Journal of Southeast University (English Edition) Vol. 26, No. 3, pp. 475 – 479 Sept. 2010 ISSN 1003—7985

Simulation of pavement roughness based on time domain model

Niu Kaijian Li Chang

(School of Transportation, Southeast University, Nanjing 210096, China)

Abstract: In order to describe pavement roughness more intuitively and effectively, a method of pavement roughness simulation, i. e., the stochastic sinusoidal wave, is introduced. The method is based on the primary idea that pavement roughness is denoted as the sum of numerous sines or cosines with stochastic phases, and uses the discrete spectrum to approach the target stochastic process. It is a discrete numerical method used to simulate pavement roughness. According to a given pavement power spectral density (PSD) coefficient, under the condition that the character of displacement frequency based on the time domain model is in accordance with the given pavement surface spectrum, the pavement roughness is optimized to stochastic equivalent vibrations by computer simulation, and the curves that describe pavement roughness under each grade are obtained. The results show that the stochastic sinusoidal wave is suitable for simulation of measured pavement surface spectra based on the time domain model. The method of the stochastic sinusoidal wave is important to the research on vehicle ride comfort due to its rigorous mathematical derivation, extensive application range and intuitive simulation curve. Finally, a roughness index defined as the nominal roughness index (NRI) is introduced, and it has correlation with the PSD coefficient.

Key words: pavement roughness; stochastic sinusoidal wave; stochastic process; power spectral density (PSD) coefficient; time domain; nominal roughness index (NRI)

The measurement methods have shown that pavement roughness can be modeled to the stochastic field. Pavement roughness varies not only in amplitude but also in frequency at different road sections. Neither the three-meter ruler nor the international roughness index (IRI) can directly describe the stochastic variations of pavement roughness. The statistical model in the stochastic vibration theory can describe comprehensive performance of pavement roughness better[1−2].

In electronic engineering, the power spectral density (PSD) is used to describe the distribution of electric power in frequency bands. In road and vehicle engineering, PSD roughness is primarily used to evaluate vehicle response, suspension optimization and control, dynamic pavement loading, and energy consumption[3−7]. As an intuitive representation method, PSD roughness can describe the variations of pavement roughness more explicitly than the IRI. Recently, PSD roughness has been proposed to replace the IRI[1, 8]. In general cases, some assumptions will be put forward when PSD roughness is applied to structure the stochastic statistical model. Pavement roughness is regarded as conforming to stationarity assumptions of the vibration-source stochastic field, which is a zero mean Gaussian isotropic random field in the spatial domain and is a normal stationary ergodic random process in the time domain[9]. For many years, researchers have proposed various pavement roughness models based on either time domain or spatial domain with their respective advantages and disadvantages. Among them, the method of the stochastic sinusoidal wave is based on the idea of making the discrete spectrum approach the target stochastic process and it is a discrete numerical method used to simulate pavement roughness. So, this method can be used to simulate the stationarity stochastic process with any sharp spectrum density.

1 Stochastic Process and Its Relevant Concepts

1.1 Stochastic process and stochastic vibration

In nature, there are two kinds of variation processes: One is a deterministic process which can be described with a deterministic function; the other is not deterministic and cannot be described with a deterministic function. This kind of variation process is called a stochastic process.

Stochastic vibration is defined as a dynamic response caused by stochastic excitation. In structural dynamics, the excitation is called load. In many engineering examples, excitations are uncertain. For instance, when a car is running on the road, the excitation transmitted from pavement to vehicle is not a deterministic excitation.

1.2 Stationarity stochastic process

In practice, we often encounter a kind of stochastic process such as water stochastic flows with their amplitudes and vibration characteristics almost invariant. Such a process is called a stationarity stochastic process, or stationarity process for short.

Generally, the stochastic process in the dynamic system is not stationarity at first, i. e., the so-called transition process. When this process disappears and becomes stationary, it can be regarded as a stationarity stochastic process.

The stationarity stochastic process has some characteristics in numerical properties:

1) The mathematical expectation of the stationarity process is a constant;

2) The variance is also a constant, unchanging over time;

3) The auto-correlation function is just related to a single variable.

1.3 Ergodicity

As we know, the statistical characteristics of the stationarity process are independent of the selection of the time ori-

Received 2010-04-02.

Biographies: Niu Kaijian (1985—), male, graduate; Li Chang (corresponding author), male, doctor, associate professor, lichang_seu@163.com.

Citation: Niu Kaijian, Li Chang. Simulation of pavement roughness based on time domain model[J]. Journal of Southeast University (English Edition), 2010, 26(3): 475 −479.

gin. A sample function of the stochastic process can represent the whole stochastic process when the length of recording time is enough. So, the average value of any sample function in the whole timeline can displace the ensemble average. It is proved that when some conditions are met, a stochastic process possesses the characteristic of so-called ergodicity.

It can be verified theoretically whether a process is ergodic. But the verification is difficult. So far as we know, most of the stochastic processes in engineering are ergodic. For instance, the sample which comes from pavement longitudinal profile height data is usually a stationarity stochastic sample with the characteristic of ergodicity.

2 Pavement Roughness and PSD Roughness

2.1 Pavement roughness

Pavement roughness is an important index which is used to evaluate the quality of pavement. Pavement roughness is related to driving comfort and safety, and it is a comprehensive index related with people, vehicles and roads. The American Society of Testing Materials (ASTM) defines pavement roughness as that the traveled surface roughness is the vertical deviation of the pavement surface relative to the ideal plane. This deviation will affect vehicle dynamic characteristics, driving quality, dynamic load and drainage[10]. Three roughness evaluation indices: ruler maximum clearance h, roughness standard deviation σ and the IRI are given in Ref. [11]

Because pavement roughness is a non-physical quantity and the definition of this quantity is related to the measurement method, different results come out due to different definitions or different measurement methods. At present, the common measurement methods are section type, reaction type and subjective evaluation type.

2.2 Description of PSD roughness

For a stochastic process, its sample function usually does not satisfy the absolutely integrable condition of the Fourier transform. So, a stochastic process cannot be directly used with a Fourier transform. The usual method is used to research the PSD roughness and now it becomes a practical and effective method.

Pavement roughness is not a function of time frequency or angular frequency, but it is a power spectral density based on spatial frequency. When the variable of the time domain is time, the time domain has a unit of s. When the variable is road length, i. e., the so-called spatial domain, the spatial domain has a unit of m. The frequency of the time variable is represented as f with the unit of Hz, while the frequency of the length variable is represented as n with the unit of m^{-1}.

The spatial frequency n is the reciprocal of the pavement surface spatial wavelength λ; i. e. $\lambda = 1/n$. The relationship among spatial frequency, time frequency and angular frequency is such that

$$n = \frac{f}{v} = \frac{\omega}{2\pi v} \tag{1}$$

where ω represents the angular frequency, and v is the speed.

It is indicated by many measurements that pavement roughness ξ is a zero mean Gaussian isotropic random field in the spatial domain, which is regarded as meeting the assumptions of the vibration source stochastic field and it becomes a normal stationary ergodic random process in the time domain. According to the Wienner-Khintchine theory, the following forms constitute a couple of the Fourier transform:

$$S_\xi(n) = \int_{-\infty}^{\infty} R_\xi(X)\,\mathrm{e}^{-\mathrm{i}2\pi nX}\mathrm{d}X \tag{2a}$$

$$R_\xi(X) = \int_{-\infty}^{\infty} S_\xi(n)\,\mathrm{e}^{\mathrm{i}2\pi nX}\mathrm{d}n \tag{2b}$$

where X represents the distance of two points along the pavement; $S_\xi(n)$ is the PSD roughness; n represents the spatial frequency. $R_\xi(X)$ is the spatial auto-correlation function and is defined as

$$R_\xi(X) = E[\xi(x)\xi(x+X)] \tag{3}$$

where $E[\cdot]$ represents the expectation of a stochastic process that can be estimated from

$$E[\xi(x)] = \lim_{X\to\infty}\frac{1}{X}\int_0^X \xi(x)\,\mathrm{d}x \tag{4}$$

If a stochastic process which describes pavement roughness is $\xi(t)$, PSD roughness $S_\xi(f)$, the function of time frequency f, is described by

$$S_\xi(f) = \int_{-\infty}^{\infty} R_\xi(t)\,\mathrm{e}^{-\mathrm{i}2\pi ft}\mathrm{d}t \tag{5}$$

where t represents the time lag, and $R_\xi(t)$ is the temporal auto-correlation function.

As $\omega = 2\pi f$, PSD roughness $S_\xi(\omega)$, the function of angular frequency ω, is described by

$$S_\xi(\omega) = (2\pi)^{-1}\int_{-\infty}^{\infty} R_\xi(t)\,\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}t \tag{6}$$

and $S_\xi(f) = 2\pi S_\xi(\omega)$.

Assume that a vehicle moves along a pavement of length X at constant speed v. The relationship of length and speed is $X = vt$, where t is the time. Based on Eq. (1), we obtain

$$S_\xi(f) = \int_{-\infty}^{\infty} R_\xi\left(\frac{X}{v}\right)\mathrm{e}^{-\mathrm{i}2\pi(vn)X/v}\frac{1}{v}\mathrm{d}X = \frac{1}{v}\int_{-\infty}^{\infty} R_\xi\left(\frac{X}{v}\right)\mathrm{e}^{-\mathrm{i}2\pi nX}\mathrm{d}X = \frac{1}{v}S_\xi(n) \tag{7}$$

And the relationship among $S_\xi(n)$, $S_\xi(f)$ and $S_\xi(\omega)$ is

$$S_\xi(n) = vS_\xi(f) = 2\pi v S_\xi(\omega) \tag{8}$$

2.3 Classification of PSD

The file ISO/TC108/SC2N67 put forward by the International Organization for Standardization and the Representative Model of the Vehicle Vibration—the Plainness of the Road Surface (GB 7031—1986)[12] drafted by Changchun Vehicle Institute all suggest that the pavement power spectral density $G_\xi(n)$ should be described by

$$G_\xi(n) = G_\xi(n_0)\left(\frac{n}{n_0}\right)^{-m} \tag{9}$$

where n represents the spatial frequency, which is the reciprocal of the spatial wavelength λ; n_0 is the referenced spatial frequency, i. e. $n_0 = 0.1\ \text{m}^{-1}$; $G_\xi(n_0)$ is the PSD coefficient with the unit of m^2/m^{-1}; m is the frequency index. In this paper, the value of m is 2.

The two files mentioned above state that pavement roughness can be classified into eight classes according to PSD roughness. Tab. 1 lists the geometric means of the PSD coefficient $G_\xi(n_0)$ when $n_0 = 0.1\ \text{m}^{-1}$ and the PSD root mean square σ_ξ in the range of $0.011\ \text{m}^{-1} < n < 2.83\ \text{m}^{-1}$. The classification of pavement roughness is shown in Fig. 1. Statistics show that the road surface spectrum in China is almost in the range of grades A, B and C.

Tab. 1 Classification standard of pavement roughness

Grade	$G_\xi(n_0)/\text{cm}^3$	σ_ξ/cm^2
A	16	38. 1
B	64	76. 1
C	256	152. 3
D	1 024	304. 5
E	4 096	609. 0
F	16 384	1 218. 0
G	65 535	2 436. 1
H	262 144	4 872. 2

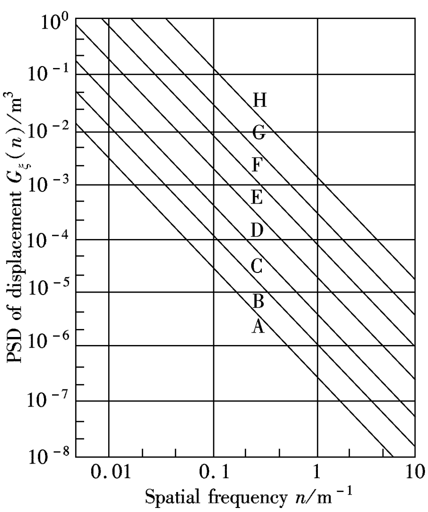

Fig. 1 Classification of pavement roughness

3 Establishment of Time Domain Model

3.1 Simulation

The method of the stochastic sinusoidal wave is based on the idea of making the discrete spectrum approximate to the target stochastic process. It is a discrete numerical method used to simulate pavement roughness. The primary idea is that the pavement roughness is denoted as the sum of numerous sines or cosines with stochastic phases. This model can simulate stochastic processes with any shape spectrum density, and the results of samples are continuous.

Assume that the displacement spectrum density is $G_\xi(f)$ when the time frequency is $f_1 < f < f_2$. According to the stochastic process theory, variance of pavement roughness can be described as

$$\sigma_\xi^2 = \int_{f_1}^{f_2} G_\xi(f)\,\mathrm{d}f \tag{10}$$

The interval (f_1, f_2) should be divided into n small intervals. The value of $G_\xi(f)$ in the whole small interval is substituted by the spectrum density value $G_\xi(f_{\text{mid},i})$ with a middle frequency $f_{\text{mid},i}(i = 1, 2, \ldots, n)$ in each small interval. So, after discretization, Eq. (10) can be written as

$$\sigma_\xi^2 = \sum_{i=1}^{n} G_\xi(f_{\text{mid},i})\,\Delta f_i \tag{11}$$

where Δf_i represents the frequency interval. In each small interval, we can find such sinusoidal functions, which have $f_{\text{mid},i}$ $(i = 1, 2, \ldots, n)$ as their frequencies and $\sqrt{G_\xi(f_{\text{mid},i})\Delta f_i}$ as their standard deviations. After all the sinusoidal functions are accumulated, the pavement stochastic displacement based on the time domain model can be described as

$$S(t) = \sum_{i=1}^{n} \sqrt{2G_\xi(f_{\text{mid},i})\Delta f_i}\sin(2\pi f_{\text{mid},i}t + \theta_i) \tag{12}$$

where θ_i represents the random number in the interval of $[0, 2\pi]$.

It can be proved that the frequency characteristics of the stochastic displacement input generated by Eq. (12) are consistent with the given road surface spectra. Thus, $S(t)$ can represent the stochastic displacement input at the current vehicle speed.

3.2 Analysis and discussion

Substituting Eqs. (8) and (9) into Eq. (12), we obtain

$$S(t) = \sum_{i=1}^{n} \sqrt{\frac{2vn_0^2\,G_\xi(n_0)}{f_{\text{mid},i}^2}\Delta f}\sin(2\pi f_{\text{mid},i}t + \theta_i) \tag{13}$$

and

$$f_{\text{mid},i} = f_1 + \frac{2i-1}{2}\Delta f \qquad i = 1, 2, \ldots, n \tag{14}$$

Although θ_i is independent of the PSD, it relates to $S(t)$. The improper selection of θ_i will lead to too large an $S(t)$ and make it have no practical value. In fact, by establishing a constraint equation which meets the probability distribution function and by introducing the penalty factor of the optimization function, we can effectively improve the practical effect of the time domain model[13].

As discussed above, the road surface spectra in China are almost in the range of grades A, B and C. So, these three kinds of pavements are researched. Tab. 2 lists some parameters in the model, where $n_0 = 0.1\ \text{m}^{-1}$.

Assume that the lower limit of the time frequency $f_1 = 0.5$ Hz, the upper limit $f_2 = 30$ Hz, the invariable interval $\Delta f_i \equiv \Delta f = 0.01$ Hz and speed $v = 30$ m/s. The length of the simulated pavement $L = 30$ m, and 1 500 discrete points are selected.

Tab. 2 PSD coefficients of grade A, B, C

Grade	PSD coefficient/cm^3
A	16
B	64
C	256

Figs. 2, 3 and 4 are respectively the pavement roughness inputs of grades A, B and C obtained by computer simulation.

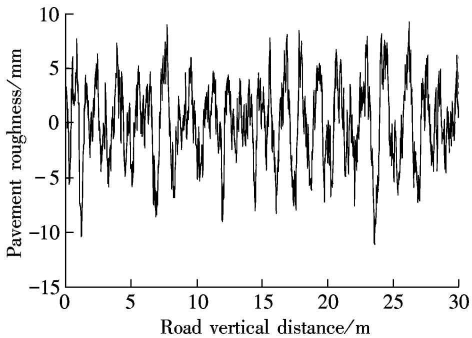

Fig. 2 Pavement roughness curve of grade A

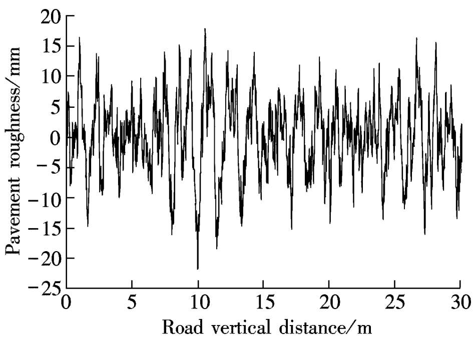

Fig. 3 Pavement roughness curve of grade B

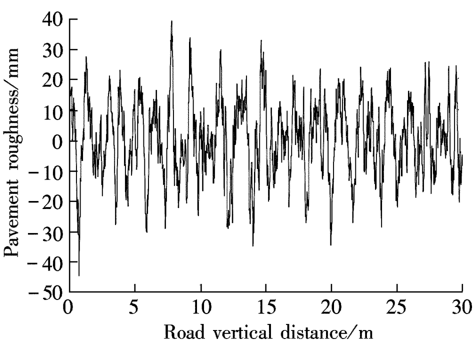

Fig. 4 Pavement roughness curve of grade C

It can be found that as the grade of pavement decreases, pavement roughness becomes increasingly worse. The variance and amplitude of pavement roughness become increasingly larger with the increase in the PSD coefficient.

The pavement roughness curves can be integrated to obtain the curve length S. Then, pavement length L is divided by the curve length S to obtain a dimensionless value. This value is an index to measure pavement roughness which is defined as the nominal roughness index (NRI). Tab. 3 lists the NRI of grades A, B and C in this model.

The physical meaning of the NRI is the average curve length per unit pavement length. The value of the NRI is related to the PSD coefficient and the number of discrete points per meter and speed. Given the conditions of 50 discrete points per meter and $v = 30$ m/s, there is a one to one correspondence between the PSD coefficient and the NRI. The numerical simulation is shown in Tab. 4.

Tab. 3 NRI of grade A, B and C

Grade	NRI
A	1.001 94
B	1.007 77
C	1.029 77

Tab. 4 PSD coefficient and NRI

Grade	PSD coefficient/cm^3	NRI
A	16	1.001 94
B	64	1.007 77
C	256	1.029 77
D	1 024	1.109 99
E	4 096	1.359 32
F	16 384	1.999 77
G	65 535	3.415 39
H	262 144	6.561 48

Though there is a correspondence between the PSD coefficient and the NRI, the physical meaning of the NRI is more explicit than that of the PSD coefficient. By simulation, pavement roughness curves can describe the variations in pavement roughness more intuitively and effectively than those of PSD roughness. The method of the stochastic sinusoidal wave is also important to the research on vehicle ride comfort due to its rigorous mathematical derivation, extensive application range and intuitive simulation curve.

4 Conclusions

1) The method proposed in this paper can generate a stochastic vibration signal that accords with PSD roughness better. And it is more intuitive and effective than the traditional method.

2) Because the sinusoidal signal is ergodic, multiple samples can be combined to simulate the stochastic signal.

3) As the grade of pavement decreases, pavement roughness becomes increasingly worse. The variance and amplitude of pavement roughness become greater when the PSD coefficient increases.

4) The nominal roughness index (NRI) is proposed to measure pavement roughness.

5) Though the pavement roughness input based on the time domain model has been solved by the Fourier transform, the precision of the model needs to be further verified.

References

[1] Deng Xuejun, Sun Lu. *The dynamics of vehicle-pavement structures systems* [M]. Beijing: China Communications Press, 2000. (in Chinese)

[2] Kropác O, Múcka P. Be careful when using the international roughness index as an indicator of road unevenness [J]. *Journal of Sound and Vibration*, 2005, **287**(4/5): 989 − 1003.

[3] Dodds C J. Laboratory simulation of vehicle service stress [J]. *American Society of Mechanical Engineers*, 1974, **73**(4): 391 − 398.

[4] Sun Lu. Simulation of pavement roughness and IRI based on power spectral density [J]. *Mathematics and Computers in Simulation*, 2003, **61**(2): 77 - 88.
[5] Sun Lu, Greenberg B S. Dynamic response of linear systems to moving stochastic sources [J]. *Journal of Sound and Vibration*, 2000, **229**(4): 957 - 972.
[6] Sun Lu, Deng Xuejun. Recent advancement to spectral analysis in studying of pavement roughness: theory and applications [C]//*The* 14*th Proc CSME Forum* 1998 *Symp on Advances in Transportation Systems*. Toronto, Canada, 1998.
[7] Deng Xuejun, Sun Lu. Recent advancement to spectral analysis in studying of pavement roughness: measurement and analysis [C]//*The* 14*th Proc CSME Forum* 1998 *Symp on Advances in Transportation Systems*. Toronto, Canada, 1998.
[8] Zhou Xiaoqing, Yan Li, Sun Lijun. Study and validation of the relationship between international roughness index and power spectral density [J]. *China Civil Engineering Journal*, 2007, **40**(1): 99 - 104. (in Chinese)
[9] Bogsjö K. Evaluation of stochastic models of parallel road tracks [J]. *Probabilistic Engineering Mechanics*, 2007, **22**(4): 362 - 370.
[10] American Society for Testing and Materials (ASTM). E867—02 Terminology relating to vehicle-pavement systems [S]. West Conshohocken: ASTM International, 2002.
[11] Research Institute of Highway Ministry Transport. JTGF80/1—2004 Quality inspection and evaluation standards for highway engineering [S]. Beijing: China Communications Press, 2004. (in Chinese)
[12] Changchun Vehicle Institute. GB 7031—1986 Representative model of vehicle vibration—the plainness of the road surface [S]. Beijing: Standards Press of China, 1986. (in Chinese)
[13] Sun Ning, Li Guixian. A simple method for generating the simulating signals of random vibrations [J]. *Journal of Vibration and Shock*, 2000, **19**(2): 50 - 63. (in Chinese)

基于时域模型的路面平整度模拟

钮凯健　　李　昶

(东南大学交通学院,南京 210096)

摘要:为了更加直观有效地描述路面平整度,提出了一种路面平整度的模拟方法,即随机正弦波法. 该方法将路面平整度表示成大量具有随机相位的正弦或余弦之和,采用离散谱逼近目标随机过程,是一种离散化数值模拟路面平整度的方法. 根据给定的路面功率谱系数,在时域路面随机位移输入的频率特征与给定的路面谱相一致的情况下,通过计算机模拟将路面平整度优化成为随机振动的等效信号,得到各等级下路面的平整度曲线. 结果表明:随机正弦波法适用于实测道路谱的时域模拟,由于该方法数学推导严密,使用范围广泛,且模拟曲线直观,对于汽车平顺性研究具有十分重要的意义. 最后,提出一个与路面功率谱系数相关的平整度指标——名义平整度指数.

关键词:路面平整度;随机正弦波;随机过程;功率谱系数;时域;名义平整度指数
中图分类号:U416.2