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Improved results on synchronization in arrays of coupled delayed neural networks with hybrid coupling

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Abstract: In order to investigate the influence of hybrid coupling on the synchronization of delayed neural networks, by choosing an improved delay-dependent Lyapunov-Krasovskii functional, one less conservative asymptotical criterion based on linear matrix inequality (LMI) is established. The Kronecker product and convex combination techniques are employed. Also the bounds of time-varying delays and delay derivatives are fully considered. By adjusting the inner coupling matrix parameters and using the Matlab LMI toolbox, the design and applications of addressed coupled networks can be realized. Finally, the efficiency and applicability of the proposed results are illustrated by a numerical example with simulations.

Key words: delayed neural networks; hybrid coupling; synchronization; Lyapunov-Krasovskii functional; linear matrix inequality (LMI)

In the past decade, synchronization of various chaotic systems has gained considerable attention since the pioneering works appeared[1−2]. Especially, since chaos synchronization in arrays of linearly coupled dynamical systems was first considered by Wu et al.[3], various coupled chaotic systems have received much attention as they can exhibit some interesting phenomena, and many elegant results have been derived[3−10]. As one typical complex system, delayed neural networks (DNNs) are verified to exhibit the complex and unpredictable behaviors such as stable equilibria, bifurcation, and chaotic attractors. Wu et al.[3−10] studied chaos synchronization for coupled DNNs. Together with some effective techniques in Refs. [3 − 7], the researchers studied various synchronizations for delayed complex networks. However, the above results were only presented via some complicated inequalities, which make them difficult to be checked and applied to real cases. Through employing the Kronecker product and the LMI technique, the global synchronization was studied for DNNs including robust and discrete-time ones with various couplings in Refs. [8 − 10], and some easy-to-test sufficient conditions were established. For hybrid coupling, though the authors proposed some elegant results in Refs. [8 − 10], those methods still seem to be conservative and need some improvements. Thus, it is important and challenging to derive some less conservative results for arrays of DNNs with hybrid coupling. This constitutes the main focus of the present work.

In this paper, the global asymptotical synchronization of N identical DNNs with both variable interval delay and hybrid coupling is considered and one novel LMI-based condition is derived by utilizing the Kronecker product and free-weighting matrix technique. It shows that the chaos synchronization is ensured by a suitable design of the inner coupled linking matrix and delayed linking ones. Finally, the efficiency of the synchronization criteria can be demonstrated by utilizing one numerical example.

1 Problem Formulations

Suppose that nodes are coupled with states $\boldsymbol{x}_i(t)\,(i=1,2,\ldots,N)$. Then the DNNs can be written as

$$\dot{\boldsymbol{x}}_i(t) = -\boldsymbol{C}\boldsymbol{x}_i(t) + \boldsymbol{A}\boldsymbol{f}(\boldsymbol{x}_i(t)) + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{x}_i(t-\tau(t))) + \boldsymbol{I}(t) + \sum_{j=1}^{N} l_{ij}\boldsymbol{F}\boldsymbol{x}_j(t) + \sum_{j=1}^{N} l_{ij}\boldsymbol{K}\boldsymbol{x}_j(t-\tau(t)) + \sum_{j=1}^{N} l_{ij}\boldsymbol{J}\int_{t-v(t)}^{t}\boldsymbol{x}_j(s)\,\mathrm{d}s \tag{1}$$

where $\boldsymbol{x}_i(t)=[\boldsymbol{x}_{i1}(t),\ldots,\boldsymbol{x}_{in}(t)]^{\mathrm{T}}$ are the state vectors; $\boldsymbol{f}(\boldsymbol{x}_i(\cdot))=[\boldsymbol{f}_1(\boldsymbol{x}_{i1}(\cdot)),\ldots,\boldsymbol{f}_n(\boldsymbol{x}_{in}(\cdot))]^{\mathrm{T}}$ is the activation function of the neurons; $\boldsymbol{I}(t)=[\boldsymbol{I}_1(t),\ldots,\boldsymbol{I}_n(t)]^{\mathrm{T}}\in\mathbf{R}^n$ is the external input vector; $\boldsymbol{A}=[a_{ij}]_{n\times n}$, $\boldsymbol{B}=[b_{ij}]_{n\times n}$, $\boldsymbol{C}=\mathrm{diag}\{c_1,\ldots,c_n\}>\boldsymbol{0}$, $\boldsymbol{F}=[f_{ij}]_{n\times n}$, $\boldsymbol{K}=[k_{ij}]_{n\times n}$, and $\boldsymbol{J}=[j_{ij}]_{n\times n}$ are respectively the inner coupling matrices between the connected nodes i and j at time t and $t-\tau(t)$.

For system (1), the following assumptions and definition are adopted.

A1) $\tau(t)$ and $v(t)$ denote two interval time-varying delays satisfying $0\leqslant\tau_0\leqslant\tau(t)\leqslant\tau_m$, $\dot{\tau}(t)\leqslant\mu<+\infty$, $0\leqslant v_0\leqslant v(t)\leqslant v_m$. And we set $\bar{\tau}_m=\tau_m-\tau_0$ and $\bar{v}_m=v_m-v_0$.

A2) $\boldsymbol{L}=[l_{ij}]_{N\times N}$ is the configuration matrix that is irreducible and satisfies $l_{ij}=l_{ji}, i\neq j, l_{ii}=-\sum_{j=1,j\neq i}^{N} l_{ij}$. $l_{ij}>0$ if there is a connection between node i and node j; otherwise, $l_{ij}=0$.

A3) For any $\alpha,\beta\in\mathbf{R}$ and the constants σ_i^+ and σ_i^-, the nonlinear function $f_i(\cdot)$ satisfies $f_i(0)=0$, and

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$$[f_i(\alpha)-f_i(\beta)-\sigma_i^{+}(\alpha-\beta)][f_i(\alpha)-f_i(\beta)-\sigma_i^{-}(\alpha-\beta)]\leqslant 0 \qquad i=1,2,\dots,n \tag{2}$$

Here, we introduce $\boldsymbol{\Sigma}_1=\operatorname{diag}\{\sigma_1^{+}\sigma_1^{-},\dots,\sigma_n^{+}\sigma_n^{-}\}$ and $\boldsymbol{\Sigma}_2=\operatorname{diag}\left\{\frac{\sigma_1^{+}+\sigma_1^{-}}{2},\dots,\frac{\sigma_n^{+}+\sigma_n^{-}}{2}\right\}$.

Definition 1 Dynamic networks (1) is said to be asymptotically synchronized, if for any initial conditions $\phi_i(s),\phi_j(s)\in \boldsymbol{C}([t_0-\tau_m,t_0],\mathbf{R}^n)$, $i,j=1,2,\dots,N$, there exists $T>t_0$ such that $\|\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t)\|<\varepsilon$ for $t>T$.

2 Main Results

The following lemmas are essential for deriving the synchronization criterion.

Lemma 1 Let $\boldsymbol{U}=[u_{ij}]_{N\times N}$, $\boldsymbol{P}\in\mathbf{R}^{n\times n}$, $\boldsymbol{x}=[\boldsymbol{x}_1^{\mathrm T},\boldsymbol{x}_2^{\mathrm T},\dots,\boldsymbol{x}_N^{\mathrm T}]^{\mathrm T}$, and $\boldsymbol{y}=[\boldsymbol{y}_1^{\mathrm T},\boldsymbol{y}_2^{\mathrm T},\dots,\boldsymbol{y}_N^{\mathrm T}]^{\mathrm T}$ with $\boldsymbol{x}_i,\boldsymbol{y}_i\in\mathbf{R}^n$, $i=1,2,\dots,N$. If $\boldsymbol{U}=\boldsymbol{U}^{\mathrm T}$ and each row sum of $\boldsymbol{U}$ is 0, then

$$\boldsymbol{x}^{\mathrm T}(\boldsymbol{U}\otimes\boldsymbol{P})\boldsymbol{y}=-\sum_{1\leqslant i\leqslant j\leqslant N}u_{ij}(\boldsymbol{x}_i-\boldsymbol{x}_j)^{\mathrm T}\boldsymbol{P}(\boldsymbol{y}_i-\boldsymbol{y}_j)$$

Lemma 2 Suppose that $\boldsymbol{\Omega},\boldsymbol{\Xi}_{1i},\boldsymbol{\Xi}_{2i}(i=1,2)$ are constant matrices, $\alpha,\beta\in[0,1]$. $\boldsymbol{\Omega}+[\alpha\boldsymbol{\Xi}_{11}+(1-\alpha)\boldsymbol{\Xi}_{12}]+[\beta\boldsymbol{\Xi}_{21}+(1-\beta)\boldsymbol{\Xi}_{22}]<\mathbf{0}$ holds, if $\boldsymbol{\Omega}+\boldsymbol{\Xi}_{ij}+\boldsymbol{\Xi}_{kl}<\mathbf{0}$ $(i,j,k,l=1,2)$ hold simultaneously.

Together with the Kronecker product, we can reformulate system (1) as the following form equivalently,

$$\begin{aligned}\dot{\boldsymbol{x}}(t)=&-(\boldsymbol{I}_N\otimes\boldsymbol{C})\boldsymbol{x}(t)+(\boldsymbol{I}_N\otimes\boldsymbol{A})\boldsymbol{f}(\boldsymbol{x}(t))+(\boldsymbol{I}_N\otimes\boldsymbol{B})\boldsymbol{f}(\boldsymbol{x}(t-\tau(t)))+\\&(\boldsymbol{L}\otimes\boldsymbol{F})\boldsymbol{x}(t)+(\boldsymbol{L}\otimes\boldsymbol{K})\boldsymbol{x}(t-\tau(t))+(\boldsymbol{L}\otimes\boldsymbol{J})\int_{t-v(t)}^{t}\boldsymbol{x}(s)\,\mathrm{d}s+\boldsymbol{I}(t)\end{aligned}\tag{3}$$

where $\boldsymbol{x}(t)=[\boldsymbol{x}_1^{\mathrm T}(t),\dots,\boldsymbol{x}_N^{\mathrm T}(t)]^{\mathrm T}$, $\boldsymbol{f}(\boldsymbol{x}(\cdot))=[\boldsymbol{f}^{\mathrm T}(\boldsymbol{x}_1(\cdot)),\dots,\boldsymbol{f}^{\mathrm T}(\boldsymbol{x}_N(\cdot))]^{\mathrm T}$, and $\boldsymbol{I}(t)=[\boldsymbol{I}^{\mathrm T}(t),\dots,\boldsymbol{I}^{\mathrm T}(t)]^{\mathrm T}$.

Then by utilizing the most improved techniques in Ref. [11] for achieving the stability criteria, we state and establish one new delay-dependent sufficient condition on synchronization for system (3).

Theorem 1 Suppose that assumptions A1) to A3) hold, and the following dynamic system (3) is asymptotically synchronized. There exist $n\times n$ matrices $\boldsymbol{P}>\mathbf{0},\boldsymbol{S}>\mathbf{0},\boldsymbol{Z}>\mathbf{0},\boldsymbol{P}_l>\mathbf{0},\boldsymbol{Q}_l>\mathbf{0}(l=1,2,3),\boldsymbol{Z}_i>\mathbf{0},\boldsymbol{L}_i(i=1,2)$, $n\times n$ diagonal matrices $\boldsymbol{U}>\mathbf{0},\boldsymbol{V}>\mathbf{0},\boldsymbol{W}>\mathbf{0},\boldsymbol{H}>\mathbf{0}$, and $12n\times n$ matrices $\boldsymbol{N}_i(i=1,2,3)$ such that the LMIs in (4) hold.

$$\begin{bmatrix}\boldsymbol{\Omega}_{ij}+\boldsymbol{\varsigma}+\boldsymbol{\varsigma}^{\mathrm T}-\boldsymbol{I}_k^{\mathrm T}\boldsymbol{Z}_2\boldsymbol{I}_k & \sqrt{\tau_0}\boldsymbol{N}_1 & \sqrt{\bar\tau_m}\boldsymbol{N}_2\\ * & -\boldsymbol{S} & \mathbf{0}\\ * & * & -\boldsymbol{Z}\end{bmatrix}<\mathbf{0},\ \begin{bmatrix}\boldsymbol{\Omega}_{ij}+\boldsymbol{\varsigma}+\boldsymbol{\varsigma}^{\mathrm T}-\boldsymbol{I}_k^{\mathrm T}\boldsymbol{Z}_2\boldsymbol{I}_k & \sqrt{\tau_0}\boldsymbol{N}_1 & \sqrt{\bar\tau_m}\boldsymbol{N}_3\\ * & -\boldsymbol{S} & \mathbf{0}\\ * & * & -\boldsymbol{Z}\end{bmatrix}<\mathbf{0}\qquad \forall 1\leqslant i\leqslant j\leqslant N; k=1,2 \tag{4}$$

where $\boldsymbol{\varsigma}=[\boldsymbol{N}_1\quad \boldsymbol{N}_2-\boldsymbol{N}_1\quad -\boldsymbol{N}_3\quad \mathbf{0}\quad \boldsymbol{N}_3-\boldsymbol{N}_2\quad \mathbf{0}]$, $\boldsymbol{I}_1=[\mathbf{0}\quad \boldsymbol{I}_n\quad \mathbf{0}]$, $\boldsymbol{I}_2=[\mathbf{0}\quad \boldsymbol{I}_n\quad \mathbf{0}]$, and

$$\boldsymbol{\Omega}_{ij}=\begin{bmatrix}
\boldsymbol{\Xi}_{11} & \mathbf{0} & \mathbf{0} & \boldsymbol{\Xi}_{14} & \mathbf{0} & \mathbf{0} & \boldsymbol{\Xi}_{17} & \boldsymbol{\Xi}_{18} & \boldsymbol{\Xi}_{19} & \boldsymbol{\Xi}_{1,10} & \mathbf{0} & \boldsymbol{L}_1^{\mathrm T}\boldsymbol{B}\\
* & \boldsymbol{\Xi}_{22} & \mathbf{0} & \mathbf{0} & \boldsymbol{W\Sigma}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\
* & * & -\boldsymbol{P}_3-\boldsymbol{H\Sigma}_1 & \mathbf{0} & \mathbf{0} & \boldsymbol{H\Sigma}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\
* & * & * & \boldsymbol{\Xi}_{44} & \mathbf{0} & \mathbf{0} & \boldsymbol{A}^{\mathrm T}\boldsymbol{L}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\
* & * & * & * & \boldsymbol{\Xi}_{55} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\
* & * & * & * & * & -\boldsymbol{Q}_3-\boldsymbol{H} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\
* & * & * & * & * & * & \boldsymbol{\Xi}_{77} & \boldsymbol{\Xi}_{78} & \boldsymbol{\Xi}_{79} & \boldsymbol{\Xi}_{7,10} & \mathbf{0} & \boldsymbol{L}_2^{\mathrm T}\boldsymbol{B}\\
* & * & * & * & * & * & * & \boldsymbol{\Xi}_{88} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{V\Sigma}_2\\
* & * & * & * & * & * & * & * & -\boldsymbol{Z}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0}\\
* & * & * & * & * & * & * & * & * & -\boldsymbol{Z}_2 & \mathbf{0} & \mathbf{0}\\
* & * & * & * & * & * & * & * & * & * & -\boldsymbol{Z}_2 & \mathbf{0}\\
* & * & * & * & * & * & * & * & * & * & * & \boldsymbol{\Xi}_{12,12}
\end{bmatrix}$$

with

$$\begin{gathered}
\boldsymbol{\Xi}_{11}=-\boldsymbol{L}_1^{\mathrm T}\boldsymbol{C}-\boldsymbol{C}^{\mathrm T}\boldsymbol{L}_1-l_{ij}N(\boldsymbol{L}_1^{\mathrm T}\boldsymbol{F}+\boldsymbol{F}^{\mathrm T}\boldsymbol{L}_1)-\boldsymbol{U\Sigma}_1+\boldsymbol{P}_2+v_0^2\boldsymbol{Z}_1+\overline{V}_m^2\boldsymbol{Z}_2,\ \boldsymbol{\Xi}_{14}=\boldsymbol{U\Sigma}_2+\boldsymbol{L}_1^{\mathrm T}\boldsymbol{A}\\
\boldsymbol{\Xi}_{17}=\boldsymbol{P}-\boldsymbol{L}_1^{\mathrm T}-\boldsymbol{C}^{\mathrm T}\boldsymbol{L}_2-l_{ij}N\boldsymbol{F}^{\mathrm T}\boldsymbol{L}_2,\ \boldsymbol{\Xi}_{18}=-l_{ij}N\boldsymbol{L}_1^{\mathrm T}\boldsymbol{K},\ \boldsymbol{\Xi}_{19}=-l_{ij}N\boldsymbol{L}_1^{\mathrm T}\boldsymbol{J},\ \boldsymbol{\Xi}_{1,10}=-l_{ij}N\boldsymbol{L}_1^{\mathrm T}\boldsymbol{J}\\
\boldsymbol{\Xi}_{22}=-\boldsymbol{P}_2+\boldsymbol{P}_1+\boldsymbol{P}_3-\boldsymbol{W\Sigma},\ \boldsymbol{\Xi}_{44}=-\boldsymbol{U}+\boldsymbol{Q},\ \boldsymbol{\Xi}_{55}=-\boldsymbol{W}-\boldsymbol{Q}_2+\boldsymbol{Q}_1+\boldsymbol{Q}_3\\
\boldsymbol{\Xi}_{77}=-\boldsymbol{L}_2^{T}-\boldsymbol{L}_2+\tau_0\boldsymbol{S}+\bar\tau_{\mathrm m}\boldsymbol{Z},\ \boldsymbol{\Xi}_{78}=-l_{ij}N\boldsymbol{L}_2^{\mathrm T}\boldsymbol{K},\ \boldsymbol{\Xi}_{79}=-l_{ij}N\boldsymbol{L}_2^{\mathrm T}\boldsymbol{J}\\
\boldsymbol{\Xi}_{7,10}=-l_{ij}N\boldsymbol{L}_2^{\mathrm T}\boldsymbol{J},\ \boldsymbol{\Xi}_{88}=-(1-\mu)\boldsymbol{P}_1-\boldsymbol{V\Sigma}_2,\ \boldsymbol{\Xi}_{12,12}=-(1-\mu)\boldsymbol{Q}_1-\boldsymbol{V}
\end{gathered}$$

Proof Together with $\boldsymbol{U}=[u_{ij}]_{N\times N}=\begin{bmatrix}N-1 & \dots & -1\\ \vdots & & \vdots\\ -1 & \dots & N-1\end{bmatrix}$, we construct the Lyapunov-Krasovskii functional:

$$V(\boldsymbol{x}(t)) = V_1(\boldsymbol{x}(t)) + V_2(\boldsymbol{x}(t)) + V_3(\boldsymbol{x}(t)) \tag{5}$$

where

$$\begin{aligned} V_1(\boldsymbol{x}(t)) = {} & \boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{U}\otimes\boldsymbol{P})\boldsymbol{x}(t) + \int_{t-\tau(t)}^{t-\tau_0}[\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{U}\otimes\boldsymbol{P}_1)\boldsymbol{x}(s) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))(\boldsymbol{U}\otimes\boldsymbol{Q}_1)\boldsymbol{f}(\boldsymbol{x}(s))]\,\mathrm{d}s + \\ & \int_{t-\tau_0}^{t}[\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{U}\otimes\boldsymbol{P}_2)\boldsymbol{x}(s) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))(\boldsymbol{U}\otimes\boldsymbol{Q}_2)\boldsymbol{f}(\boldsymbol{x}(s))]\,\mathrm{d}s + \\ & \int_{t-\tau_m}^{t-\tau_0}[\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{U}\otimes\boldsymbol{P}_3)\boldsymbol{x}(s) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))(\boldsymbol{U}\otimes\boldsymbol{Q}_3)\boldsymbol{f}(\boldsymbol{x}(s))]\,\mathrm{d}s \\ V_2(\boldsymbol{x}(t)) = {} & \int_{t-\tau_0}^{t}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)(\boldsymbol{U}\otimes\boldsymbol{S})\dot{\boldsymbol{x}}(s)\,\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_m}^{-\tau_0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)(\boldsymbol{U}\otimes\boldsymbol{Z})\dot{\boldsymbol{x}}(s)\,\mathrm{d}s\mathrm{d}\theta \\ V_3(\boldsymbol{x}(t)) = {} & \int_{-v_0}^{0}\int_{t+\theta}^{t}v_0\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{U}\otimes\boldsymbol{Z}_1)\boldsymbol{x}(s)\,\mathrm{d}s\mathrm{d}\theta + \int_{-v_m}^{-v_0}\int_{t+\theta}^{t}\bar{v}_m\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{U}\otimes\boldsymbol{Z}_2)\boldsymbol{x}(s)\,\mathrm{d}s\mathrm{d}\theta \end{aligned}$$

Now, by directly estimating the derivative of $V(\boldsymbol{x}(t))$ along the trajectory of system (3), it can be deduced that

$$\begin{aligned} \dot{V}_1(\boldsymbol{x}(t)) \leqslant {} & 2\boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{U}\otimes\boldsymbol{P})\dot{\boldsymbol{x}}(t) + [\boldsymbol{x}^{\mathrm{T}}(t-\tau_0)(\boldsymbol{U}\otimes(\boldsymbol{P}_1-\boldsymbol{P}_2+\boldsymbol{P}_3))\boldsymbol{x}(t-\tau_0) + \\ & \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau_0))(\boldsymbol{U}\otimes(\boldsymbol{Q}_1-\boldsymbol{Q}_2+\boldsymbol{Q}_3))\boldsymbol{f}(\boldsymbol{x}(t-\tau_0))] - (1-\mu)[\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))(\boldsymbol{U}\otimes\boldsymbol{P}_1)\boldsymbol{x}(t-\tau(t)) + \\ & \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t)))(\boldsymbol{U}\otimes\boldsymbol{Q}_1)\boldsymbol{f}(\boldsymbol{x}(t-\tau(t)))] + [\boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{U}\otimes\boldsymbol{P}_2)\boldsymbol{x}(t) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))(\boldsymbol{U}\otimes\boldsymbol{Q}_2)\boldsymbol{f}(\boldsymbol{x}(t))] - \\ & [\boldsymbol{x}^{\mathrm{T}}(t-\tau_m)(\boldsymbol{U}\otimes\boldsymbol{P}_3)\boldsymbol{x}(t-\tau_m) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau_m))(\boldsymbol{U}\otimes\boldsymbol{Q}_3)\boldsymbol{f}(\boldsymbol{x}(t-\tau_m))] \end{aligned} \tag{6}$$

$$\dot{V}_2(\boldsymbol{x}(t)) = \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\boldsymbol{U}\otimes(\tau_0\boldsymbol{S}+\bar{\tau}_m\boldsymbol{Z}))\dot{\boldsymbol{x}}(t) - \int_{t-\tau_0}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)(\boldsymbol{U}\otimes\boldsymbol{S})\dot{\boldsymbol{x}}(s)\,\mathrm{d}s - \int_{t-\tau_m}^{t-\tau_0}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)(\boldsymbol{U}\otimes\boldsymbol{Z})\dot{\boldsymbol{x}}(s)\,\mathrm{d}s \tag{7}$$

$$\begin{aligned} \dot{V}_3(\boldsymbol{x}(t)) \leqslant {} & \boldsymbol{x}(t)(\boldsymbol{U}\otimes(v_0^2\boldsymbol{Z}_1))\boldsymbol{x}(t) - \Big[\int_{t-v_o}^{t}\boldsymbol{x}(s)\,\mathrm{d}s\Big]^{\mathrm{T}}(\boldsymbol{U}\otimes\boldsymbol{Z}_1)\Big[\int_{t-v_0}^{t}\boldsymbol{x}(s)\,\mathrm{d}s\Big] + \boldsymbol{x}(t)(\boldsymbol{U}\otimes(\bar{v}_m^2\boldsymbol{Z}_2))\boldsymbol{x}(t) - \\ & \left(1+\frac{v_m-v(t)}{\bar{v}_m}\right)\Big[\int_{t-v(t)}^{t-v_0}\boldsymbol{x}(s)\,\mathrm{d}s\Big]^{\mathrm{T}}(\boldsymbol{U}\otimes\boldsymbol{Z}_2)\Big[\int_{t-v(t)}^{t-v_0}\boldsymbol{x}(s)\,\mathrm{d}s\Big] - \\ & \left(1+\frac{v(t)-v_0}{\bar{v}_m}\right)\Big[\int_{t-v_m}^{t-v(t)}\boldsymbol{x}(s)\,\mathrm{d}s\Big]^{\mathrm{T}}(\boldsymbol{U}\otimes\boldsymbol{Z}_2)\Big[\int_{t-v_m}^{t-v(t)}\boldsymbol{x}(s)\,\mathrm{d}s\Big] \end{aligned} \tag{8}$$

For any $n\times n$ matrices $\boldsymbol{L}_i (i=1,2)$, noting that $\boldsymbol{UL}=\boldsymbol{NL}$ and checking $(\boldsymbol{U}\otimes\boldsymbol{L}_i^{\mathrm{T}})(\boldsymbol{L}\otimes\boldsymbol{F}) = (\boldsymbol{NL})\otimes(\boldsymbol{L}_i^{\mathrm{T}}\boldsymbol{F})$, $(\boldsymbol{U}\otimes\boldsymbol{L}_i^{\mathrm{T}})(\boldsymbol{L}\otimes\boldsymbol{K}) = (\boldsymbol{NL})\otimes(\boldsymbol{L}_i^{\mathrm{T}}\boldsymbol{K})$, $(\boldsymbol{U}\otimes\boldsymbol{L}_i^{\mathrm{T}})(\boldsymbol{L}\otimes\boldsymbol{J}) = (\boldsymbol{NL})\otimes(\boldsymbol{L}_i^{\mathrm{T}}\boldsymbol{J})$ for $i=1,2$, one can deduce

$$\begin{aligned} 0 = {} & 2[\boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{U}\otimes\boldsymbol{L}_1^{\mathrm{T}}) + \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\boldsymbol{U}\otimes\boldsymbol{L}_2^{\mathrm{T}})][-\dot{\boldsymbol{x}}(t) - (\boldsymbol{I}_N\otimes\boldsymbol{C})\boldsymbol{x}(t) + (\boldsymbol{I}_N\otimes\boldsymbol{A})\boldsymbol{f}(\boldsymbol{x}(t)) + \\ & (\boldsymbol{I}_N\otimes\boldsymbol{B})\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) + \boldsymbol{I}(t)] + 2[\boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{NL}\otimes\boldsymbol{L}_1^{\mathrm{T}}) + \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\boldsymbol{NL}\otimes\boldsymbol{L}_2^{\mathrm{T}})]\boldsymbol{Fx}(t) + \\ & 2[\boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{NL}\otimes\boldsymbol{L}_1^{\mathrm{T}}) + \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\boldsymbol{NL}\otimes\boldsymbol{L}_2^{\mathrm{T}})]\boldsymbol{Kx}(t-\tau(t)) + 2[\boldsymbol{x}^{T}(t)(\boldsymbol{NL}\otimes\boldsymbol{L}_1^{\mathrm{T}}) + \\ & \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\boldsymbol{NL}\otimes\boldsymbol{L}_2^{\mathrm{T}})]\Big[\boldsymbol{J}\int_{t-v_o}^{t}\boldsymbol{x}(s)\,\mathrm{d}s + \boldsymbol{J}\int_{t-v(t)}^{t-v_o}\boldsymbol{x}(s)\,\mathrm{d}s\Big] \end{aligned} \tag{9}$$

In the following, we can denote the terms to simplify the subsequent proof

$$\boldsymbol{x}_{ij}(\cdot) = \boldsymbol{x}_i(\cdot) - \boldsymbol{x}_j(\cdot),\ \boldsymbol{f}(\boldsymbol{x}_{ij}(\cdot)) = \boldsymbol{f}(\boldsymbol{x}_i(\cdot)) - \boldsymbol{f}(\boldsymbol{x}_j(\cdot)) \qquad \forall 1\leqslant i\leqslant j\leqslant N \tag{10}$$

Then for any $n\times n$ diagonal matrices $\boldsymbol{U}>\boldsymbol{0}$, $\boldsymbol{V}>\boldsymbol{0}$, $\boldsymbol{W}>\boldsymbol{0}$, $\boldsymbol{H}>\boldsymbol{0}$, and $\boldsymbol{\Sigma}_i$ $(i=1,2)$ in A3), it follows from (2) that

$$\begin{aligned} 0 \leqslant 2\sum_{1\leqslant i\leqslant j\leqslant N}\{ & -[\boldsymbol{x}_{ij}^{\mathrm{T}}(t)\boldsymbol{U\Sigma}_1\boldsymbol{x}_{ij}(t) - 2\boldsymbol{x}_{ij}^{\mathrm{T}}(t)\boldsymbol{U\Sigma}_2\boldsymbol{f}(\boldsymbol{x}_{ij}(t)) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}_{ij}(t))\boldsymbol{Uf}(\boldsymbol{x}_{ij}(t))] - \\ & [\boldsymbol{x}_{ij}^{\mathrm{T}}(t-\tau(t))\boldsymbol{V\Sigma}_1\boldsymbol{x}_{ij}(t-\tau(t)) - 2\boldsymbol{x}_{ij}^{\mathrm{T}}(t-\tau(t))\boldsymbol{V\Sigma}_2\boldsymbol{f}(\boldsymbol{x}_{ij}(t-\tau(t))) + \\ & \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}_{ij}(t-\tau(t)))\boldsymbol{Vf}(\boldsymbol{x}_{ij}(t-\tau(t)))] - [\boldsymbol{x}_{ij}^{\mathrm{T}}(t-\tau_0)\boldsymbol{W\Sigma}_1 x_{ij}(t-\tau_0) - \\ & 2\boldsymbol{x}_{ij}^{\mathrm{T}}(t-\tau_0)\boldsymbol{W\Sigma}_2\boldsymbol{f}(\boldsymbol{x}_{ij}(t-\tau_0)) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}_{ij}(t-\tau_0))\boldsymbol{Wf}(\boldsymbol{x}_{ij}(t-\tau_0))] - \\ & [\boldsymbol{x}_{ij}^{\mathrm{T}}(t-\tau_m)\boldsymbol{H\Sigma}_1\boldsymbol{x}_{ij}(t-\tau_m) - 2\boldsymbol{x}_{ij}^{\mathrm{T}}(t-\tau_m)\boldsymbol{H\Sigma}_2\boldsymbol{f}(\boldsymbol{x}_{ij}(t-\tau_m)) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}_{ij}(t-\tau_m))\boldsymbol{Hf}(x_{ij}(t-\tau_m))]\} \end{aligned} \tag{11}$$

For any $12n\times n$ constant matrices $\boldsymbol{N}_i (i=1,2,3)$, it follows from the Newton-Leibniz formula that

$$\begin{aligned} 0 = 2\sum_{1\leqslant i<j\leqslant N}\boldsymbol{\zeta}_{ij}^{\mathrm{T}}(t)\Big\{ & \boldsymbol{N}_1\Big[\boldsymbol{x}_{ij}(t) - \boldsymbol{x}_{ij}(t-\tau_0) - \int_{t-\tau(t)}^{t-\tau_0}\dot{\boldsymbol{x}}_{ij}(s)\Big] + \boldsymbol{N}_2\Big[\boldsymbol{x}_{ij}(t-\tau_0) - \boldsymbol{x}_{ij}(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_0}\dot{\boldsymbol{x}}_{ij}(s)\Big] + \\ & \boldsymbol{N}_3\Big[\boldsymbol{x}_{ij}(t-\tau(t)) - \boldsymbol{x}_{ij}(t-\tau_m) - \int_{t-\tau_m}^{t-\tau(t)}\dot{\boldsymbol{x}}_{ij}(s)\Big]\Big\} \end{aligned} \tag{12}$$

where

$$\boldsymbol{\zeta}_{ij}^{\mathrm{T}}(t) = \Big[\boldsymbol{x}_{ij}^{\mathrm{T}}(t), \boldsymbol{x}_{ij}^{\mathrm{T}}(t-\tau_0), \ \boldsymbol{x}_{ij}^{\mathrm{T}}(t-\tau_m), \ \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}_{ij}(t)), \ \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}_{ij}(t-\tau_0)), \ \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}_{ij}(t-\tau_m)), \dot{\boldsymbol{x}}_{ij}^{\mathrm{T}}(t),$$
$$\boldsymbol{x}_{ij}^{\mathrm{T}}(t-\tau(t)) \left(\int_{t-v_0}^{t} h(\boldsymbol{x}_{ij}(s))\,\mathrm{d}s\right)^{\mathrm{T}} \left(\int_{t-v(t)}^{t-v_0} h(\boldsymbol{x}_{ij}(s))\,\mathrm{d}s\right)^{\mathrm{T}} \left(\int_{t-v_m}^{t-v(t)} h(\boldsymbol{x}_{ij}(s))\,\mathrm{d}s\right)^{\mathrm{T}} \boldsymbol{f}^{T}(\boldsymbol{x}_{ij}(t-\tau(t)))\Big]$$

Together with lemma 1, $(\boldsymbol{U}\otimes\boldsymbol{L}_i^{\mathrm{T}})\boldsymbol{I}(t)=\boldsymbol{0}(i=1,2)$, and combining terms (6) to (12), we can deduce

$$\dot{V}(\boldsymbol{x}(t)) \leqslant \sum_{1\leqslant i<j\leqslant N} \boldsymbol{\zeta}_{ij}^{\mathrm{T}}(t)[\boldsymbol{\Omega}_{ij} + \boldsymbol{\varsigma} + \boldsymbol{\varsigma}^{\mathrm{T}} + \tau_0\boldsymbol{N}_1\boldsymbol{S}^{-1}\boldsymbol{N}_1^{\mathrm{T}} + [\tau(t)-\tau_0]\boldsymbol{N}_2\boldsymbol{Z}^{-1}\boldsymbol{N}_2^{\mathrm{T}} + [\tau_m-\tau(t)]\boldsymbol{N}_3\boldsymbol{Z}^{-1}\boldsymbol{N}_3^{\mathrm{T}} - \frac{v_m - v(t)}{\bar{v}_m}\boldsymbol{I}_1^{\mathrm{T}}\boldsymbol{Z}_2\boldsymbol{I}_1 - \frac{v(t)-v_0}{\bar{v}_m}\boldsymbol{I}_2^{\mathrm{T}}\boldsymbol{Z}_2\boldsymbol{I}_2\Big]\boldsymbol{\zeta}_{ij}(t) := \sum_{1\leqslant i<j\leqslant N}\boldsymbol{\zeta}_{ij}^{\mathrm{T}}(t)\boldsymbol{\Delta}_{ij}(t)\boldsymbol{\zeta}_{ij}(t)$$

Through using lemma 2 and the Schur-complement, the LMIs in (4) can guarantee $\boldsymbol{\Delta}_{ij}(t)<\boldsymbol{0}$ to be true and, thus, there must exist a positive scalar $\chi>0$ such that $\boldsymbol{\Delta}_{ij}(t)\leqslant -\chi\boldsymbol{I}<\boldsymbol{0}$. Therefore, it can be obtained that

$$\dot{V}(\boldsymbol{x}(t)) \leqslant \sum_{1\leqslant i<j\leqslant N}\boldsymbol{\zeta}_{ij}^{\mathrm{T}}(t)\boldsymbol{\Delta}_{ij}(t)\boldsymbol{\zeta}_{ij}(t) \leqslant -\chi\sum_{1\leqslant i<j\leqslant N}\|\boldsymbol{x}_i-\boldsymbol{x}_j\|^2 < 0 \qquad \forall\,\boldsymbol{x}_{ij}(t)\neq\boldsymbol{0}$$

which indicates that $\|\boldsymbol{x}_i-\boldsymbol{x}_j\|\rightarrow 0$ for all $t\rightarrow+\infty$ and $1\leqslant i<j\leqslant N$. Therefore we can conclude that system (3), i. e., system (1) can reach the global asymptotical synchronization, and it completes the proof.

Remark 1 Yuan et al. [9-10] considered the global synchronization for arrays of coupled DNNs with hybrid coupling and in this paper, we extend constant delay to a time-varying one. Moreover, the conditions are presented via LMIs, therefore, by using LMI in Matlab Toolbox, it is straightforward and convenient to check the feasibility without tuning any parameters. Moreover, we estimate the distributed delay term in (8) more effectively than the ones in Ref. [10].

3 Numerical Example

In this section, one example is provided to illustrate the effectiveness of the proposed results.

Example 1 Consider a two-dimensional delayed neural network system as follows:

$$\dot{\boldsymbol{x}}(t) = -\boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{A}\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) \tag{13}$$

where $\boldsymbol{C}=\begin{bmatrix}1.5 & 0\\ 0 & 1.5\end{bmatrix}$, $\boldsymbol{A}=\begin{bmatrix}2 & -0.2\\ -0.3 & 3\end{bmatrix}$, $\boldsymbol{B}=\begin{bmatrix}-1.5 & -0.1\\ -0.2 & -2.5\end{bmatrix}$, $\tau(t)=0.6+0.45\sin(8t)+0.05\cos^2(20t)$, $v(t)=0$, and $\boldsymbol{f}_i(\boldsymbol{x}_i)=\tanh(\boldsymbol{x}_i)$ for $i=1,2$. Choosing the following inner linking matrix $\boldsymbol{L}=\begin{bmatrix}-3 & 1 & 2\\ 1 & -2 & 1\\ 2 & 1 & -3\end{bmatrix}$ and the inner coupling matrices $\boldsymbol{F}=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}$, $\boldsymbol{K}=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}$, and $\boldsymbol{J}=\boldsymbol{0}$, we consider a dynamic system consisting of three coupled identical networks with delayed coupling as

$$\dot{\boldsymbol{x}}_i(t) = -\boldsymbol{C}\boldsymbol{x}_i(t) + \boldsymbol{A}\boldsymbol{f}(\boldsymbol{x}_i(t)) + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{x}_i(t-\tau(t))) + \boldsymbol{I}(t) + \sum_{j=1}^{3} l_{ij}\boldsymbol{F}\boldsymbol{x}_j(t) + \sum_{j=1}^{3} l_{ij}\boldsymbol{K}\boldsymbol{x}_j(t-\tau(t)) \tag{14}$$

Fig. 1 shows that the system has a chaotic attractor. Based on theorem 1, there does exist a feasible solution to the LMIs in (4), and we can verify the global asymptotical synchronization for system (14). The total error of system (14) is defined as $\mathrm{error}(t) = \sum_{i=1}^{2}\sqrt{[\boldsymbol{x}_{1i}(t)-\boldsymbol{x}_{2i}(t)]^2+[\boldsymbol{x}_{2i}(t)-\boldsymbol{x}_{3i}(t)]^2}$, and the total synchronous error can be depicted in Fig. 2 with the initial conditions $\boldsymbol{x}_1=[-0.5 \quad -0.3]^{\mathrm{T}}$, $\boldsymbol{x}_2=[0.3 \quad 0.7]^{\mathrm{T}}$, and $\boldsymbol{x}_3=[-0.5 \quad -0.6]^{\mathrm{T}}$.

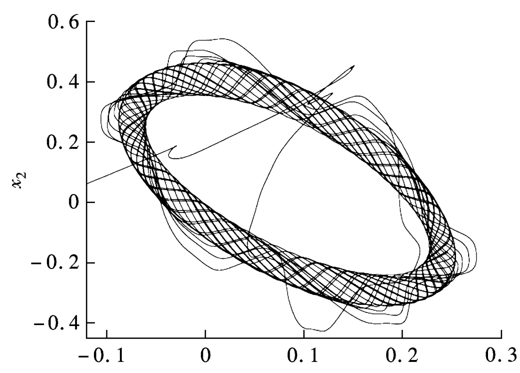

Fig. 1 The synchronized trajectory of system (13)

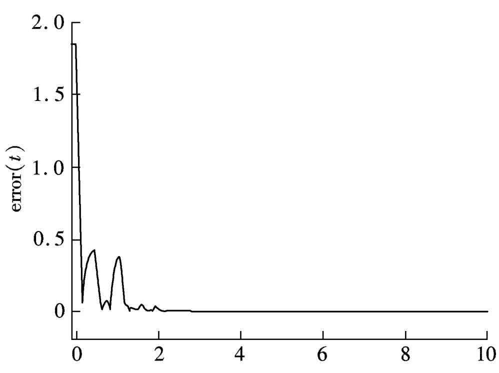

Fig. 2 The total synchronous error of system (14)

4 Conclusion

This paper investigates the global synchronization for the coupled DNNs with hybrid coupling. One novel condition is established by employing the Lyapunov-Krasovskii functional and the generalized convex combination. It is worth pointing out that some good mathematical techniques are employed, which can improve the earlier methods. The synchronization criterion is presented in terms of LMIs, which can be easily checked by utilizing LMI in Matlab Toolbox. Finally, one numerical example is utilized to illustrate the effectiveness of simulation results.

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具有混杂耦合的耦合时滞神经网络同步的进一步分析

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摘要：为了探讨混杂耦合现象对时滞神经网络系统同步的影响，通过选取一个改进的时滞依赖 Lyapunov-Krasovskii 泛函，基于线性矩阵不等式（LMI），建立了一个保守性更小的渐近同步准则．该准则采用矩阵 Kronecker 积与凸组合等方法，充分考虑了变时滞与其导函数的上下界．通过调整准则中的耦合矩阵参数并运用 Matlab 工具箱 LMI 进行求解，能够实现所探讨耦合神经系统的设计与应用．最后，通过一个仿真算例说明了所得结论的有效性与适用性．

关键词：时滞神经网络；混杂耦合；全局同步；Lyapunov-Krasovskii 泛函；线性矩阵不等式
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