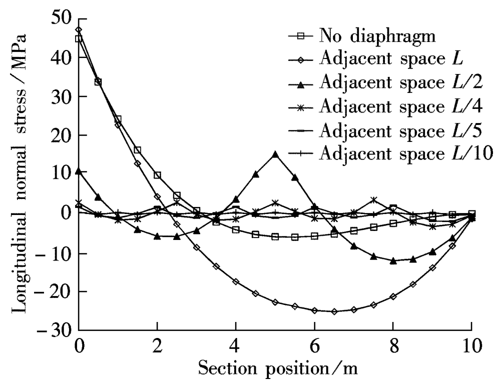

Fig. 7 Distortion normal stress distribution under distortion load

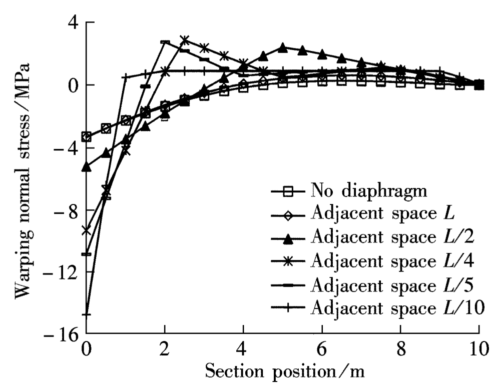

Fig. 8 Warping normal stress distribution under torsion load

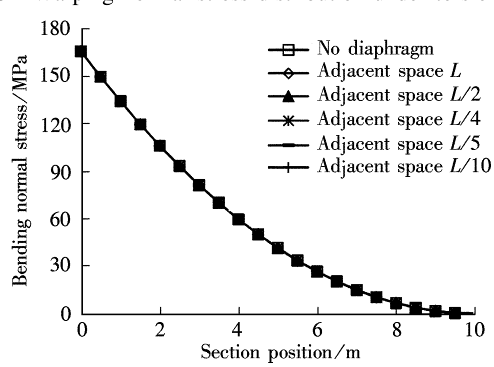

Fig. 9 Bending normal stress distribution under bending load

normal stress under torsion load, which lies in the fixed end of the thin-walled box beam, increases. While the warping normal stress tends to zero in 2 to 10 m of the beam, the distortion normal stress is much less than the warping normal stress and the bending normal stress. There is little influence of the diaphragms on the bending normal stress. Therefore, the influence of the diaphragms on the distortion normal stress mainly reflects the influence of the diaphragms on the longitudinal normal stress of the thin-walled box beams under eccentric load.

3.2 Influences on distortion normal stresses of fixed supported and simple supported box beams

The above results of the cantilever beam show that the influence of the distortion normal stress mainly reflects the influence of the diaphragms on the longitudinal normal stress of the thin-walled box beam. The influences of the diaphragms on the distortion normal stress of fixed supported beams and simple supported beams are investigated in this section. The beam model and the load in this section is the same as the cantilever beam, and the difference is only in the boundary conditions. Using the program of this paper, the influences of the diaphragms on the distortion normal stresses of the fixed supported beam and the simple supported beam are obtained, as shown in Fig. 10 and Fig. 11.

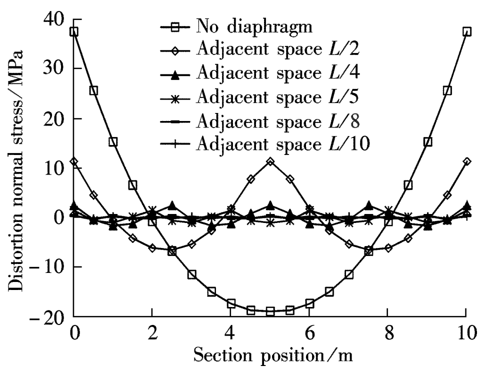

Fig. 10 Distortion normal stress distribution of fixed supported beam under distortion load

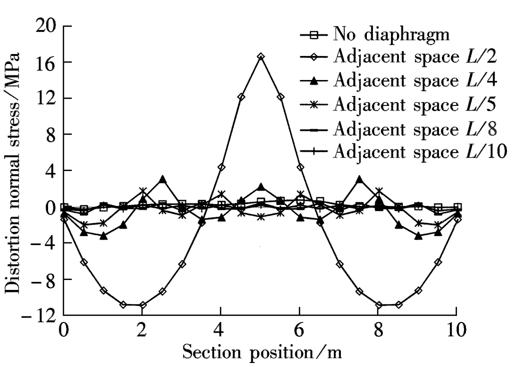

Fig. 11 Distortion normal stress distribution of simple supported beam under distortion load

From Fig. 10 and Fig. 11, it can be seen that the diaphragms have the same trend of influences on the distortion normal stresses in the fixed supported beam and the simple supported beam. The directions of the distortion normal stresses will be opposite when diaphragms are set. The distortion normal stress is parabolically distributed between two adjacent diaphragms. With the decrease in diaphragm space, the parabolic peak is reduced. When the diaphragm space is less than one-eighth of the beam length, the distortion normal stresses in tow beams tend to zero. The influences of section distortion can be ignored.

4 Simplified Calculation Method of Thin-Walled Box Beams Considering Section Distortion

Since the computation by means of the finite element program is complex, it is not convenient to estimate the stress of a member directly. Therefore, a simplified formula for calculating the normal stress of thin-walled box beams under eccentric load is proposed by means of numerical regression. The longitudinal normal stress of a thin-walled box beam under the eccentric load is denoted as

$$\sigma_Z = \sigma_M + \sigma_W + \sigma_D \tag{2}$$

where σ_M is the bending normal stress; σ_W is the warping normal stress; and σ_D is the distortion normal stress.

A distortion-warping coefficient γ is introduced. Eq. (2) can be simplified as

$$\sigma_Z = \gamma(\sigma_M + \sigma_W) \tag{3}$$

The distortion-warping coefficient γ is the function of diaphragm adjacent space L and the maximal dimension H of the beam cross section. The calculation formula of distortion-warping coefficient γ can be obtained by regressing the numerical results.

The distortion-warping coefficient of the cantilever beam is shown as

$$\gamma = 0.039\frac{L}{H} + 0.943 \tag{4}$$

The distortion-warping coefficient of the fixed supported box beam and the simple supported box beam is given as

$$\gamma = 0.013\frac{L^2}{H} + 0.063\frac{L}{H} + 0.932 \tag{5}$$

Fig. 12 shows the fitting curves of the distortion-warping coefficient γ.

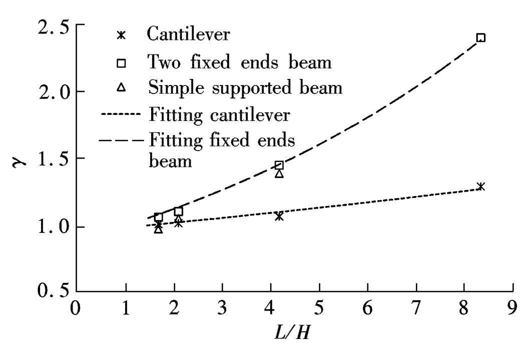

Fig. 12 Fitting curve of the distortion-warping coefficient γ

It can be concluded from Fig. 12 that:

1) Setting diaphragms have similar influences on the distortion normal stress in fixed supported beams and simple supported beams. The influence is obviously greater than the one in the cantilever beams.

2) When L/H is less than 2, the distortion-warping coefficient γ of the thin-walled box beam tends to one. It means that the distortion normal stress of the thin-walled box beam tends to zero, and the effect of the section distortion can be

ignored.

3) The longitudinal normal stress considering the section distortion can be simply calculated by the bending normal stress σ_M plus warping stress σ_W and then multiplying distortion-warping coefficient γ. σ_M and σ_W can be obtained by restrained torsion analysis with the traditional assumption that the box-beam has a rigid periphery when loading.

5 Conclusions

From the above investigation of the loading behavior of the thin-walled box beams, the following conclusions can be drawn:

1) Setting diaphragms have a great influence on the distortion normal stress, while there is very little influence on the bending normal stress.

2) When the ratio L/H is less than 2, the distortion normal stress of the thin-walled box beams tends to zero, and the effect of section distortion can be ignored.

3) Diaphragms have the same trend of influences on the distortion normal stresses in the fixed supported beams and the simple supported beams. And it is obviously greater than the one in cantilever beams.

4) The simplified formula for calculating the normal stress of thin-walled box beams under eccentric load is proposed by introducing the distortion-warping coefficient γ.

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考虑畸变效应的梁单元刚度矩阵应用

李海锋　　罗永峰

（同济大学建筑工程系，上海 200092）

摘要：基于势能驻值原理和广义坐标法原理推导出考虑截面畸变效应的梁单元刚度矩阵. 根据此梁单元刚度矩阵，编制了计算薄壁钢箱梁结构的有限元程序，且此程序可以考虑截面畸变效应和扭转效应. 运用自编程序，分析横隔板间距对薄壁钢箱梁受力性能的影响. 通过数值计算，得出布置横隔板对畸变正应力的影响最大，对弯曲正应力的影响很小. 畸变荷载作用下，只有当横隔板布置间距小到一定数值时，薄壁箱梁的畸变正应力才会减小. 最后，引入畸变翘曲影响系数 γ，简化了薄壁箱形梁纵向正应力的计算方法. 当横隔板布置间距 L 与薄壁箱梁横截面最大尺寸 H 的比值小于 2 时，畸变翘曲影响系数 γ 趋于 1，表明薄壁箱形梁的畸变正应力趋于零，可以忽略畸变效应的影响.

关键词：薄壁箱梁；势能驻值；广义坐标法；畸变；翘曲；畸变翘曲影响系数

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