Journal of Southeast University (English Edition) Vol. 26, No. 3, pp. 505 – 508 Sept. 2010 ISSN 1003—7985

A generalization of co- $*^n$ -modules

Yao Lingling Chen Jianlong

(Department of Mathematics, Southeast University, Nanjing 211189, China)

Abstract: A module is called a co- $*^\infty$ -module if it is co-selfsmall and ∞ -quasi-injective. The properties and characterizations are investigated. When a module U is a co- $*^\infty$ -module, the functor $\mathrm{Hom}_{_RU}(-, U)$ is exact in $\mathrm{Copres}^\infty(U)$. A module U is a co- $*^\infty$ -module if and only if U is co-selfsmall and for any exact sequence $0\to M\to U^I\to N\to 0$ with $M\in\mathrm{Copres}^\infty(U)$ and I is a set, $N\in\mathrm{Copres}^\infty(U)$ is equivalent to $\mathrm{Ext}^1_R(N, U)\to\mathrm{Ext}^1_R(U^I, U)$ is a monomorphism if and only if U is co-selfsmall and for any exact sequence $0\to L\to M\to N\to 0$ with L, $N\in\mathrm{Copres}^\infty(U)$, $N\in\mathrm{Copres}^\infty(U)$ is equivalent to the induced sequence $0\to\Delta(N)\to\Delta(M)\to\Delta(L)\to 0$ which is exact if and only if U induces a duality Δ_{U_S}: ${}^\perp U_S\Leftrightarrow\mathrm{Copres}^\infty(U):\Delta_{_RU}$. Moreover, U is a co- $*^n$-module if and only if U is a co- $*^\infty$ -module and $\mathrm{Copres}^\infty(U)=\mathrm{Copres}^n(U)$.

Key words: co- $*^\infty$ -module; ∞ -quasi-injective; co-selfsmall; co- $*^n$-module

Both quasi-progenerators[1] and tilting modules over arbitrary rings [2–5] induce equivalences between certain categories of modules that are generalizations of Morita equivalence. In Ref. [6], Menini and Orsatti introduced a generalization of these modules which is called $*$ -modules nowadays. Colpi et al. [4,7–9] investigated this concept and some others worked over a dual notion of quasi-progenerators, called quasi-duality modules, cotilting modules (the dual notion of tilting modules), costar modules (the dual notion of $*$ -modules), which have been a popular topic in module theory. These modules induce various generalizations of Morita duality.

Recently, Wei[10] generalized the notion of $*$ -modules to $*^n$-modules which can be viewed as a generalization of both $*$ -modules and tilting modules of projective dimensions $\leqslant n$. Just as costar modules to $*$ -modules done by Colby and Fuller [11], Yao et al. [12] introduced the dual situation of $*^n$-modules: co- $*^n$-modules. Then the properties and characterizations of co- $*^n$-modules are studied and it is proved that co-selfsmall n-cotilting modules are co- $*^n$-modules.

In this paper, we mainly consider a more general setting. We introduce the notion of co- $*^\infty$ -modules which generalizes co- $*^n$-modules. The most important results on co- $*^n$-modules are extended to co- $*^\infty$ -modules. In particular, we present a characterization of the co- $*^\infty$ -module $_RU$ with $S=\mathrm{End}(_RU)$ in terms of category duality:

$$\Delta_{U_S}=\mathrm{Hom}_S(-, U): C\Leftrightarrow D: \Delta_{_RU}=\mathrm{Hom}_R(-, U)$$

between full subcategories $C\subseteq S$-Mod and $D\subseteq R$-Mod where C consists of modules N such that $\mathrm{Ext}^i_S(N, U)=0$ for all $i\geqslant 1$, and D consists of modules ∞ -copresented by U.

1 Preliminaries

All the rings have non-zero identity and all the modules are unitary. For every ring R, R-Mod (Mod-R) denotes the category of all the left (right) R-modules. Let $_RU\in R$-Mod. A left R-module $_RN$ is called n-copresented by $_RU$ if there exists an exact sequence $0\to {_RN}\to U^{I_0}\to U^{I_1}\to\ldots\to U^{I_{n-2}}\to U^{I_{n-1}}$ where $I_i(0\leqslant i\leqslant n-1)$ are index sets. We say that $N\in R$-Mod is ∞ -copresented by U if it is n-copresented by U for all n. Denote $\mathrm{Copres}^n(_RU)$ and $\mathrm{Copres}^\infty(_RU)$ with the category of all the modules that are n-copresented and ∞ -copresented, respectively by $_RU$. It is clear to see that there is an inclusion between categories: $\mathrm{Copres}^\infty(_RU)\subseteq\mathrm{Copres}^{n+1}(_RU)\subseteq\mathrm{Copres}^n(_RU)$. Denote $\mathrm{Copres}^2(_RU)$ by $\mathrm{Copres}(_RU)$ and $\mathrm{Copres}^1(_RU)$ by $\mathrm{Cogen}(_RU)$ as usual.

Let R and S be rings and $_RU_S$ be a given bimodule. We denote $\Delta_{_RU}$ the functor $\mathrm{Hom}_R(-, U)$ and Δ_{U_S} the functor $\mathrm{Hom}_S(-, U)$, both of which can be simply denoted as Δ_U or Δ if no confusion appears. It is well known that $(\Delta_{_RU}, \Delta_{U_S})$ is a pair of adjoint contravariant functors with the canonical morphism:

$$\delta_X: X\to\Delta^2X \text{ with } x\mapsto(f\mapsto f(x))$$

A left R-module $_RU$ is said to be selfsmall if for any set X there is the canonical isomorphism $\mathrm{Hom}_R(U, U^{(X)})\cong\mathrm{Hom}_R(U, U)^{(X)}$. $_RU$ is called n-quasi-projective if for any exact sequence $0\to M\to U^{(X)}\to N\to 0$ in R-Mod with $_RM\in\mathrm{Pres}^{n-1}(_RU)$, and the induced sequence $0\to\mathrm{Hom}_R(U, M)\to\mathrm{Hom}_R(U, U^{(X)})\to\mathrm{Hom}_R(U, N)\to 0$ is exact. Dually, a left R-module $_RU$ is called co-selfsmall if for any set I there is the canonical isomorphism $\mathrm{Hom}_R(U, U)^{(I)}\cong\mathrm{Hom}_R(U^I, U)$. $_RU$ is n-quasi-injective if for any exact sequence $0\to M\to U^I\to N\to 0$ with $_RN\in\mathrm{Copres}^{n-1}(_RU)$, and the induced sequence $0\to\Delta(N)\to\Delta(U^I)\to\Delta(M)\to 0$ is exact.

Lemma 1[13] For any X in R-Mod or Mod-S, and a bimodule $_RU_S$, δ_X is a monomorphism if and only if $X\in\mathrm{Cogen}(U)$.

By taking a free resolution of N_S, one can easily obtain the following result.

Lemma 2 Let $_RU\in R$-Mod and $S=\mathrm{End}(_RU)$. Then $\Delta_{U_S}(N)\in\mathrm{Copres}(_RU)$ for any $N_S\in$ Mod-S. If, moreover, $\mathrm{Ext}^i_S(N, U)=0$ for $1\leqslant i\leqslant n$, then $\Delta_{U_S}(N)\in\mathrm{Copres}^{n+2}(_RU)$.

Received 2009-11-06.

Biographies: Yao Lingling (1982—), female, graduate; Chen Jianlong (corresponding author), male, doctor, professor, jlchen@seu.edu.cn.

Foundation items: The National Natural Science Foundation of China (No. 10971024), Specialized Research Fund for the Doctoral Program of Higher Education (No. 200802860024).

Citation: Yao Lingling, Chen Jianlong. A generalization of co- $*^n$-modules [J]. Journal of Southeast University (English Edition), 2010, 26(3): 505 – 508.

Let ${}_RU \in R$-Mod. ${}_RU$ is called a co-$*^n$-module if ${}_RU$ is co-selfsmall, $(n+1)$-quasi-injective, and $\mathrm{Copres}^{n+1}({}_RU) = \mathrm{Copres}^n({}_RU)$. In Ref. [12], the following result is proved.

Theorem 1 Let ${}_RU$ be a left R-module and $S = \mathrm{End}({}_RU)$. Then the following conditions are equivalent:

1) ${}_RU$ is a co-$*^n$-module;

2) ${}_RU$ is co-selfsmall and for any exact sequence $0\to M\to U^I\to N\to 0$ in R-Mod, where $M \in \mathrm{Copres}^n(U)$ and I is a set, we have $N \in \mathrm{Copres}^n(U)$ if and only if $\mathrm{Ext}_R^1(N, U)\to \mathrm{Ext}_R^1(U^I, U)$ is a canonical monomorphism;

3) ${}_RU$ is co-selfsmall and for any exact sequence $0\to L\to M\to N\to 0$ where $L, M \in \mathrm{Copres}^n({}_RU)$, we have $N \in \mathrm{Copres}^n({}_RU)$ if and only if $0\to\Delta(N)\to\Delta(M)\to\Delta(L)\to 0$ is exact;

4) ${}_RU$ induces a duality $\Delta_{U_S}: {}^{\perp}U_S \Leftrightarrow \mathrm{Copres}^n({}_RU): \Delta_{_RU}$, where ${}^{\perp}U_S = \{N \mid \mathrm{Ext}_S^i(N, U) = 0 \text{ for any } i\geqslant 1\}$.

2 Co-$*^{\infty}$-Modules

Motivated by the idea of $*^{\infty}$-modules[14] and the co-$*^n$-modules[12], a more general setting is considered in this paper. First we introduce the following notion.

Definition 1 An R-module U is said to be ∞-quasi-injective if for any $M \in \mathrm{Copres}^{\infty}(U)$ and any infinite U-copresentation of M: $0\to M\to U^{I_1}\to U^{I_2}\to\ldots$, the induced sequence $\Delta(U^{I_1})\to\Delta(M)\to 0$ is exact.

Lemma 3 The following are equivalent for an R-module U:

1) U is ∞-quasi-injective;

2) For any infinite exact sequence $0\to M\to U^{I_1}\to U^{I_2}\to\ldots$, the induced sequence $\Delta(U^{I_n})\to\ldots\to\Delta(U^{I_1})\to\Delta(M)\to 0$ is exact, where $n\geqslant 1$;

3) Any infinite exact sequence $0\to M\to U^{I_1}\to U^{I_2}\to\ldots$ is also exact after the functor Δ.

Proof 3) $\Rightarrow$ 2) $\Rightarrow$ 1) are clear.

1) $\Rightarrow$ 3) For any $i\geqslant 1$, let $K_i = \mathrm{Ker}(U^{I_i}\to U^{I_{i+1}})$. It is easy to see that $0\to K_i\to U^{I_i}\to U^{I_{i+1}}\to\ldots$ be an infinite U-copresentation of K_i. Hence we obtain that $\Delta(U^{I_i})\to\Delta(K_i)\to 0$ is also exact. Then it follows that the induced infinite sequence $\ldots\to\Delta(U^{I_2})\to\Delta(U^{I_1})\to\Delta(M)\to 0$ is exact.

Now we can give the definition of co-$*^{\infty}$-modules.

Definition 2 An R-module U is said to be co-$*^{\infty}$-module if U is co-selfsmall and ∞-quasi-injective.

From definition 2 we clearly see that all the co-selfsmall injective (or more generally, n-quasi-injective) modules are co-$*^{\infty}$-modules. In particular, all the co-$*^n$-modules are co-$*^{\infty}$-modules but the converse fails clearly. In fact, we have the following results.

Proposition 1 $U \in R$-Mod is a co-$*^n$-module if and only if U is a co-$*^{\infty}$-module and $\mathrm{Copres}^n(U) = \mathrm{Copres}^{\infty}(U)$.

Proof The necessity is obvious. Conversely, it suffices to show that U is $(n+1)$-quasi-injective. Let $0\to M\to U^I\to N\to 0$ be exact with $N \in \mathrm{Copres}^n(U)$, where I is a set. By hypothesis, $M \in \mathrm{Copres}^{\infty}(U)$, and U is ∞-quasi-injective. Hence, we have $\Delta(U^I)\to\Delta(M)\to 0$. Therefore, U is $(n+1)$-quasi-injective and U is a co-$*^n$-module.

Proposition 2 Let U be a co-$*^{\infty}$-module. Then

1) δ_M is an isomorphism for any $M \in \mathrm{Copres}^{\infty}(U)$;

2) $\Delta_{_RU}(\mathrm{Copres}^{\infty}(U)) = {}^{\perp}U_S = \{N_S \mid \mathrm{Ext}_S^i(N, U) = 0 \text{ for any } i\geqslant 1\}$, where $S = \mathrm{End}({}_RU)$.

Proof 1) For any $M \in \mathrm{Copres}^{\infty}(U)$, we have an exact sequence $0\to M\to U^I\to N\to 0$ with $N \in \mathrm{Copres}^{\infty}(U)$, where I is a set. By hypothesis, U is ∞-quasi-injective. Hence, the induced sequence is exact. It is easy to see that the following diagram commutes:

$$\begin{tikzcd}
0 \arrow[r] & \Delta^2(M) \arrow[r] & \Delta^2(U^I) \arrow[r] & \Delta^2(N) \arrow[r] & \mathrm{Ext}_S^1(\Delta(M), U) \\
0 \arrow[r] & M \arrow[r] \arrow[u, "\delta_M"] & U^I \arrow[r] \arrow[u, "\delta_{U^I}"] & N \arrow[r] \arrow[u, "\delta_N"] & 0
\end{tikzcd}$$

By lemma 1, δ_N is a monomorphism. Since δ_{U^I} is a natural isomorphism, δ_M is an isomorphism.

2) For any $M \in \mathrm{Copres}^{\infty}(U)$, we consider again the above exact commutative diagram. Notice that $N \in \mathrm{Copres}^{\infty}(U)$, so δ_N is also an isomorphism by 1). It follows that $\mathrm{Ext}_S^i(\Delta(M), U) = 0$. Applying the same arguments to N, we also have $\mathrm{Ext}_S^i(\Delta(N), U) = 0$. Now we derive that $\mathrm{Ext}_S^i(\Delta(M), U) = 0$ for all $i\geqslant 1$ from the fact that $\mathrm{Ext}_S^i(\Delta(N), U) \cong \mathrm{Exi}_S^{i+1}(\Delta(M), U)$ for any $i\geqslant 1$. Hence, we have $\Delta_{_RU}(\mathrm{Copres}^{\infty}(U)) \subseteq {}^{\perp}U_S$.

On the other hand, for any $M \in {}^{\perp}U_S$, we have $\Delta(M) \in \mathrm{Copres}^{\infty}(U)$ by lemma 2. Take the exact sequence $0\to L\to S^{(I)}\to M\to 0$, where I is a set. Then $L \in {}^{\perp}U$ and $\Delta(L) \in \mathrm{Copres}^{\infty}(U)$. Clearly, the induced sequence $0\to\Delta(M)\to\Delta(S^{(I)})(\cong U^I)\to\Delta(L)\to 0$ is exact. Since U is a co-$*^{\infty}$-module, we have the following induced commutative diagram with exact rows:

$$\begin{tikzcd}
0 \arrow[r] & L \arrow[r] \arrow[d, "\delta_L"] & S^{(I)} \arrow[r] \arrow[d, "\delta_{S^{(I)}}"] & M \arrow[r] \arrow[d, "\delta_M"] & 0 \\
0 \arrow[r] & \Delta^2(L) \arrow[r] & \Delta^2(S^{(I)}) \arrow[r] & \Delta^2(M) \arrow[r] & 0
\end{tikzcd}$$

Notice that $\delta_{S^{(I)}}$ is a natural isomorphism, so δ_M is an epimorphism. Similarly, δ_L is also an epimorphism. It follows that δ_M is an isomorphism. Therefore, $M \cong \Delta^2(M) = \Delta(\Delta(M)) \in \Delta(\mathrm{Copres}^{\infty}(U))$. This shows that ${}^{\perp}U_S \subseteq \Delta_{_RU}(\mathrm{Copres}^{\infty}(U))$. Hence we obtain the conclusion.

Lemma 4 Let U be a co-$*^{\infty}$-module. Then $\Delta_{_RU}$ is an exact functor in $\mathrm{Copres}^{\infty}(U)$.

Proof Considering any exact sequence $0\to L\to M\to N\to 0$ in $\mathrm{Copres}^{\infty}(U)$, we have an induced exact sequence $0\to\Delta(N)\to\Delta(M)\to\Delta(L)\to X_S\to 0$, where $X_S = \mathrm{Im}(\Delta(L)\to\mathrm{Ext}_R^1(N, U))$. Then we obtain two short exact sequences: $0\to\Delta(N)\to\Delta(M)\to Y\to 0$ and $0\to Y\to\Delta(L)\to X\to 0$ with $Y = \mathrm{Im}(\Delta(M)\to\Delta(L))$. Applying the functor Δ_{U_S} to the first sequence, we obtain the following commutative diagram with exact rows:

$$\begin{tikzcd}
0 \arrow[r] & L \arrow[r] \arrow[d] & M \arrow[r] \arrow[d, "\delta_M"] & N \arrow[r] \arrow[d, "\delta_N"] & 0 \\
0 \arrow[r] & \Delta(Y) \arrow[r] & \Delta^2(M) \arrow[r] & \Delta^2(N) \arrow[r] & \mathrm{Ext}_S^1(Y, U)
\end{tikzcd}$$

Since U is a co-$*^{\infty}$-module and $M, N \in \mathrm{Copres}^{\infty}(U)$, we have that δ_M and δ_N are isomorphisms and that $\mathrm{Ext}_S^i(\Delta(M), U) = 0 = \mathrm{Exi}_S^i(\Delta(N), U)$ for any $i \geqslant 1$ by proposition 2. Hence we obtain that $\mathrm{Ext}_S^i(Y, U) = 0 =$ for all $i \geqslant 1$ and that $L \cong \Delta(Y)$. Thus $Y \in {}^{\perp}U_S = \Delta_{_U}(\mathrm{Copres}^{\infty}(U))$, also by proposition 2. Put $Y = \Delta D$ for some $D \in \mathrm{Copres}^{\infty}(U)$. Then we obtain

$$Y = \Delta(D) \cong \Delta(\Delta^2(D)) \cong \Delta^2(\Delta(D)) = \Delta^2 Y$$

It follows that

$$X_S = \mathrm{Coker}(Y \to \Delta(L)) \cong \mathrm{Coker}(\Delta^2 Y \to \Delta(\Delta(Y))) = 0$$

Hence $0 \to \Delta(N) \to \Delta(M) \to \Delta(L) \to 0$ is exact.

We are now in a position to give some characterizations of co-$*^{\infty}$-modules which are similar to the characterizations of co-$*^n$-modules.

Theorem 2 Let $U \in R$-Mod and $S = \mathrm{End}({}_RU)$. Then the following conditions are equivalent:

1) U is a co-$*^{\infty}$-module;

2) U is co-selfsmall and for any exact sequence $0 \to M \to U^I \to N \to 0$ with $M \in \mathrm{Copres}^{\infty}(U)$ and I is a set, we obtain that $N \in \mathrm{Copres}^{\infty}(U)$ if and only if $\mathrm{Ext}_R^1(N, U) \to \mathrm{Ext}_R^1(U^I, U)$ is a monomorphism canonically;

3) U is co-selfsmall and for any exact sequence $0 \to L \to M \to N \to 0$ with L, $N \in \mathrm{Copres}^{\infty}(U)$, we obtain that $N \in \mathrm{Copres}^{\infty}(U)$ if and only if the induced sequence $0 \to \Delta(N) \to \Delta(M) \to \Delta(L) \to 0$ is exact;

4) U induces a duality $\Delta_{U_S}: {}^{\perp}U_S \Leftrightarrow \mathrm{Copres}^{\infty}(U) : \Delta_{_U}$.

Proof 1) $\Rightarrow$ 3) The necessity follows from lemma 4. We now prove the sufficiency. Assume that the induced sequence $0 \to \Delta(N) \to \Delta(M) \to \Delta(L) \to 0$ is exact. Applying the functor Δ_{U_S}, we have an exact sequence $\mathrm{Ext}_S^i(\Delta(M), U) \to \mathrm{Exi}_S^i(\Delta(N), U) \to \mathrm{Ext}_S^{i+1}(\Delta(L), U)$ for any $i \geqslant 1$. Since U is a co-$*^{\infty}$-module, $\mathrm{Ext}_S^i(\Delta(M), U) = 0 = \mathrm{Exi}_S^{i+1}(\Delta(L), U)$ for all $i \geqslant 1$ by proposition 2. Hence we obtain that $\mathrm{Ext}_S^i(\Delta(N), U) = 0$ for all $i \geqslant 1$. It follows that $N \in \mathrm{Copres}^{\infty}(U)$ by lemma 2.

3) $\Rightarrow$ 2) is clear.

2) $\Rightarrow$ 1) We only need to prove that U is ∞-quasi-injective. For any exact sequence $0 \to M \to U^I \to N \to 0$ with $N \in \mathrm{Copres}^{\infty}(U)$ and I is a set, we obtain that $\Delta(U^I) \to \Delta(M) \to 0$ is exact by hypothesis 2).

1) $\Rightarrow$ 4) By proposition 2 and lemma 2.

4) $\Rightarrow$ 1) Since $S^{(I)} \in {}^{\perp}U_S$, $S^{(I)}$ is reflexive and we obtain that $(\mathrm{Hom}_R(U, U))^{(I)} = S^{(I)} \cong \Delta^2(S^{(I)}) = \Delta(\Delta(S^{(I)})) \cong \Delta(U^I) = \mathrm{Hom}_R(U^I, U)$.

Thus U is co-selfsmall. For any exact sequence $0 \to M \to U^I \to N \to 0$ with $N \in \mathrm{Copres}^{\infty}(U)$, we have the induced exact sequence $0 \to \Delta(N) \to \Delta(U^I) \to \Delta(M) \to X_S \to 0$ where $X_S = \mathrm{Im}(\Delta(M) \to \mathrm{Ext}_R^1(N, U))$. As in the proof of lemma 4, we obtain the following commutative diagram with exact rows:

$$\begin{tikzcd}
0 \arrow[r] & M \arrow[r] \arrow[d] & U^I \arrow[r] \arrow[d, "\delta_{U^I}"] & N \arrow[r] \arrow[d, "\delta_N"] & 0 \\
0 \arrow[r] & \Delta(Y) \arrow[r] & \Delta^2(U^I) \arrow[r] & \Delta^2(N) \arrow[r] & \mathrm{Ext}_S^1(Y, U)
\end{tikzcd}$$

where $Y = \mathrm{Im}(\Delta(U^I) \to \Delta(M))$. Then after the same procedure, we obtain that $X_S = 0$; i. e., U is ∞-quasi-injective. Therefore, U is a co-$*^{\infty}$-module.

Corollary 1 Let U be a co-$*^{\infty}$-module with $S = \mathrm{End}({}_RU)$ and $M \in$ Mod-S. If $N \in \mathrm{KerExt}_S^{i \geqslant 0}(-, U)$, then $N = 0$.

Proof By theorem 2, we obtain that $N \cong \Delta(\Delta(N)) = 0$.

Proposition 3 Let U be a co-$*^{\infty}$-module. Then $\mathrm{Copres}^{\infty}(U)$ is closed under extensions if and only if $\mathrm{Copres}^{\infty}(U) \subseteq {}_RU^{\perp 1} = \{{}_RM \mid \mathrm{Ext}_R^1(M, U) = 0\}$.

Proof The necessity. For any $M \in \mathrm{Copres}^{\infty}(U)$ and any extension of U by M: $0 \to U \xrightarrow{f} N \to M \to 0$, we obtain that $N \in \mathrm{Copres}^{\infty}(U)$ by assumption. By theorem 2, the induced sequence $0 \to \Delta(M) \to \Delta(N) \to \Delta(U) \to 0$ is exact. Therefore, there exists a morphism g: $N \to_R U$ such that $gf = 1_{_U}$. This shows that $\mathrm{Copres}^{\infty}(U) \subseteq {}_RU^{\perp 1}$.

The sufficiency. For any extension of L by M: $0 \to L \to N \to M \to 0$, where $L, M \in \mathrm{Copres}^{\infty}(U)$. By assumption we obtain that the induced sequence $0 \to \Delta(M) \to \Delta(N) \to \Delta(L) \to 0$ is exact. According to proposition 2, both δ_L and δ_M are isomorphisms and $\Delta(L), \Delta(M) \in {}^{\perp}U$. Then we obtain that δ_N is an isomorphism and $\Delta(N) \in {}^{\perp}U$. Hence, $N \cong \Delta^2(N) = \Delta(\Delta(N)) \in \mathrm{Copres}^{\infty}(U)$ by lemma 2, i. e., $\mathrm{Copres}^{\infty}(U)$ is closed under extensions.

References

[1] Fuller K R. Density and equivalence [J]. *J Algebra*, 1974, **29**(3): 528 - 550.

[2] Angeleri Hügel L, Coelho F U. Infinitely generated tilting modules of finite projective dimension [J]. *Forum Math*, 2001, **13**(2): 239 - 250.

[3] Bazzoni S. A characterization of *n*-cotilting and *n*-tilting modules [J]. *J Algebra*, 2004, **273**(1): 359 - 372.

[4] Colpi R. Tilting modules and $*$-modules [J]. *Comm Algebra*, 1993, **21**(4): 1095 - 1102.

[5] Miyashita Y. Tilting modules of finite projective dimension [J]. *Math Z*, 1986, **193**(1): 113 - 146.

[6] Menini C, Orsatti A. Representable equivalences between categories of modules and applications [J]. *Rend Sem Mat Univ Padova*, 1989, **82**: 203 - 231.

[7] Colpi R, Menini C. On the structure of $*$-modules [J]. *J Algebra*, 1993, **158**(2): 400 - 419.

[8] Fuller K R. $*$-modules over ring extensions [J]. *Comm Algebra*, 1997, **29**(9): 2839 - 2860.

[9] Trlifaj J. $*$-modules are finitely generated [J]. *J Algebra*, 1994, **169**(2): 392 - 398.

[10] Wei J, Huang Z, Tong W, et al. Tilting modules of finite projective dimension and a generalization of $*$-modules [J]. *J Algebra*, 2003, **268**(2): 404 - 418.

[11] Colby R R, Fuller K R. Costar modules [J]. *J Algebra*, 2001, **242**(1): 146 - 159.

[12] Yao L L, Chen J L. Co-$*^n$-modules [J]. *Algebra Colloq*, 2010, **17**(3): 447 - 456.

[13] Colby R R, Fuller K R. *Equivalence and duality for module categories* [M]. Cambridge, UK: Cambridge University Press, 2004.

[14] Wei J. Equivalences and the tilting theory [J]. *J Algebra*, 2005, **283**(2): 584 - 595.

余星 n 模的推广

姚玲玲　　陈建龙

(东南大学数学系, 南京 211189)

摘要: 如果一个模余自小和无穷拟内射称其为余星无穷模. 研究了其性质及等价刻画. 当一个模为余星无穷模时, 函子 $\mathrm{Hom}_{_RU}(-,U)$ 在 $\mathrm{Copres}^{\infty}(U)$ 中正合. 一个模是余星无穷模当且仅当 U 余自小, 对任意的正合列 $0\to M\to U^I\to N\to 0$ 满足 $M\in\mathrm{Copres}^{\infty}(U)$ 且 I 是一个集合, $N\in\mathrm{Copres}^{\infty}(U)$ 等价于 $\mathrm{Ext}_R^1(N,U)\to\mathrm{Ext}_R^1(U^I,U)$ 是一个单同态当且仅当 U 余自小并且对于任意的正合列 $0\to L\to M\to N\to 0$ 满足 $L, N\in\mathrm{Copres}^{\infty}(U)$, $N\in\mathrm{Copres}^{\infty}(U)$ 等价于导出的列 $0\to\Delta(N)\to\Delta(M)\to\Delta(L)\to 0$ 是正合的当且仅当 U 通过函子 Δ_{U_S} 和 $\Delta_{_RU}$ 导出了子范畴 ${}^{\perp}U_S$ 和 $\mathrm{Copres}^{\infty}(U)$ 之间的对偶. 并且证明了一个模为余星 n 模当且仅当它是余星无穷模且 $\mathrm{Copres}^{\infty}(U)=\mathrm{Copres}^{n}(U)$.

关键词: 余星无穷模; 无穷拟内射; 余自小; 余星 n 模

中图分类号: O154.2