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Network efficiency analysis of Chinese inter-bank market

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Abstract: The inter-bank market network models are constructed based on the inter-bank credit lending relationships, and the network efficiency characters of the Chinese inter-bank market are studied. Since it is impossible to obtain the specific credit data among banks, this paper estimates the inter-bank lending matrix based on the partial information of banks. Thus, directed network models of the Chinese inter-bank market are constructed by using the threshold method. The network efficiency measures and the effects of random attacks and selective attacks on the global efficiency of the inter-bank network are analyzed based on the network models of the inter-bank market. Empirical results suggest that the efficiency measures are sensitive to the threshold, and that the global efficiency is little affected by random attacks, while it is highly sensitive to selective attacks. Properties such as inter-bank market network efficiency would be useful for risk management and stability of the inter-bank market.

Key words: inter-bank market; network efficiency; attack

After the innovative research of a small-world network by Wattz and Strogatz[1] and a scale-free network by Barabasi and Albert[2], complex networks have been a general method to study common properties of complex systems in the real world, and have been applied in statistical physics, social sciences, biological sciences and many other different fields. Recently financial networks have received much attention mainly as a method to visualize the relationships among financial entities, such as the correlation network of stock price returns[3-5], commercial credit among firms, financial credit from banks to firms and inter-bank credit[6].

For the inter-bank market, the inter-bank lending relationships, discounts, acceptances, guarantees and other forms of relationships make it a complex network. In fact, there is a lot of literature justifying the emergence of the complex network in the inter-bank market. Some interesting empirical results are provided by scholars[7-9] who have recently shown that inter-bank networks present typical characteristics of complex networks, such as being scale-free, having a hierarchical structure, a small world and a high degree of heterogeneity. A complete understanding of the structure of relationships in the inter-bank market is of primary importance, because it plays a crucial role in the inter-bank contagion risks and domino effects characterized by credit relationships. The concept of a network can be used to model the market for inter-bank lending and to study contagion risk. Applications of this idea can be found in Refs. [10 – 14].

The study of the structural properties of the inter-bank network is very important in the understanding of its functions as well as the response of the inter-bank market to external factors such as the spreading of unexpected shocks over the network. However, the concept of efficiency in a network is strongly related to its structural properties[15-17]. Some relevant definitions of the network efficiency are introduced in Ref. [16]. Furthermore, Crucitti et al. [18] studied the influence of error and attack tolerance on the efficiency of scale-free networks, and found that the global and the local efficiency are not affected by the failure of some randomly chosen nodes, though they are extremely sensitive to the removal of the few nodes which play a crucial role in maintaining the network's connectivity.

This paper uses a threshold method to construct directed network models of the inter-bank market based on the inter-bank relationships, and then analyzes the Chinese inter-bank network efficiency properties.

1 Network Modeling of Inter-Bank Market

A network G consists of a non-empty set of elements V called vertices, and a list of unordered pairs of these elements called edges E. If i and j are vertices of G, an edge of the form ij is said to join or connect i and j[9]. Next, we construct the inter-bank networks based on the inter-bank lending relationships.

1.1 Computing lending matrix

The lending relationships in the inter-bank market can be represented by matrix $\boldsymbol{X} = (x_{ij})_{N\times N}$:

$$\boldsymbol{X}_{N\times N} = \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & & \vdots & & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & x_{NN} \end{bmatrix} \begin{matrix} a_1 \\ \vdots \\ a_N \end{matrix}$$
$$\begin{matrix} l_1 & \cdots & l_j & \cdots & l_N \end{matrix}$$

where x_{ij} denotes bank i's exposure to bank j ($x_{ii} = 0$, $i = 1, 2, \dots, N$, because a bank cannot be exposed to itself), and N denotes the number of banks, while a_i and l_j denote bank i's inter-bank assets and bank j's inter-bank liabilities respectively, where $a_i = \sum_{j=1}^{N} x_{ij}$ and $l_j = \sum_{i=1}^{N} x_{ij}$.

In many countries, such as China, the Netherlands and Germany, we do not know all the information on inter-bank trading transactions, but we do know the sum of each bank's inter-bank loans and deposits. Still, we can estimate the bilateral exposure matrix by minimizing the entropy[19]. In order to estimate $\boldsymbol{X}$, we have to minimize the relative entropy of $\boldsymbol{X}$ with respect to the matrix with elements $x_{ij}^0 = a_i l_j$ for $i \neq j$ and 0 for $i = j$,

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$$\min \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij} \ln\left(\frac{x_{ij}}{x_{ij}^{0}}\right)$$

s. t.

$$a_i = \sum_{j=1}^{N} x_{ij},\ l_j = \sum_{i=1}^{N} x_{ij},\ x_{ij} \geqslant 0$$

with the conventions that $x_{ij}=0$ if and only if $x_{ij}^{0}=0$, and $0 \ln(0/0)=0$. More details about the above method can be seen in Ref. [19].

1.2 Constructing the network

Through the calculation of the inter-bank lending matrix, we can obtain the inter-bank lending size. But the estimation method is biased over the inter-bank market towards a "complete structure of claims"[20], which does not conform with the actual situation. Therefore, in this paper we use a threshold method to construct inter-bank networks. The vertices in the inter-bank networks denote banks, and the edge density between the vertices denotes inter-bank lending relationships. When the ratio x'_{ij}, i. e., $x_{ij}/\sum_{i=1}^{N}\sum_{j=1}^{N} x_{ij}$, is greater than or equal to the threshold value $c \in [0, 1]$, there is an edge for vertices, or there is not an edge for vertices.

To use the language of graph theory, the inter-bank network can be described as follows: Let graph $G(V, E)$ represent the inter-bank network, where V and E are the sets of network vertices and edges, respectively. E is defined as

$$E = \begin{cases} e_{ij} = 1 & i \neq j \text{ and } x'_{ij} \geqslant c \\ e_{ij} = 0 & i = j \end{cases}$$

2 Data

For Chinese banks, the amounts of lending in city commercial banks, urban credit cooperatives, rural credit cooperatives and postal savings institutions in the inter-bank lending market are relatively small or the data are difficult to collect, so the network we construct in this paper does not include these financial institutions but only includes 3 state policy banks, 4 state-owned banks and 12 shareholding banks. The data we use for estimating the inter-bank lending matrix are from the *China Financial Statistical Yearbook* in 2008. In this paper, we estimate the inter-bank lending matrix by the LINGO software. As the credit lending data occupy a large space, this article does not set out many specific results, but the results are given in related distribution analyses.

Fig. 1 shows the distribution of ratio x'_{ij} in 2007. We can

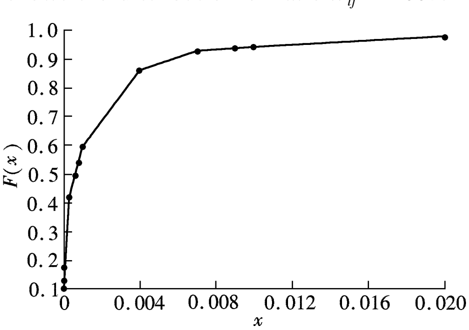

Fig. 1 The distribution $F(x)$ of ratio x'_{ij}

see that most ratios are at intervals of (0, 0.01]. In addition, it is obvious that the number of edges in the inter-bank network decreases as the threshold value c increases. From Fig. 2, we know that the edge density drops sharply as c increases, where the edge density of the network represents the existing edge number divided by the maximal edge number. It is manifest that the edge density drops sharply from 99.42% to 6.43% as c increases from 0 to 0.01, which similarly shows that most ratios are at intervals of (0, 0.01].

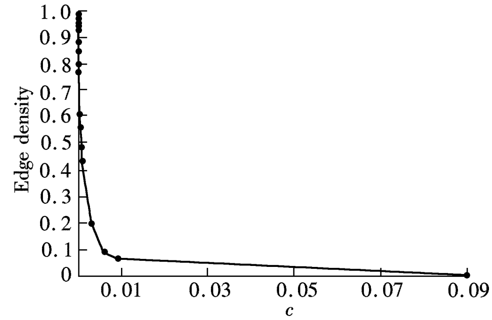

Fig. 2 The edge density of inter-bank network

3 Empirical Analysis

3.1 Efficiency measures analysis

The concept of efficiency in a complex network plays the role of measuring its ability of the exchange of information and its response for the spread of perturbations in diverse applications. For the inter-bank network, the indices that describe efficiency mainly include in-efficiency, out-efficiency and global efficiency.

The in-efficiency and the out-efficiency of the vertices i are defined respectively as[9]

$$\frac{\sum_{j=1}^{N} 1/d_{ji}}{N-1},\quad \frac{\sum_{j=1}^{N} 1/d_{ij}}{N-1}$$

where d_{ij} is the minimal number of paths needed to reach bank i from bank j. The global efficiency of a network can be defined as[16]

$$E_{\text{glob}} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}}$$

In Fig. 3, we plot the global efficiency as a function of the threshold c, and find that the global efficiency of the inter-bank network decreases as the threshold c increases. The

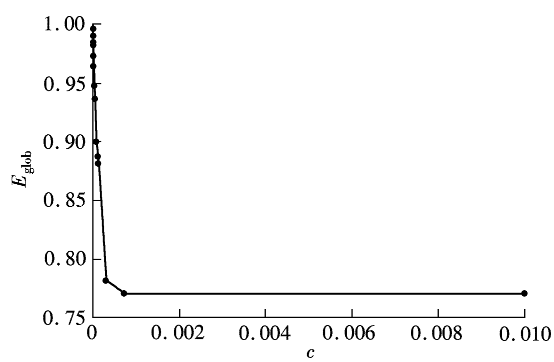

Fig. 3 Global efficiency of inter-bank market network under different thresholds

reason is that the increase of the threshold c makes some edges disappear and, consequently, excludes some paths that, before the increase of the threshold c, contributed to the connectivity of the inter-bank network. Fig. 4 shows the in-efficiency and the out-efficiency of vertices when $c = 0.0000075$, 0.0007 and 0.01. When c equals other values, we can obtain similar results. As it can be seen, the in-efficiency and the out-efficiency of vertices are highly affected by the threshold c.

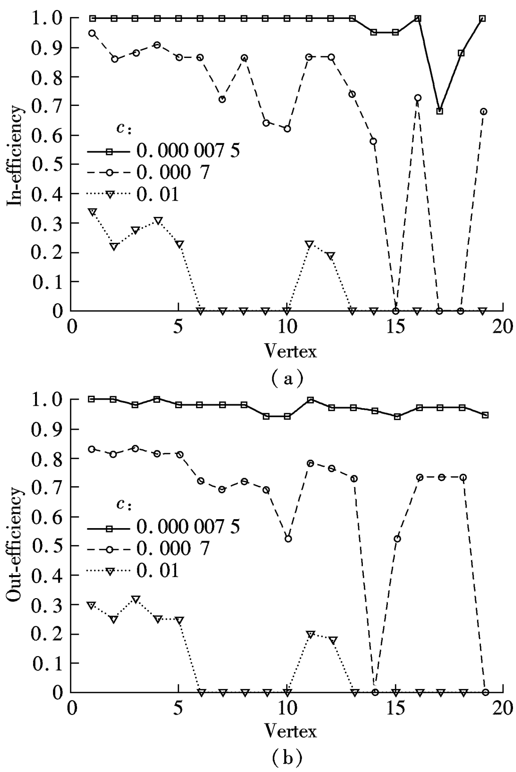

Fig. 4 In-efficiency and out-efficiency of vertices under different thresholds. (a) In-efficiency of vertices; (b) Out-efficiency of vertices

3. 2 Effects of random attacks and selective attacks

For the inter-bank network, we adopt deleting vertices to analyze the effect of attacks on its global efficiency, which corresponds to the bankruptcy of some banks, such as the bankruptcy of Washington Mutual Bank, Silver State Bank and other banks in the recent U. S. sub-prime mortgage crisis. We adopt the following two methods of deleting vertices: random attacks and selective attacks. Random attacks mean removing some vertices from the network in a stochastic manner, while selective attacks refer to removing the vertices in a certain order, namely, from the vertices with the largest degree to vertices with comparatively smaller degrees and so on. We only analyze the interval $[4\times10^{-6}, 2\times10^{-3}]$, because under other circumstances, the networks are too dense and are not suitable for analysis.

In Fig. 5, we plot the global efficiency as functions of the fraction of removed vertices when $c = 0.000009$, 0.000075, 0.00011 and 0.002, where f denotes the fraction of removed vertices. We can see from Fig. 5 that the inter-bank network shows a highly different behavior with respect to random attacks and selective attacks. The global efficiency is little affected by random attacks, while it is highly sensitive to selective attacks. If the threshold is equal to other values in the interval $[4\times10^{-6}, 2\times10^{-3}]$, the general rules of global efficiency are not changed. So we only demonstrate the aforesaid four cases in Fig. 5.

The reason for this behavior is rooted in the heterogeneity of the vertices. The results also indicate that the vertices with high degrees play an important role in the inter-bank networks.

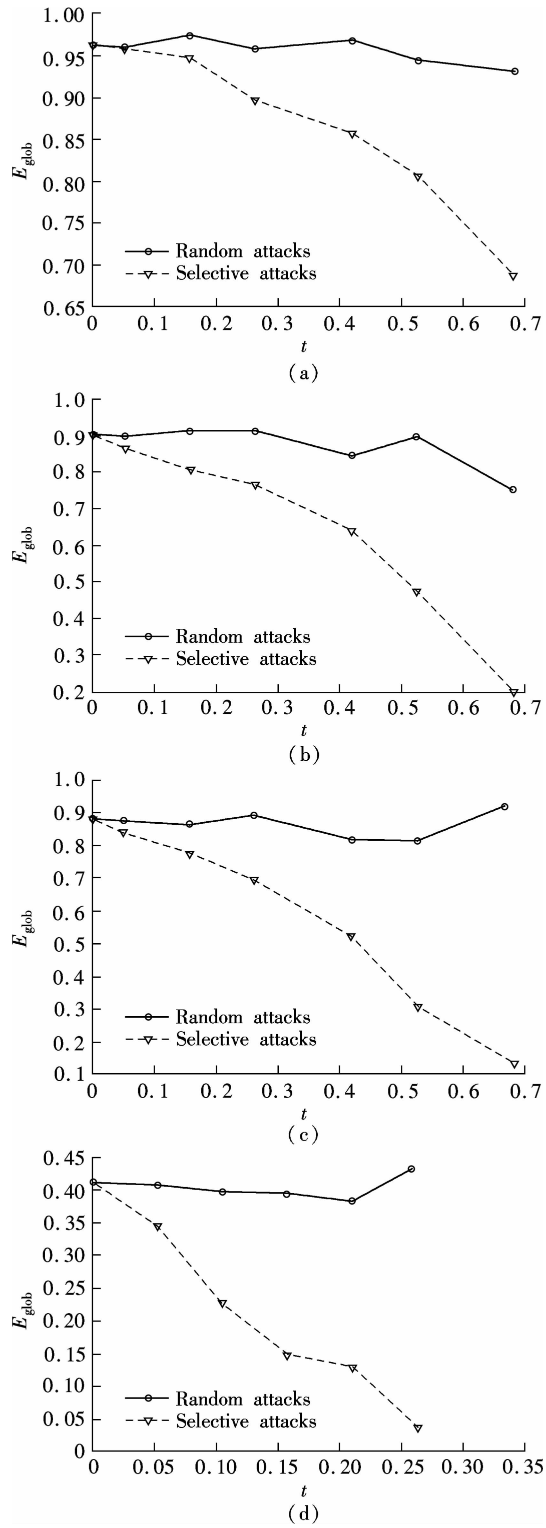

Fig. 5 Effects of random attacks and selective attacks on global efficiency. (a) $c = 0.000009$; (b) $c = 0.000075$; (c) $c = 0.00011$; (d) $c = 0.002$

4 Conclusion

Complex theory is a powerful tool to analyze the functions and structural properties of the inter-bank market. In this paper, based on the partial information of the inter-bank market, we construct directed network modes of the inter-bank market by using a threshold method. Moreover, based on the data of some Chinese banks, we analyze the Chinese inter-bank networks. We investigate the effects of random attacks and selective attacks on the global efficiency of the inter-bank network. For the global efficiency, we find that it is comparatively sensitive to selective attacks but it is little affected by random attacks. This means that the inter-bank network has a low stability against selective attacks, while it has a high stability against random attacks. Properties such as inter-bank network efficiency would also be useful for risk management and stability of the inter-bank market.

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我国银行间市场的网络效率分析

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摘要:基于银行间信用拆借关系,构建了银行间市场的网络模型,研究了我国银行间市场的网络效率特征. 由于无法获得具体的银行间信用拆借规模,利用银行间市场部分信息估测了银行间信用拆借矩阵,进而采用阈值法构建了我国银行间市场的有向网络模型. 在银行间市场的网络模型基础上,分析了我国银行间市场的网络效率指标性质以及随机性攻击和选择性攻击对银行间市场网络全局效率的影响. 实证结果表明:银行间网络的效率指标对阈值非常敏感;银行间网络全局效率受随机性攻击影响不大,但受选择性攻击影响较大. 该结果有助于银行间市场的风险管理,维护银行间市场的稳定.

关键词:银行间市场;网络效率;攻击

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