

Iterative regularization method for image denoising with adaptive scale parameter

Li Wenshu^{1,2} Luo Jianhua¹ Liu Qiegen¹ He Fangfang² Wei Xiujin²

(¹School of Life Sciences and Biotechnology, Shanghai Jiao Tong University, Shanghai 200240, China)

(²College of Informatics and Electronics, Zhejiang Sci-Tech University, Hangzhou 310018, China)

Abstract: In order to decrease the sensitivity of the constant scale parameter, adaptively optimize the scale parameter in the iteration regularization model (IRM) and attain a desirable level of applicability for image denoising, a novel IRM with the adaptive scale parameter is proposed. First, the classic regularization item is modified and the equation of the adaptive scale parameter is deduced. Then, the initial value of the varying scale parameter is obtained by the trend of the number of iterations and the scale parameter sequence vectors. Finally, the novel iterative regularization method is used for image denoising. Numerical experiments show that compared with the IRM with the constant scale parameter, the proposed method with the varying scale parameter can not only reduce the number of iterations when the scale parameter becomes smaller, but also efficiently remove noise when the scale parameter becomes bigger and well preserve the details of images.

Key words: iterative regularization model (IRM); total variation; varying scale parameter; image denoising

The search for efficient image denoising methods is still a valid challenge at the crossing of functional analysis and statistics. During the last decade^[1–3], the relationships between the variational regularization method and the wavelet shrinkage have become one of the most active areas of research. In this paper, we are motivated by the following classical denoising problem of images degraded by the additive white Gaussian noise.

Given a noisy image $f(x, y): \Omega \rightarrow \mathbf{R}$, where Ω is a bounded open subset of \mathbf{R}^2 , we obtain a decomposition equation:

$$f(x, y) = g(x, y) + n(x, y) \quad (1)$$

where $g(x, y)$ is the true image and $n(x, y)$ is the noise with $(x, y) \in \Omega$ and $n(x, y) \sim (0, \delta^2)$.

The most classical variational model is

$$u = \arg \min_{u \in BV(\Omega)} \{J(u) + \lambda \|f - u\|_2^2\} \quad (2)$$

For some scale parameter $\lambda > 0$, where $BV(\Omega)$ denotes the space of functions with a bounded variation on Ω , $\|\cdot\|_2$ is L_2 norm. $J(u)$ is the regularization item and $\|f - u\|_2^2$ is

the fitting item. λ is chosen to balance the inconsistency and the deviation from the noise image $f(x, y)$. Therefore, much research is concentrated on the regularization item $J(u)$. The total variation model of Rudin-Osher-Fatemi (ROF) for image denoising is considered to be one of the best denoising models. But there are two serious issues concerning the ROF model^[4–9]. First, it is very complicated to compute the solutions of the optimization problems induced by the variational method. Secondly, it is difficult to extract textures from images by using the ROF model. An iterative regularization method (IRM)^[10–11], which replaces the regularization term by a generalized Bregman distance^[12], is proposed. This model is

$$u_{k+1} = \arg \min_{u \in BV(\Omega)} \left\{ J(u) + \frac{\lambda}{2} \|f + v_k - u\|_2^2 \right\} \quad (3a)$$

$$v_{k+1} = v_k + f - u_{k+1} \quad (3b)$$

A large λ corresponds to very little noise removal, and hence $u(x, y)$ is quickly close to $f(x, y)$ and the quality of image denoising is not effective. A small λ yields an over-smoothed $u(x, y)$ and the iterated times are enhanced. In spite of the sophistication of the recently proposed methods, most algorithms have not yet attained a desirable level of applicability^[13–15].

In this paper, we propose a new denoising method with the varying scale parameter, where the regularization item is

$$J(u) = \iint_{\Omega} |\nabla u| dx dy. \text{ We propose a method to obtain the}$$

scale parameter from the iterative regularization. Finally, some numerical examples are presented and show that the proposed method can improve the quality of the image denoising and reduce the optimal number of iterations.

1 Iterative Regularization Method

The iterative regularization method makes use of some signals in the removed residual part for the denoising algorithms. For $p \in \partial J(v)$, we define the non-negative quantity,

$$D^p(u, v) \equiv D_f^p(u, v) \equiv J(u) - J(v) - \langle p, u - v \rangle \quad (4)$$

Then, the equivalent representation of Eq. (3) is

$$u_{k+1} = \arg \min_{u \in BV(\Omega)} \left\{ D_f^p(u, v_k) + \frac{\lambda}{2} \|f + v_k - u\|_2^2 \right\} \quad (5a)$$

$$p_{k+1} = p_k + f - u_{k+1} \quad (5b)$$

where $u_0 = 0$, $v_0 = 0$ and $D_f^p(u, v)$ is the Bregman distance between u and v . As the optimal number of iterations k increases, u is close to the noisy image f . The scale parameter λ tunes the weight between the regularization term and the

Received 2009-11-17.

Biography: Li Wenshu (1975—), male, doctor, associate professor, wshlee@163.com.

Foundation items: The National Natural Science Foundation of China (No. 60702069), the Research Project of Department of Education of Zhejiang Province (No. 20060601), the Natural Science Foundation of Zhejiang Province (No. Y1080851), Shanghai International Cooperation on Region of France (No. 06SR07109).

Citation: Li Wenshu, Luo Jianhua, Liu Qiegen, et al. Iterative regularization method for image denoising with adaptive scale parameter [J]. Journal of Southeast University (English Edition), 2010, 26(3): 453 – 456.

fidelity term. The iterative refinement method yields a well-defined sequence of minimizers $\{u_k\}$ which satisfies $\|u_k - f\|_{L^2}^2 \leq \|u_{k-1} - f\|_{L^2}^2$ and if $f \in \text{BV}(\Omega)$, then $\|u_k - f\|_{L^2}^2 \leq J(f)/k$, i. e., u_k monotonically converges to f in $L^2(\Omega)$ at a rate of $1/\sqrt{k}$. For $g \in \text{BV}(\Omega)$ and $\gamma > 1$, we obtain

$$\begin{aligned} D(g, u_k) &\leq D(g, u_{k-1}) \\ \text{s. t. } \|u_k - f\|_{L^2} &\geq \gamma \|g - f\|_{L^2} \end{aligned}$$

Thus, the distance between a restored image u_k and a possible exact image g decreases until the L^2 -distance of f and u_k is greater than the L^2 -distance of f and g . This result can be used to construct a stopping rule for our iterative procedure.

2 IRM with Varying Scale Parameter

We know that for the IRM, the bigger the scale parameter λ is, the smaller the number of iterations is to the stopping criterion, but u is quickly close to the noise image f , the quality of the image denoising is not ideal. When the scale parameter λ is smaller, the number of the iterations will be enhanced. Therefore, it is important to choose an optimal value λ .

2.1 Varying scale parameter

Let $J(u) = \iint_{\Omega} |\nabla u| dx dy$, and differentiate both sides with respect to u for Eq. (3a). We obtain

$$\nabla \left(\frac{1}{|\nabla u|} \nabla u \right) + \lambda (f + v_k - u) = 0 \quad (6)$$

Multiplying Eq. (6) by $\nabla \left(\frac{1}{|\nabla u|} \nabla u \right)$ and integrating over x and y , we obtain

$$\begin{aligned} \iint_{\Omega} \nabla \left(\frac{1}{|\nabla u|} \nabla u \right) \nabla \left(\frac{1}{|\nabla u|} \nabla u \right) dx dy + \\ \lambda \iint_{\Omega} \nabla \left(\frac{1}{|\nabla u|} \nabla u \right) (f + v_k - u) dx dy = 0 \end{aligned} \quad (7)$$

Then, we obtain the following equation:

$$\lambda = \frac{\iint_{\Omega} \nabla \left(\frac{1}{|\nabla u|} \nabla u \right) \nabla \left(\frac{1}{|\nabla u|} \nabla u \right) dx dy}{\iint_{\Omega} \nabla \left(\frac{1}{|\nabla u|} \nabla u \right) (u - f - v_k) dx dy} \quad (8)$$

In numerical implementations, we accordingly use λ_{k+1} to denote λ in Eq. (8). Applying the proposed scale parameter to the IRM with initial values $u^0 = 0$, $v^0 = 0$, we obtain different scale parameters λ_{k+1} for different iterations. Eq. (3) can be written as

$$\lambda_{k+1} = \frac{\iint_{\Omega} \nabla \left(\frac{1}{|\nabla u_k|} \nabla u_k \right) \nabla \left(\frac{1}{|\nabla u_k|} \nabla u_k \right) dx dy}{\iint_{\Omega} \nabla \left(\frac{1}{|\nabla u_k|} \nabla u_k \right) (u_k - f - v_k) dx dy} \quad (9a)$$

$$u_{k+1} = \arg \min_{u \in \text{BV}(\Omega)} \{ |\nabla u| + \lambda_{k+1} \|f + v_k - u\|_2^2 \} \quad (9b)$$

$$v_{k+1} = v_k + (f - u_{k+1}) \quad (9c)$$

Eq. 9 (a) gives an adaptive value λ_{k+1} , which appears to converge as $k \rightarrow \infty$.

2.2 Initial scale parameter

By the numerical experiment, we find that the quality of image denoising is not ideal when the initial values $u^0 = 0$, $v^0 = 0$. If the initial condition holds, Eq. (9a) will be divided by zero.

If an initial scale parameter value λ_0 is randomly given, we calculate $\tilde{\lambda}_k$ by

$$\tilde{\lambda}_k = \frac{\iint_{\Omega} \nabla \left(\frac{1}{|\nabla u_k|} \nabla u_k \right) \nabla \left(\frac{1}{|\nabla u_k|} \nabla u_k \right) dx dy}{\iint_{\Omega} \nabla \left(\frac{1}{|\nabla u_k|} \nabla u_k \right) (u_k - f - v_k) dx dy} \quad (10)$$

After the iterations take some steps, we obtain the sequence vectors $\{\tilde{\lambda}_k\}$ and find that λ_k has some properties as follows:

1) The sequence vectors $\{\tilde{\lambda}_k\}$ monotonically decrease as the number of iterations k increases (see Fig. 1(c)).

2) As the number of iterations k increases, the sequence vectors $\{\tilde{\lambda}_k\}$ will first decrease, and then increase closely to

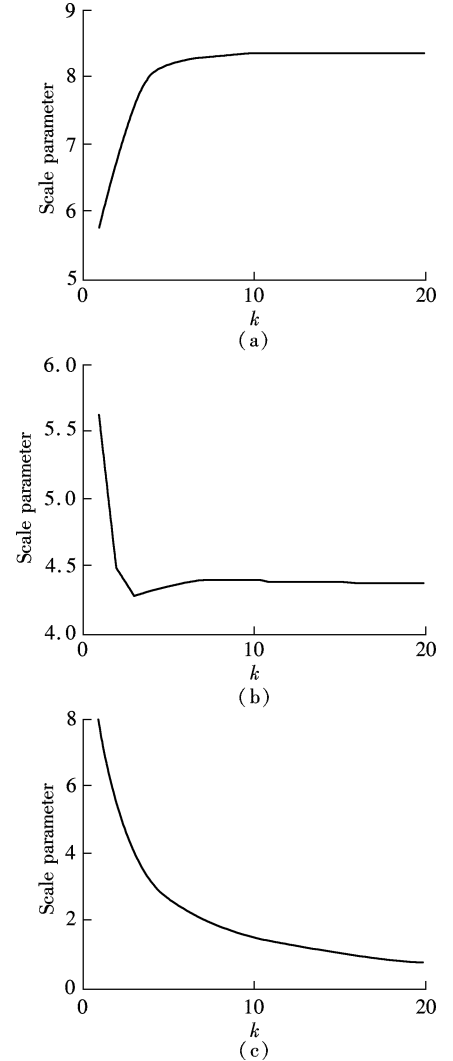


Fig. 1 Trend of the scale parameter λ_k as the number of iterations k increases. (a) Monotonically increasing; (b) Decreasing with change-point; (c) Monotonically decreasing

λ_0 (see Fig. 1(b)).

3) The sequence vectors $\{\tilde{\lambda}_k\}$ monotonically increase as the number of iterations k increases (see Fig. 1(a)).

Therefore, we can obtain the initial value of the varying scale parameter by the trend of the sequence vectors $\{\tilde{\lambda}_k\}$. In Fig. 1, taking the “Barbara” images for an example, the trend of the sequence vector $\{\tilde{\lambda}_k\}$ is obtained when the scale parameter λ_0 is 8.33, 4.34 and 0.013, respectively.

2.3 IRM framework with varying scale parameter

According to the above analyses in sections 2.1 and 2.2, our general iterative regularization procedure can be formulated as

$$\begin{aligned} \mathbf{u}_{j+1} &= \arg \min_{\mathbf{u} \in \text{BV}(\Omega)} \left\{ \mathbf{J}(\mathbf{u}) + \frac{\lambda_0}{2} \|\mathbf{f} + \mathbf{v}_j - \mathbf{u}\|_2^2 \right\} \\ \mathbf{v}_{j+1} &= \mathbf{v}_j + \mathbf{f} - \mathbf{u}_{j+1} \end{aligned} \quad (11)$$

Step 1 We randomly select λ_0 , and let $\mathbf{u}_0 = \mathbf{0}$, $\mathbf{v}_0 = \mathbf{0}$ and $j = 0, 1, 2, \dots$

1) According to Eq. (11) and Eq. (10), we calculate \mathbf{u}_{j+1} , \mathbf{v}_{j+1} and $\tilde{\lambda}_j$ by the number of iteration j . Generally, $j = 2$.

2) We observe the trend of the sequence vectors $\{\tilde{\lambda}_k\}$. According to the properties of section 2.2, we obtain the initial value λ_1 of the proposed method.

Step 2 Let $\mathbf{u}_0 = \mathbf{0}$, $\mathbf{v}_0 = \mathbf{0}$ and $k = 1, 2, \dots$

1) According to Eq. (9) and the initial value λ_1 , we cal-

culate \mathbf{u}_{k+1} , \mathbf{v}_{k+1} and λ_{k+1} by the number of iterations k ;

2) We obtain image \mathbf{u}_k and stop the iteration when $\|\mathbf{f} - \mathbf{u}_k\| \leq \sigma$.

3 Result and Discussion

All the solutions to the variational problem are obtained using the gradient descent in a standard fashion. Now we use the Chambolle algorithm^[8]. The only nontrivial difficulty comes when $|\nabla \mathbf{u}| \approx 0$. We solve the problem, as is usual, by perturbing $\mathbf{J}(\mathbf{u}) = \int_{\Omega} |\nabla \mathbf{u}| \, dxdy$ to $\mathbf{J}_{\varepsilon}(\mathbf{u}) =$

$\int_{\Omega} \sqrt{|\nabla \mathbf{u}|^2 + \varepsilon^2} \, dxdy$, where ε is the small positive number.

In our calculation, we set $\varepsilon = 10^{-12}$ and the iteration unit interval $\tau = 0.2$ for the Chambolle algorithm. The performance of the denoising algorithm is measured in terms of peak signal-to-noise-ratio (PSNR), which can be defined as

$$\text{PSNR} = 10 \lg \left(\frac{255^2}{\frac{1}{MN} \sum_{m=1}^M \sum_{n=1}^N (f_{mn} - u_{mn})^2} \right) \quad (12)$$

where \mathbf{f} is the original image and \mathbf{u} is the denoising image.

Fig. 2 and Fig. 3 display the results with the constant and the varying scale parameters of the IRM with the “Camera-men” images adding the Gauss noise $\sigma = 20$ when λ is smaller and bigger, respectively.

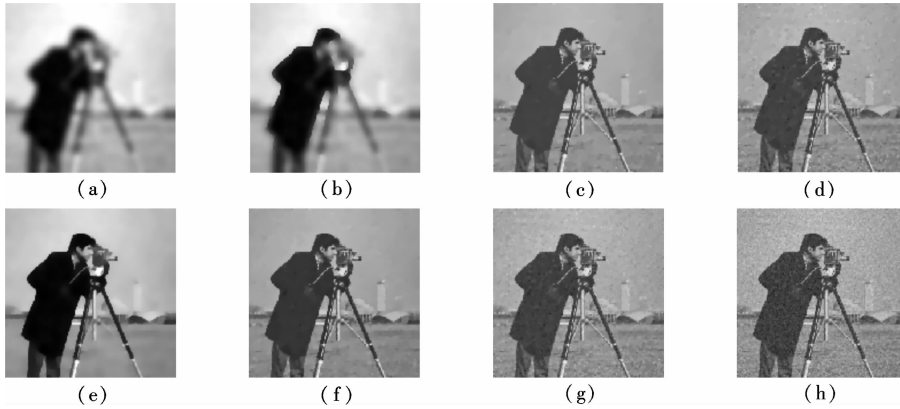


Fig. 2 Denoising images obtained by constant scale parameter for IRM and proposed method when λ becomes smaller. (a) $k = 1$, PSNR = 19.7 dB; (b) $k = 5$, PSNR = 25.1 dB; (c) $k = 15$, PSNR = 30 dB; (d) $k = 21$, PSNR = 28.8 dB; (e) $k = 1$, PSNR = 25 dB; (f) $k = 2$, PSNR = 30.2 dB; (g) $k = 3$, PSNR = 28.3 dB; (h) $k = 5$, PSNR = 25.4 dB

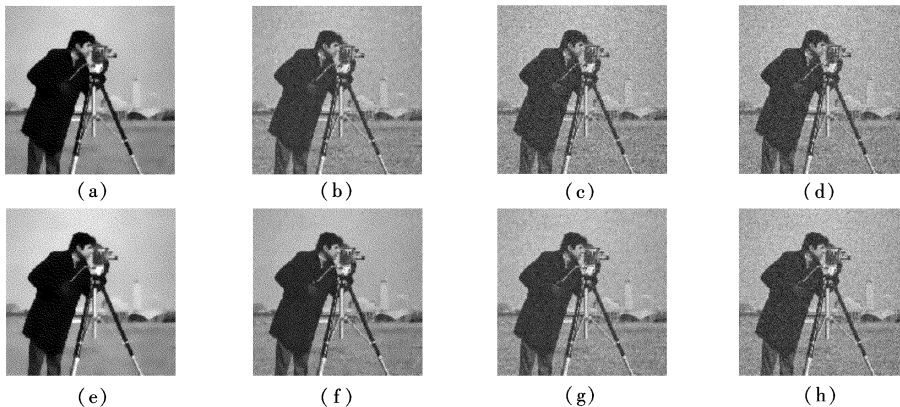


Fig. 3 Denoising images obtained by constant scale parameter for IRM and proposed method when λ becomes greater. (a) $k = 1$, PSNR = 27.2 dB; (b) $k = 2$, PSNR = 28.7 dB; (c) $k = 15$, PSNR = 23.5 dB; (d) $k = 21$, PSNR = 23.5 dB; (e) $k = 1$, PSNR = 26.3 dB; (f) $k = 2$, PSNR = 30.2 dB; (g) $k = 3$, PSNR = 28.4 dB; (h) $k = 5$, PSNR = 25.6 dB

Figs. 2 (a) to (d) show that more iteration steps are required to stop criterion with the smaller scale parameter $\lambda_0 = 0.67$; Figs. 2 (e) to (h) show that the proposed method requires fewer iteration steps to obtain the optimal denoising results. First, we use the constant scale parameter $\lambda_0 = 0.67$ to iterate three times and obtain a decreased sequence vector $\{\tilde{\lambda}_k\}$. According to Eq. (11), we obtain the initial value $\lambda_1 = \tilde{\lambda}_2 = 5.74$. The last two plots show that $\|\mathbf{f} - \mathbf{u}_k\|_{L_2}$ monotonically decreases with the iterations, first dropping below σ at the optimal iterate $k = 12$ and 3 , respectively. It shows that the proposed method converges faster than the IRM with the constant scale parameter.

As shown in Fig. 3, with the large scale parameter $\lambda_0 = 10$, the original IRM converges to the noisy image \mathbf{f} quickly, and only one iteration is needed to reach the requirement. Obviously, the denoising result is not satisfied. However, a promising result is obtained by the varying scale parameter strategy where the initial value $\lambda_1 = \tilde{\lambda}_1 = 7.84$ according to Eq. (11).

4 Conclusion

We propose a novel iterative regularization model with the varying scale parameter. A new iterative scale parameter λ is obtained according to the trend of the sequence vectors. Generally, we can obtain the initial scale parameter λ using three steps of the iteration. By practical examples, we can see that the proposed method can reduce the number of iterations. Thus, a fast and robust method is obtained.

References

- [1] Rudin L, Osher S, Fatemi E. Nonlinear total variation based noise removal algorithms [J]. *Physica D*, 1992, **60**(1/2/3/4): 259–268.
- [2] DeVore R, Jawerth B, Lucier B. Image compression through wavelet transform coding [J]. *IEEE Trans Information Theory*, 1992, **38**(2): 719–746.
- [3] Donoho D L. Denoising by soft-threshold [J]. *IEEE Trans Information Theory*, 1995, **41**(3): 613–627.
- [4] Osher S, Solé A, Vese L. Image decomposition and restoration using total variation minimization and the $H - 1$ norm [J]. *SIAM Journal on Multiscale Modeling and Simulation*, 2003, **1**(3): 349–370.
- [5] Burger M, Gilboa G, Osher S, et al. Nonlinear inverse scale space methods [J]. *Communications in Mathematical Science*, 2006, **4**(1): 175–208.
- [6] Chambolle A, DeVore R A, Lee N Y, et al. Nonlinear wavelet image processing: variational problems, compression, and noise removal through wavelet shrinkage [J]. *IEEE Trans Image Processing*, 1998, **7**(3): 319–335.
- [7] Chambolle A, Lucier B J. Interpreting translation-invariant wavelet shrinkage as a new image smoothing scale space [J]. *IEEE Trans Image Processing*, 2001, **10**(7): 993–1000.
- [8] Steidl G, Weickert J, Brox T, et al. On the equivalence of soft wavelet shrinkage, total variation diffusion, total variation regularization, and SIDs [J]. *SIAM Journal on Numerical Analysis*, 2004, **42**(2): 686–713.
- [9] Xu J, Osher S. Iterative regularization and nonlinear inverse scale space applied to wavelet-based denoising [J]. *IEEE Trans Image Processing*, 2007, **16**(2): 534–544.
- [10] Meyer Y. *Oscillating patterns in image processing and nonlinear evolution equations* [M]. Boston: American Mathematical Society, 2001: 42–45.
- [11] Osher S, Burger M, Goldfarb D, et al. An iterative regularization method for total variation based image restoration [J]. *Multiscale Modeling and Simulation*, 2005, **4**(2): 460–489.
- [12] Bregman L. The relaxation method of finding the common points of convex sets and its application to the solution of problems in convex programming [J]. *USSR Computational Mathematics and Mathematical Physics*, 1967, **7**(3): 200–217.
- [13] Hao B B, Li M, Feng X C. Wavelet iterative regularization for image restoration with varying scale parameter [J]. *Signal Processing: Image Communication*, 2008, **23**(6): 433–441.
- [14] Liu B, King K, Steckner M, et al. Regularized sensitivity encoding (SENSE) reconstruction using Bregman iterations [J]. *Magnetic Resonance in Medicine*, 2009, **61**(1): 145–152.
- [15] Chambolle A. An algorithm for total variation minimization and applications [J]. *Journal of Math, Imaging and Vision*, 2004, **20**(1/2): 89–97.

一种变尺度参数的迭代正则去噪算法

李文书^{1,2} 骆建华¹ 刘且根¹ 何芳芳² 魏秀金²

(¹ 上海交通大学生命科学技术学院, 上海 200240)

(² 浙江理工大学信电学院, 杭州 310018)

摘要: 为了降低迭代正则化中定尺度参数对快速收敛的敏感性、自适应地优化尺度参数并提高其去噪效果, 提出了一种变尺度参数的迭代正则化去噪算法。首先, 修改了经典的正则化项, 并推导出尺度参数公式; 然后, 通过研究迭代次数与尺度参数序列的变化趋势, 得到变尺度参数的初始值; 最后, 进行正则化去噪。数值实验表明: 相对于恒定尺度参数的 IRM 算法, 变尺度参数 IRM 算法比选取尺度参数偏小的 IRM 算法迭代次数大大减少; 比选取尺度参数偏大的 IRM 算法去噪效果更为明显, 并较好地保持了图像的细节。

关键词: 迭代正则化模型 (IRM); 总变差; 变尺度参数; 图像去噪

中图分类号: TP391.4