

Optimal subcarrier allocation for cognitive radio with multi-user OFDM and distributed antenna

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Abstract: The subcarrier allocation problem in cognitive radio (CR) networks with multi-user orthogonal frequency-division multiplexing (OFDM) and distributed antenna is analyzed and modeled for the flat fading channel and the frequency selective channel, where the constraint on the secondary user (SU) to protect the primary user (PU) is that the total throughput of each PU must be above the given threshold instead of the “interference temperature”. According to the features of different types of channels, the optimal subcarrier allocation schemes are proposed to pursue efficiency (or maximal throughput), using the branch and bound algorithm and the 0-1 implicit enumeration algorithm. Furthermore, considering the tradeoff between efficiency and fairness, the optimal subcarrier allocation schemes with fairness are proposed in different fading channels, using the pegging algorithm. Extensive simulation results illustrate the significant performance improvement of the proposed subcarrier allocation schemes compared with the existing ones in different scenarios.

Key words: cognitive radio; subcarrier allocation; multi-user OFDM; distributed antenna; branch and bound algorithm; implicit enumeration algorithm; pegging algorithm

Cognitive radio (CR), as an effective technology to solve the problem of the scarcity of wireless spectrum resources, has been developed rapidly in recent years^[1]. According to the Federal Communications Commission regulations, when primary users (PUs) and secondary users (SUs) share the same spectrum bands simultaneously in CR networks, the interference from SU to PU is not allowed to exceed a threshold, which is called the “interference temperature”, in order to guarantee the normal transmission of PUs^[2]. Thus, the “interference temperature” restrains the radio resource management (RRM) in CR networks^[3]. Additionally, orthogonal frequency-division multiplexing (OFDM) is one of the most promising solutions for high data rate transmission^[4]. Accordingly, studies on the RRM, especially subcarrier allocation, in OFDM-based CR networks have an increasingly significant meaning.

Much work has been done on subcarrier allocation in OFDM-based CR networks. In Ref. [5], a maximum likelihood detection model was developed to detect the presence

and locations of licensed user signals in the frequency domain. In Ref. [6], the authors formulated the resource allocation as a multi-dimensional knapsack problem and proposed a low-complexity, greedy max-min algorithm to solve it. In Ref. [7], the authors proposed a cognitive radio-based multi-user resource allocation framework for mobile ad hoc networks using multi-carrier CDMA modulation over a frequency-selective fading channel. In Ref. [8], considering the availability of the subcarriers and the limits on total interference generated to the PUs, the authors presented a solution to an energy-efficient resource allocation problem which maximizes the cognitive radio link capacity. In Ref. [9], the authors presented a wireless unlicensed system that successfully coexists with the licensed systems in the same spectrum range, and a distributed optimization problem was formulated and solved as a dynamic selection of spectrum patterns and power allocations that are better for the available spectrum range without degrading the licensed system performance. In Ref. [10], energy efficient spectrum access was considered for a wireless cognitive radio ad hoc network, where each node is equipped with cognitive radios and has limited energy, and the network is an OFDMA system operating on time slots. Most of the existing studies mentioned above used “interference temperature” to protect PUs’ normal transmission, which cannot maximize the throughput of SUs due to different gains of the sub-carriers.

In this paper, the subcarrier allocation problem in OFDM-based CR networks with distributed antennas^[11] is discussed. The constraint ensuring that no PU is to be disturbed by SUs is not expressed as “interference temperature”, but it is that the throughput of PU should be beyond the given threshold. The main contributions of this work are as follows:

- 1) In different channels, the subcarrier allocation problems are modeled and solved by the branch and bound algorithm and the implicit enumeration algorithm.
- 2) In order to achieve the tradeoff between efficiency and fairness, the subcarrier allocation problems are remodeled as the separable nonlinear MDP (multi-dimensional knapsack problem) in different types of fading channels, which can be solved by the pegging algorithm.

1 System Model

In this paper, the CR network consists of a primary link (or PU), N secondary mobile stations (SUs) and a secondary base station (BS) with M distributed antennas (DAs), as illustrated in Fig. 1. The primary link includes a transmitter (PU-TX) and a receiver (PU-RX). The PU-TX transmits data to the PU-RX with the channel that is assigned to

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the PU. Under the regulation that the SUs do not cause any interference to the normal transmission of the PU, all the secondary mobile stations share the same channel with the PU in order to transmit data to the secondary BS with M DAs connected to the BS. Thus, a macro-diversity single-input multiple-output (SIMO) system is established with the SUs and the DAs, which can achieve higher throughput compared with the single-input multiple-output (SISO) system. Both the PU and the SUs are OFDM-based; thus it is supposed that the channel assigned to the PU includes K ($K \geq N$) subcarriers. As a macro-diversity SIMO, each SU needs few subcarriers than the PU in order to maintain the normal transmission.

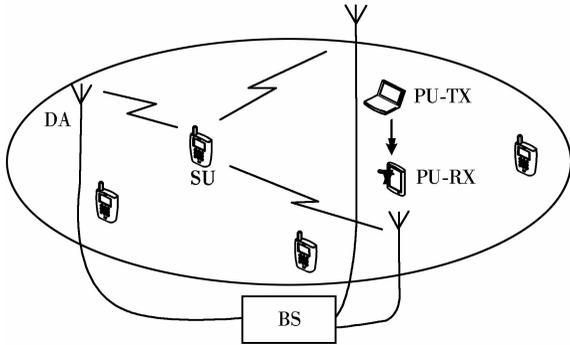


Fig. 1 System model

We consider the subcarrier allocation problem in the up-link of the CR network in this paper. As subcarriers are orthogonal with each other, we assume that h_k^{pp} is the channel gain from the PU-TX to the PU-RX in the k -th ($1 < k < K$) subcarrier. If the k -th subcarrier is assigned to the n -th SU, h_{nmk}^{ss} , h_{mk}^{ps} and h_{nk}^{sp} stand for the channel gain from the n -th SU to the m -th secondary receiving antenna in the k -th subcarrier, the channel gain from the PU-TX to the m -th secondary receiving antenna in the k -th subcarrier, and the channel gain from the n -th SU to the PU-RX in the k -th subcarrier, where $1 \leq m \leq M$ and $1 \leq n \leq N$. These channel gains are provided in channel state information (CSI) that is assumed to be perfectly known.

If we suppose to allocate the k -th subcarrier to the n -th SU, the throughput of the primary link in this subcarrier can be represented as

$$R_{kn}^p = \frac{B}{K} \log_2 \left(1 + \frac{h_k^{pp2} P_k^p}{\Gamma (h_{nk}^{sp2} P_n^s + N_0 (B/K))} \right) \quad (1)$$

where B is the total bandwidth of the channel occupied by the PU; P_k^p is the transmission power of the PU in the k -th subcarrier; P_n^s is the transmission power of the n -th SU; N_0 is the power spectrum density of AWGN; and Γ is the SNR gap between a practical system and theoretical limit, which is the function of BER, $\Gamma = -\ln(5\text{BER})/1.5$.

In the secondary BS, maximal ratio combining (MRC) is adopted to combine the receiving signals from the antennas. So the throughput of the n -th SU in the k -th subcarrier can be written as

$$R_{kn}^s = \frac{B}{K} \log_2 \left(1 + \sum_{m=1}^M \frac{h_{nmk}^{ss2} P_n^s}{\Gamma (h_{mk}^{ps2} P_k^p + N_0 (B/K))} \right) \quad (2)$$

We define $X = [x_{kn}]_{K \times N}$, where

$$x_{kn} = \begin{cases} 1 & \text{if the } k\text{-th subcarrier is assigned to the } n\text{-th SU} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Since the SU should maintain the normal transmission of the PU, the subcarrier allocation must guarantee that

$$\sum_{k=1}^K \sum_{n=1}^N x_{kn} R_{kn}^p \geq C^{\text{th}} \quad (4)$$

where C^{th} is defined as the lower bound of the PU's throughput in order to ensure it against the interference. As the SUs are not allowed to disturb the operation of the PU, generally, the SUs intend to occupy some given spectrum bands of the PU when he owns a relatively better quality of links^[31]. In other words, the PU who shares the spectrum bands with the SUs has a relatively higher SINR. Thus according to Ref. [12], Eq. (1) can be approximately transformed to

$$R_{kn}^p = \frac{B}{K} \log_2 \left(\frac{h_k^{pp2} 2P_k^p}{\Gamma (h_{nk}^{sp2} 2P_n^s + N_0 (B/K))} \right) \quad (5)$$

In the next section the subcarrier allocation problem in this model will be discussed in different channels.

2 Subcarrier Allocation

2.1 Subcarrier allocation in flat fading channel

In the flat fading wireless channel, the channel gains are similarly equivalent in different subcarriers. Thus when a subcarrier is allocated to the n -th SU, the throughputs of the PU and the SU are the same in different subcarriers and are not relevant to k .

We define the subcarrier allocation function as $K = [k_1, k_2, \dots, k_N]$, where k_n ($1 \leq n \leq N$) is the quantity of subcarriers which are assigned to the n -th SU. According to Eq. (5), we can transform Eq. (4) to

$$\sum_{n=1}^N k_n \frac{B}{K} \log_2 \left(\frac{h_k^{pp2} P_k^p}{\Gamma (h_{nk}^{sp2} P_n^s + N_0 (B/K))} \right) \geq C^{\text{th}} \quad (6)$$

which can be transformed to

$$\log_2 \left(\frac{h^{pp2} P^p}{\Gamma} \right) \sum_{n=1}^N k_n - \sum_{n=1}^N \left(k_n \log_2 \left(h_n^{sp2} P_n^s + N_0 \left(\frac{B}{K} \right) \right) \right) \geq \frac{K}{B} C^{\text{th}} \quad (7)$$

It is obvious that

$$\sum_{n=1}^N k_n = K \quad (8)$$

So Eq. (7) can be rewritten as

$$\sum_{n=1}^N \left(k_n \log_2 \left(h_n^{sp2} P_n^s + N_0 \frac{B}{K} \right) \right) \leq K \log_2 \left(\frac{h^{pp2} P^p}{\Gamma} \right) - \frac{K}{B} C^{\text{th}} \quad (9)$$

Accordingly, in order to maximize the total throughput of the SUs under the aforementioned constraints, the optimal subcarrier allocation scheme is to find out a group of quantities of subcarriers that are assigned to the SUs, which can be represented by

$$\max_K \sum_{n=1}^N k_n r_n \quad (10)$$

s. t.

$$\begin{aligned} \sum_{n=1}^N k_n w_n &\leq W \\ 1 &\leq k_n \leq K & 1 \leq n \leq N \\ \sum_{n=1}^N k_n &= K \end{aligned}$$

where

$$\begin{aligned} w_n &= \log_2 \left(h_n^{\text{sp}2} P_n^s + N_0 \frac{B}{K} \right) \\ W &= K \log_2 \left(\frac{h_k^{\text{pp}2} P_k^p}{\Gamma} \right) - \frac{K}{B} C^{\text{th}} \\ r_n &= R^s(n) \end{aligned}$$

The optimization problem (10) is an MDKP, which is also a separable linear integer programming with N dimensional degrees of freedom essentially. This problem can be solved by the branch and bound algorithm. In a typical branch and bound algorithm, there are two main steps: branching and bounding. The branching step divides a feasible set of the problem into subsets and formulates the corresponding subproblems with these subsets. The bounding step finds the upper and lower bounds for these subproblems within the corresponding subsets. Thus all the feasible combinations are used to formulate the subproblems and then a global optimal solution is obtained by removing or pruning the branches. The details of the branch and bound algorithm are described in Ref. [13].

2.2 Subcarrier allocation in frequency selective channel

In the frequency selective wireless channel, channel gains cannot be simplified as in section 2.1. As the throughputs of the PU and the SUs in different subcarriers are discrepant, it is denoted that \mathbf{X} is the subcarrier allocation function, which is defined in Eq. (3).

In order to avoid the interference and collision, it is not allowed that two or more SUs obtain the same subcarrier synchronously. In other words, each subcarrier is only allocated to one SU. Thus

$$\sum_{n=1}^N x_{kn} = 1 \quad 1 \leq k \leq K \quad (11)$$

What is more, it is guaranteed that every SU must be assigned at least one subcarrier. Namely,

$$\sum_{k=1}^K x_{kn} \geq 1 \quad 1 \leq n \leq N \quad (12)$$

As the SUs are not allowed to disturb the operation of the PU, according to Eq. (5), Eq. (4) is rewritten as

$$\sum_{k=1}^K \sum_{n=1}^N x_{kn} \frac{B}{K} \log_2 \left(\frac{h_k^{\text{pp}2} P_k^p}{\Gamma (h_{nk}^{\text{sp}2} P_n^s + N_0 (B/K))} \right) \geq C^{\text{th}} \quad (13)$$

which can be transformed to

$$\sum_{n=1}^N \left(\log_2 \left(\frac{h_k^{\text{pp}2} P_k^p}{\Gamma} \right) \sum_{n=1}^N x_{kn} \right) -$$

$$\sum_{k=1}^K \sum_{n=1}^N x_{kn} \log_2 \left(h_{nk}^{\text{sp}2} P_n^s + N_0 \frac{B}{K} \right) \geq \frac{K}{B} C^{\text{th}} \quad (14)$$

According to Eq. (12), Eq. (15) can be rewritten as

$$\sum_{k=1}^K \sum_{n=1}^N x_{kn} \log_2 \left(h_{nk}^{\text{sp}2} P_n^s + N_0 \frac{B}{K} \right) \leq \sum_{k=1}^K \log_2 \left(\frac{h_k^{\text{pp}2} P_k^p}{\Gamma} \right) - \frac{K}{B} C^{\text{th}} \quad (15)$$

Therefore, the optimal subcarrier allocation scheme is to find out a group of \mathbf{X} to maximize the total throughput of SUs under the aforementioned constraints, which can be represented as

$$\max_{\mathbf{X}} \sum_{k=1}^K \sum_{n=1}^N x_{kn} r'_{kn} \quad (16)$$

s. t.

$$\begin{aligned} \sum_{k=1}^K \sum_{n=1}^N x_{kn} w'_{kn} &\leq W' \\ \sum_{n=1}^N x_{kn} &= 1 & 1 \leq k \leq K \\ \sum_{k=1}^K x_{kn} &\geq 1 & 1 \leq n \leq N \end{aligned}$$

where

$$\begin{aligned} w'_{kn} &= \log_2 \left(h_{nk}^{\text{sp}2} P_n^s + N_0 \frac{B}{K} \right) \\ W' &= \sum_{k=1}^K \log_2 \left(\frac{h_k^{\text{pp}2} P_k^p}{\Gamma} \right) - \frac{K}{B} C^{\text{th}} \\ r'_{kn} &= R^s_{kn} \end{aligned}$$

The optimization problem (16) is another MDKP, which is essentially also a separable linear 0-1 integer programming with $K \times N$ dimensional degrees of freedom. According to Ref. [13], the branch and bound algorithm is not efficient enough for this kind of problem, as the bound is confined to $[0, 1]$ in this algorithm. The length of this interval is so small that the branch and bound algorithm just needs to judge whether the variables are 0 or 1, which is similar to the enumeration algorithm and loses the advantages of the branch and bound algorithm. Accordingly, another approach named the 0-1 implicit enumeration algorithm is used to cope with this problem. The details of this algorithm can be found in Ref. [13]. In addition, considering problem (16) is a 0-1 integer programming, the dimension of degrees of freedom is KN , which is much greater than N in problem (10). Accordingly, problem (16) is more complex than problem (10)^[13].

2.3 Optimal subcarrier allocation with fairness

As we know, efficiency and fairness are two crucial issues in RRM. In sections 2.1 and 2.2, the subcarrier allocation problems are studied in order to maximize the total throughput of the SUs, which stands for efficiency. Since the subcarrier allocation problem can be modeled as an integer programming problem that is a discrete optimization problem, not a continuous optimization problem essentially, it is unreasonable to pursue absolute fairness. Thus it is significant to study the scheme to compromise efficiency and fairness.

According to Ref. [14], in the optimal subcarrier allocation scheme with fairness, the objectives of problems (10) and (16) need to be transformed to

$$\max_K G_1 = \sum_{n=1}^N \ln(k_n r_n) \quad (17)$$

$$\max_X G'_1 = \sum_{n=1}^N \ln\left(\sum_{k=1}^K x_{kn} r'_{kn}\right) \quad (18)$$

And the constraints are the same.

Both problems (17) and (18) are separable nonlinear MDKPs. There are several approaches to cope with this kind of problem and one of the most distinguished methods is the pegging algorithm^[15]. In the pegging algorithm, the objective functions and constraint functions must be convex. Since both log-sum-exp functions and affine functions are convex, the objective functions and constraint functions in problems (17) and (18) all comply with this requirement. The fundamental idea of the pegging algorithm is that a “fixing variable” approach is involved in the branch and bound. So the slack problem corresponding to the original integer programming problem is directly solved without considering the bounds of the variables. Moreover, the variables that dissatisfy the demands of bounds are fixed in the upper and lower bounds until the variables that are not fixed in the solution of the slack problem satisfy the requirements of the bounds. Thus the pegging algorithm reduces the dimension of the slack problem dramatically in iteration, which enhances the efficiency of solving this problem. The detailed steps of this algorithm can be found in Ref. [15].

3 Simulation Results and Analysis

In this section, extensive simulation results are shown to compare the performances of different subcarrier allocation schemes in different channels.

The parameters in the simulations are chosen based on the parameters widely adopted^[3,6,12] as follows. All the wireless channels in the simulations are assumed to be Rayleigh fading. The bandwidth of each subcarrier belonging to a PU is 15 kHz. The number of SUs N is supposed to be 10. The quantity of distributed antennas M is set to be 8. The transmission power P_n^p of the PU is determined by the water filling algorithm. The transmitting powers of SUs are set to be equal in this paper in order to enable subcarrier allocation to be only determined by the radio environment, which makes it feasible. Thus P_n^s is set to be 1 W. The background noise N_0 is assumed to be -117 dBm. The threshold of PU throughput is 100 kbit/s, and BER is set to be 10^{-3} .

In Figs. 2 and 3, the efficiency and fairness of the existing subcarrier allocation scheme, the optimal subcarrier allocation scheme and the optimal subcarrier allocation scheme with fairness are compared in the flat fading channel. From the figures, we can draw the following conclusions. The total throughput of the SUs in the optimal subcarrier allocation scheme is enhanced dramatically compared with the existing subcarrier allocation scheme, but its fairness is not improved. Additionally, the total throughput of the SUs in the optimal subcarrier allocation scheme with fairness is lower than the one in the optimal subcarrier allocation scheme, but its fairness is enhanced to some degree. Besides, the opti-

mal subcarrier allocation scheme with fairness is superior to the existing subcarrier allocation scheme in both efficiency and fairness.

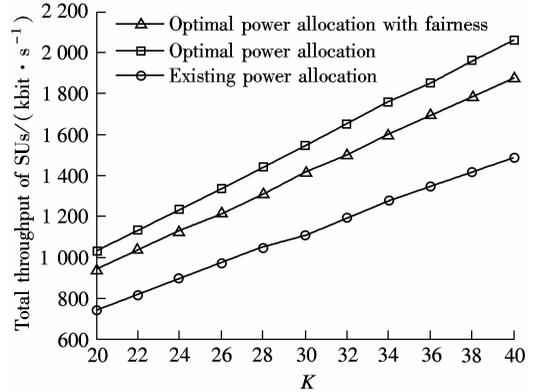


Fig. 2 Comparison of total throughput of SUs among different schemes in flat fading channel

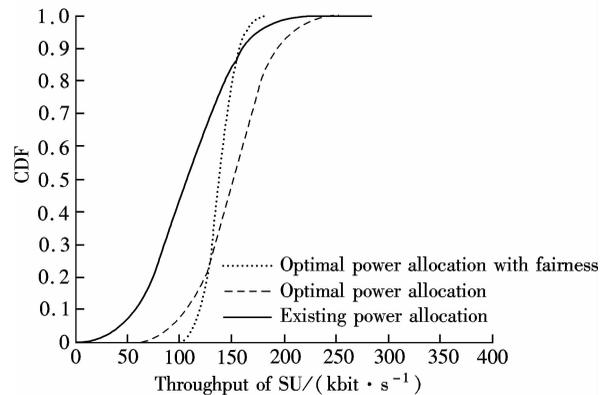


Fig. 3 Comparison of fairness among different schemes in flat fading channel ($K = 30$)

In Figs. 4 and 5, the efficiency and fairness of these three different schemes are compared in the frequency selective fading channel. The conclusions from these figures are the same as those from Figs. 2 and 3, which are not explained in detail.

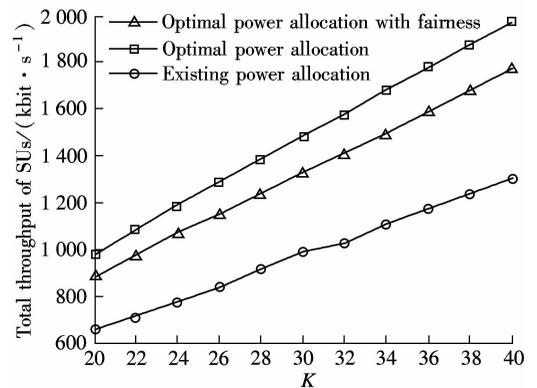


Fig. 4 Comparison of total throughput of SUs among different schemes in frequency selective fading channel

4 Conclusion

In this paper, we focus on the subcarrier allocation problem in CR networks with multi-user OFDM and distributed

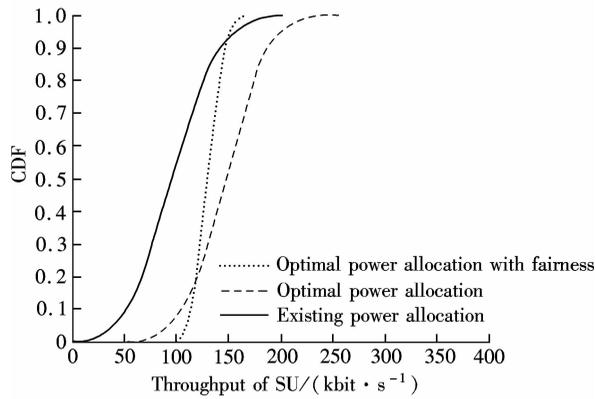


Fig. 5 Comparison of fairness among different schemes in frequency selective fading channel ($K=30$)

antennas in different types of channels. Moreover, according to the different features of the fading channels, the optimal subcarrier allocation schemes are proposed to pursue efficiency (or maximal throughput), using the branch and bound algorithm and the 0-1 implicit enumeration algorithm. Furthermore, in order to achieve the tradeoff between efficiency and fairness, the optimal subcarrier allocation scheme with fairness is proposed using the pegging algorithm. Finally, extensive simulation results illustrate the features of different subcarrier allocation schemes and indicate the performance improvement of the proposed schemes.

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基于多用户 OFDM 和分布式天线的认知无线电中最优子载波分配方案

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摘要: 分别在平坦衰落和频率选择性衰落的无线信道模型中建立了基于多用户正交频分复用技术 (OFDM) 和分布式天线的认知无线电系统中子载波分配模型. 该模型不再用“干扰温度”限制认知用户来保护主用户, 而是规定主用户在其所有子载波上的吞吐量之和需要高于一定的门限. 然后, 根据不同信道模型的特点提出了最优子载波分配方案, 该方案利用分支定界法和隐枚举法最大化系统的吞吐量. 最后, 利用 Pegging 算法提出了效率与公平折中的子载波分配方案. 仿真证明, 在不同场景中所提出的子载波分配方案与现有方案相比在吞吐量和公平性方面均具有较大的优势.

关键词: 认知无线电; 子载波分配; 多用户 OFDM; 分布式天线; 分支定界法; 隐枚举法; Pegging 算法

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