Journal of Southeast University (English Edition) Vol. 26, No. 3, pp. 471 - 474 Sept. 2010 ISSN 1003—7985

Bi-level programming model and algorithm for optimizing headway of public transit line

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Abstract: Due to the fact that headway is a key factor to be considered in bus scheduling, this paper proposes a bi-level programming model for optimizing bus headway in public transit lines. In this model, with the interests of bus companies and passengers in mind, the upper-level model's objective is to minimize the total cost, which is affected by frequency settings, both in time and economy in the transit system. The lower-level model is a transit assignment model used to describe the assignment of passengers' trips to the network based on the optimal bus headway. In order to solve the proposed model, a hybrid genetic algorithm, namely the genetic algorithm and the simulated annealing algorithm (GA-SA), is designed. Finally, the model and the algorithm are tested against the transit data, by taking some of the bus lines of Changzhou city as an example. Results indicate that the proposed model allows supply and demand to be linked, which is reasonable, and the solving algorithm is effective.

Key words: headway; bi-level model; transit assignment; hybrid genetic algorithm

As an important part of scheduling operations, headway determination of the public transit line plays a fundamental role in the public transportation system, and it determines other scheduling aspects, such as the setting of timetables, vehicle scheduling and driver assignment. Foreign bus enterprises usually operate in regions. However, the transit line operating mode is widely applied in China. According to the situation of China, the model in this paper treats bus operating cost on a per-vehicle-per-stop basis. Up until now, some researchers have made many attempts at determining transit frequency settings[1-2]. Trip frequency scheduling is studied by taking into account the present practices, limited bus fleet size and lack of parking spaces[3]. Some transit vehicle real-time scheduling models and algorithms are also studied[4]. Most of these scheduling problems are NP-complete or NP-hard problems; therefore, heuristic algorithms are usually applied. A genetic algorithm is more applicable, and a hybrid genetic algorithm is designed for the proposed model. In this paper, it is assumed that the origin-destination (OD) trip demand is given. All the travelers have full predictive information about present and future conditions in the given transit network and select paths that minimize trip costs.

1 Bi-Level Programming Model

1.1 Upper model[5]

Usually, both the passenger and the enterprise's interests are considered, although there is a conflict between the two. Generally, there are three aspects of impact factors: the distribution of passenger flow, the vehicle condition and the environmental condition. In order to analyze the process, some assumptions are made: 1) A one-way trip is selected; 2) Transit passenger demand is independent of the bus headway, as well as the operating condition; 3) All the transit vehicles have the same fixed capacity and operate precisely as specified in preset timetables; 4) Passenger queues at platforms follow the single channel FIFO rule; 5) Dwell time at each stop is determined by the number of passengers' getting on and off the bus; 6) A passenger would not leave the platform, and she or he would not wait more than two times; 7) All the vehicles operate normally, and accidents, such as congestion, traffic accidents and others, are not considered.

From the above assumptions, it is easy to obtain the departure time of vehicle j ($j = 1, 2, \dots, N$, and N is the total departure number of the transit line in a given period) from the first stop i ($i = 1, 2, \dots, V$, and there are V stops on the line), that is T_j^1, which is equal to $T^\circ + \sum_{k=1}^{j} t_k$, where T_j^i is the departure time from stop i of vehicle j, T° is the initial time of the entire period, t_j is the headway between vehicle $j-1$ and j. The ΔT is the entire period of analysis. So the j-th vehicle departure time from the i-th stop is equal to the start time from the start stop plus the traveling time between stops plus the sum of the dwell time at each stop,

$$T_j^i = T_j^1 + \sum_{l=1}^{i} S_l^j + \sum_{l=1}^{i} R^l \tag{1}$$

where S_i^j is the vehicle j's dwell time at stop i, which is equal to the maximum boarding time $uB_j^{i,i+s}$ or alighting time $d\sum_{l=1}^{i-1} W_j^{l,i}$. Here, u is the average time that passengers board the bus, and d is the average time passengers that alight from the bus. $B_j^{i,i+s}$ is the available capacity of vehicle j from stop i to stop $i+s$. $W_j^{i,i+s}$ is the number of passengers who go for stop $i+s$ when the j-th vehicle departs from stop i. R^i is the time that a vehicle spends between departure from stop $i-1$ and arrives at i, $R^1 = 0$. When the j-th vehicle arrives at stop i, passengers who wait for stop $i+s$ can be evaluated by

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Received 2009-11-11.

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Foundation items: The National Natural Science Foundation of China(No. 50978057), the National Key Technology R&D Program of China during the 11th Five-Year Plan Period (No. 2006BAJ18B03), the Scientific Research Foundation of Graduate School of Southeast University (No. YBJJ1013), the Program for Postgraduates Research Innovation in University of Jiangsu Province(No. CX09B_060Z).

Citation: Zhang Jian, Li Wenquan. Bi-level programming model and algorithm for optimizing headway of public transit line[J]. Journal of Southeast University (English Edition), 2010, 26(3): 471 - 474.

$$W_j^{i,i+s} = W_{j-1}^{i,i+s} - B_{j-1}^{i,i+s} + \sum_{s=1}^{V-i} \int_{T_{j-1}^i}^{T_j^i} D^{i,i+s}(t)\ \mathrm{d}t \qquad (2)$$

where $D^{i,i+s}(t)$ is integrable, because it is an approximately continuous progress that passengers arrive at stop i for stop $i+s$. Passengers are composed of two parts: the remaining passengers of vehicle $j-1$ due to its capacity constraints, and the new arrival of passengers. If the available capacity is larger than the number of passengers waiting for the bus, all the passengers will get on the vehicle; otherwise, only some of them will.

$$B_j^{i,i+s} = \min\left\{W_j^{i,i+s},\ W_j^{i,i+s} \frac{P - \sum_{l=1}^{i-1}\sum_{s=1}^{V-i} B_j^{l,i+s}}{\sum_{s=1}^{V-i} W_j^{i,i+s}}\right\} \qquad (3)$$

where P is the rated passenger capacity. Taking passengers' personal space into consideration, there is a maximum rated capacity P_2, so that people in the vehicle will not feel too crowded. Contrarily, there is a minimum rated passenger capacity P_1 to ensure the profit of the enterprises. That is because more personal space means more trips by bus, and a greater cost of operation. To balance these two profits, we have

$$P_1 \leqslant \frac{\sum_{j=1}^{N}\sum_{i=1}^{V-1}\sum_{l=1}^{i}\sum_{s=1}^{V-i} B_j^{l,i+s}}{NP(V-1)} \leqslant P_2 \qquad (4)$$

In the interests of the passengers, the overall waiting time is minimized to be the objective function. There are two cases: All the passengers get on the vehicle when the j-th bus arrives at stop i, or the vehicle's remaining capacity is less than that of the number of the waiting passengers. There is a function of waiting time,

$$C_{\mathrm{p}} = \sum_{i=1}^{V-1}\sum_{s=1}^{V-i}\sum_{j=1}^{N}\int_{T_{j-1}^i}^{T_j^i} D^{i,i+s}(t)(T_j^i - t)\,\mathrm{d}t + \sum_{i=1}^{V-1}\sum_{s=1}^{V-i}\sum_{j=1}^{N-1}(W_j^{i,i+s} - B_j^{i,i+s})(T_{j+1}^i - T_j^i) \qquad (5)$$

With the interests of the enterprise in mind, the variable economic cost is the function's objective, i. e.,

$$C_{\mathrm{c}} = NC_1 - \sum_{i=1}^{V-1}\sum_{s=1}^{V-i}\sum_{j=1}^{N} B_j^{i,i+s} C_2 \qquad (6)$$

where C_1 is the cost of one vehicle for one trip and C_2 is the ticket cost for one passenger once.

From the perspective of general social benefits, the final objective function is denoted as

$$\min\ X_1 C_{\mathrm{p}} + X_2 C_{\mathrm{c}} \qquad (7)$$

where X_1 and X_2 are non-negative conversion factors.

1.2 Lower model

As the upper model has taken into account congestion, the lower model is developed based on the distribution of a transit network under a non-congested condition. A passenger's trip encompasses three components: in-vehicle time, waiting time, and walking time. The waiting time that a passenger spends on a stop or station is determined by attracting lines. The principle of a passenger's chosen path is her/his expected time cost, i. e. the shortest travel time. Similarly, all the travelers are assumed to have full predictive information about the headway of each current line. The transit distribution model is as follows:

$$\min \sum_{\alpha\in A} c_\alpha v_\alpha + \sum_{i\in I} w_i$$

$$\text{s. t.}$$

$$\sum_{\alpha\in A_i^+} v_\alpha - \sum_{\alpha\in A_i^-} v_\alpha = g_i \qquad i \in I$$

$$w_i = \frac{V_i}{\sum_{\alpha\in A_i^+} f_\alpha x_\alpha}$$

$$v_\alpha \leqslant f_\alpha w_i \qquad i\in I, \alpha\in A_i^+$$

$$v_\alpha \geqslant 0 \qquad \alpha \in A$$

$$x_\alpha = 0 \text{ or } 1 \qquad \alpha\in A$$

where α is the arc segment; A is the set of arc segments; c_α is the travel time on α; v_α is the transit flow on α; i is the node; I is the set of nodes; w_i is the total waiting time at i; f_α is the united departure frequency (the sum of the frequency of attracting line on arc α); g_i is the transit flow produced from node i; V_i is the transit flow produced from node i; A_i^+ is the set of line that departs from node i; A_i^- is the set of lines that enters node i.

The new transit flow obtained from the distribution of the lower model is used to optimize the bus headway of each line. It is worth noting that the trip gives a clear end point, rather than a starting point, and the strategy[6] is a rule that allows passengers to reach their end node from each node in the network end.

2 Solution

The genetic algorithm (GA)[7] has the advantage in the overall search process, and the simulated annealing algorithm (SA) has the advantage of a strong local search ability. Thus, a hybrid genetic algorithm is presented. The basic idea of GA-SA to solve the bi-level programming model is: Code the decision variable of the upper model, calculate the fitness function for each string by the lower model, reproduce, cross, mutate, simulate annealing and pause test, and obtain the best string. Specific algorithm steps are as follows:

Step 1 Initialization: 1) Determine the population of chromosomes (individuals) for each generation, N; the crossover probability P_{c}; the mutation probability P_{m}; the biggest generation GenMax in the GA, and let gen = 0; 2) Determine the generations of internal circulation M in the SA, initialize temperature T_0, let $T = T_0$; 3) The objective function Eq. (7) is the fitness function, and the bus headway, i. e. a chromosome, is coded. N feasible solutions such as $X(1) = (\dots, x_i(1), \dots)$ are randomly produced to compose the initial population, where $x = x_{\min} + \left(\sum_{i=1}^{\beta} b_i \times 2^{i-1}\right)\frac{x_{\max} - x_{\min}}{2^n - 1}$ (b_i is the i-th gene of the binary string, $b_i = 0$ or 1), let gen = 1.

Step 2 X(gen) are introduced into the lower model, and they are calculated by the distribution method. The fitness of each individual $x_i(\text{gen})$ $(i = 1, 2, \ldots, N)$ is obtained. If gen = GenMax, the biggest fitness chromosome is the optimal solution; otherwise, return to step 3.

Step 3 Selection operation. Reproduce the population $X(\text{gen})$ according to the distribution of fitness. The Monte Carlo approach is applied.

Step 4 Crossover operation. Cross two parent strings according to the crossover probability. In this paper, we adopt the partially matched crossover (PMX) method here.

Step 5 Mutation operation. Randomly select a gene to operate, and let gen = gen + 1. Then obtain a new population $X(\text{gen})$, and calculate its fitness.

Step 6 Let $i = 1$, carry out the simulated annealing operation to population $X(\text{gen})$:

1) If $i = N$, go to step 7; otherwise let internal circulation $k = 1$, return to 2);

2) Produce a new state of the individual $x_i(\text{gen})$ by the state function; decode this new individual and introduce it into the lower model in order to obtain the objective function value of the upper model;

3) Accept the new individual by the metropolis acceptance probability;

4) If $k = M$, let $i = i + 1$, and return to 1); otherwise, let $k = k + 1$; and return to 2).

Step 7 Let $T = T_0 a^k$, a is a constant between 0 and 1; return to step 2.

3 Case Study

Parts of bus lines in Changzhou city (1, 6, 20, 62, 320, 901) in one-way direction are selected[8]. The examined period is the morning peak. The transit OD flow and the count of passengers that get on and off at stops are collected. The experiment was run on an Intel Core 2/2.71 GHz, 2 GB Ram the computer, using tool VC++ 6.0. Parameters' values of model and the algorithm are shown as follows: $N = 200$, $P_c = 0.95$, $P_m = 0.01$, GenMax = 200, $M = 100$, $T_0 = 5\,000$, $a = 0.5$, $T^0 = 700$, $\Delta T = 60$, $P_1 = 0.35$, $P_2 = 1.5$, $P = 75$ person, $C_1 = 70$ yuan/once, $C_2 = 1$ yuan/person, $X_1 = X_2 = 0.5$, $u = 5$ s, $d = 4$ s. The average frequencies of lines before and after the optimization are compared, as shown in Fig. 1.

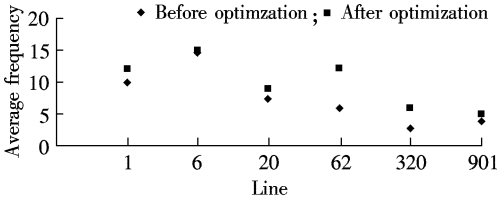

Fig. 1 Comparison of the average frequencies of routes

Results show that, according to the investigated flow data, the average headway of bus lines is at a low level in the network example. In view of the complexity of costs (for example, as different passengers have varying time costs), cost comparison is not taken into account. Due to limitations, only a set of more appropriate outcomes is adopted after many tests.

In order to analyze the proposed algorithm's performance, a test function is introduced: $X_e(s) = \dfrac{1}{T}\sum_{t=1}^{T} f_e(t)$, where $f_e(t) = \text{best}\{f_e^{(1)}, f_e^{(2)}, \ldots, f_e^{(t)}\}$. $X_e(s)$ is defined as the offline performance of strategy s in the environment e. From Fig. 2, it can be seen that the offline performance of the suggested algorithm is better.

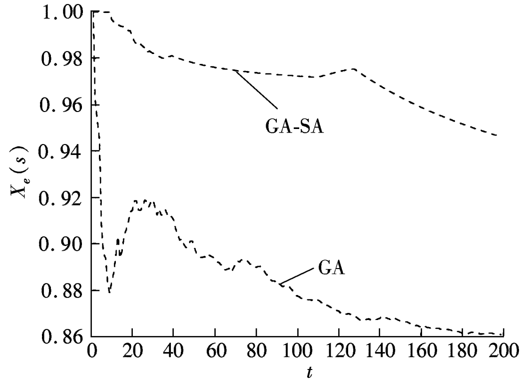

Fig. 2 Comparison of offline performances

4 Conclusion

A bi-level programming model for optimizing bus headway of a transit line in a public transit network is described. The GA-SA solving algorithm is designed, and a numerical simulation is illustrated. The transit flow decides the headway, and the headway reacts to the passengers' path choice, then the cycle repeats. The assumption of a non-congested condition merely stays at a theoretical level, but in the real world, congestion phenomena take place everywhere. The station's (stops') capacity of vehicle and traffic in an urban environment, such as at the intersections, will be studied in the future. In most cities, especially in developing countries, there are usually phenomena of a collinear nature in the transit network. If passengers know the timetable in advance, and also know the services running on the exact situation in accordance with the schedule, the public transit service will be better.

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公交线路发车间隔优化的双层规划模型与算法

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摘要:针对发车间隔这一公交车辆调度中需要考虑的关键因素,提出了一种优化线路发车间隔的双层规划模型. 该模型从公交公司和乘客两者利益出发,上层模型以最小化公交系统中因发车频率设置而产生的时间和经济总成本为目标;下层模型是公交客流分配模型,用来描述发车间隔优化后的客流分配情况. 设计了一种混合遗传算法,即模拟退火遗传算法,来求解模型. 最后,以常州市某几条线路为例,利用公交数据对模型和算法进行了检测. 结果表明:所提出的线路发车间隔优化模型体现了公交供需关系是合理的,而且求解算法是有效的.

关键词:发车间隔;双层模型;公交客流分配;混合遗传算法

中图分类号:U419