

# Volumetric extraction of porous materials based on octree algorithm

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**Abstract:** Through the octree data structure analysis, a volumetric dataset of closed-cell porous materials is converted into a dataset of hierarchical octree nodes, and then the specific traversal search algorithm on the octree nodes is depicted in details, which is involved in six steps of the volume growth model and one step of the volume decomposition model. Moreover, the conditions of both the proceeding traversal and three possibilities of terminating are given, and the traversal algorithm of completeness is proved from a theoretical perspective. Finally, using a simulated volumetric dataset of columnar pores, the extracting effectiveness of the octree traversal algorithm is verified. The results show that the volume and the distribution information of pores can be successfully extracted by the proposed algorithm, which builds a solid foundation for a more effective performance analysis of porous materials.

**Key words:** octree; closed-cell porous materials; volume of pores; traversal algorithm

Porous materials are usually composed of gaseous and solid substances and widely used for production and architecture due to their special characteristics such as elasticity, light-weight, high strength and so on. These properties mainly depend on the pore structures and distribution of the materials which sometimes are difficult to control in the production procedures. For figuring out the pore structures in the materials, conventional methods such as microscopic testing are used to check the surface of the materials and obtain the estimated parameters of pores through repeated physical measurements<sup>[1]</sup> or a mathematical statistical method.

With the development of imaging techniques, the advanced imaging instruments such as CT, MR and micro CT can easily obtain tomographic datasets in a non-invasive way, which makes it convenient in practice to explore the characteristics of pores. Based on the tomographies and volume datasets, the spatial structure of the pores is possible to be determined. If the anomalous pores are assumed to be regular spheres or cubes in processing<sup>[2]</sup>, some methods like statistic theory and the estimation method can find the exact location and size information of the pores, although the results are far away from the nature of the pores. Marching cube<sup>[3-4]</sup> is another method to calculate the parameters; however, this method cannot tell the exact volume size and lo-

cation of the pores, although it can easily separate pores from the solid substance.

Topology morphology is the most direct method which can obtain important relative characteristics of the pores. Although it is easy to separate pores from two-dimensional images<sup>[5]</sup>, it is very difficult three-dimensionally because the contour of a pore volume may exist in many transverse slices.

In order to obtain the characteristics of pores from the tomographic volume dataset, we propose a new volumetric extraction method based on morphology and the octree algorithm<sup>[6-7]</sup>. By using the recursive, separating and merging theory of the octree algorithm, the volumes and locations of all the pores can be easily determined.

## 1 Materials and Methods

### 1.1 States of pores

The pores in the solid substances have two different states: open cell and closed cell. The former means that the cells are connected to each other and there are no barriers among the boundaries of the cells, while the latter shows there is no interconnectivity among them, and they can also be distinguished easily from their morphology. In this paper, we will work on the closed cell materials.

### 1.2 Octree data structure

The data structure that we use to solve this problem is called the octree structure<sup>[8]</sup>, which is a kind of binary tree structure, that is to say, each parent node can be divided into eight child nodes. Each node has six parameters: parameter pParent is a pointer of its parent; parameter pChild  $[i]$  ( $i \in [0, 7]$ ) is a pointer array representing its eight children; parameters  $n_m$  and  $l_m$  are used to indicate the space location and the level of a certain node in the data volume; pore flag  $F_p$  indicates whether a certain node is a pore or not and traversal flag  $F_t$  indicates whether it has been counted as part of a pore or not.

### 1.3 Representation of volumetric data with octree structure

Just for the sake of discussion, the width, height and slices of the volume dataset are all assumed with  $2^m + 1$  pixels. Each cubic unit of the volume dataset is defined by a collection of eight adjacent pixels, and then we can obtain  $8^m$  small cubes totally. The procedures of building the octree structure grid are as follows:

$l_0$  is the 0th level of the octree grid, and it has  $8^m$  nodes. The sequence allocating all the nodes of the volume dataset of this level is from left to right, top to bottom and front to back. The data pointer of each node points to the front-top-

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left pixel of the above mentioned cubic unit. The location of each node  $n_0$  can be written as

$$n_0 = 4^m p_0 + 2^m h_0 + w_0 \\ 0 \leq p_0 < 2^m, 0 \leq h_0 < 2^m, 0 \leq w_0 < 2^m$$

The variant  $p_0$  is the slice where the left-top-front voxel of the small cube locates;  $h_0$  is the vertical location of the pixel and  $w_0$  is the horizontal location. Because there are no child nodes at the 0th level, the elements of the pointer array in the nodes at this level must be set empty. At the beginning, the traversal flag  $F_t$  of all the nodes is set true and the flag  $F_p$  is set true or false depending on whether it is a pore or not.

$l_1$  is the first level of the octree grid. Every eight child nodes of the 0th level compose a node of the first level. The flags  $F_t$  and  $F_p$  of each node can be set almost the same as the 0th level. As a new volumetric subset of the  $8^m$  nodes of the 0th level, the sequence of this level is the same as that of the 0th level: left to right, top to bottom and front to back. The location of each new node  $n_1$  can be represented as follows:

$$n_1 = 4^{m-1} p_1 + 2^{m-1} h_1 + w_1 \\ 0 \leq p_1 < 2^{m-1}, 0 \leq h_1 < 2^{m-1}, 0 \leq w_1 < 2^{m-1}$$

where  $p_1$ ,  $h_1$  and  $w_1$  are the slice, the vertical location and the horizontal location of the front-top-left node of the 0th level, respectively. The pointer array  $pChild[i]$  ( $i \in [0, 7]$ ) of each node at  $l_1$  points at its eight child nodes of  $l_0$ , and its eight child nodes have the same parent pointer  $pParent$  at  $l_1$ .

All the nodes of the second level  $l_2$ , the third level  $l_3$ , till the  $m$ -th level  $l_m$  can be managed as above. The last level  $l_m$  contains only one element that represents the whole volumetric dataset.

By this management, a volumetric dataset is converted into a dataset of some hierarchical octree nodes.

#### 1.4 Volumetric calculation of pores

As shown in Fig. 1, the black contour enclosing the black pixels can be regarded as the boundary of one pore. The sum of all black pixels represents the area of the zone. Expanding this idea from 2D to 3D, all the cubes enclosed by boundary surfaces are summed up to represent the volume of one pore.

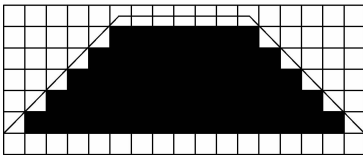


Fig. 1 Determination of volumetric pores

#### 1.5 Traversal algorithm based on octree nodes

The procedures of the traversal process algorithm through all the pores can be divided into seven steps. Steps 1 to 5 are based on the volume growth model and step 6 is on the volume decomposition model.

**Step 1** Define a volume variant  $V=0$  at the beginning.

**Step 2** Search forwards at the first node of  $l_0$ . If the traversal flag  $F_t$  is true and the porous flag  $F_p$  is false, skip over this node and go on searching forwards; else, if  $F_t$  is false and  $F_p$  is true, it means that this node belongs to a pore which has never been traversed and then go to step 3.

**Step 3** Trace upwards to the parent of this node and check if its parent node meets the criteria ( $F_t = \text{false}$ ,  $F_p = \text{true}$ ) or not. If the answer is yes, go on tracing upwards until both  $F_t$  and  $F_p$  of a certain parent node do not meet the criteria ( $F_t = \text{false}$ ,  $F_p = \text{true}$ ) which means either the boundary of the pore has been reached or some of the ambient nodes have been traversed. Now the tracing way needs to be changed for filling this pore completely. Go to step 4.

**Step 4** The stopping node is denoted as  $T$  at the  $n$ -level  $l_n$ ,  $1 \leq n \leq m$ . Trace downwards to level  $l_{n-1}$  and find its eight child nodes  $T_i$  ( $i \in [0, 7]$ ). Go to step 5.

**Step 5** Check if  $T_i$  has child nodes (At the level  $l_0$ , there are not children any more), and then it will be classified into two cases ( $l_{n-1} = l_0$  or  $l_{n-1} \neq l_0$ ):

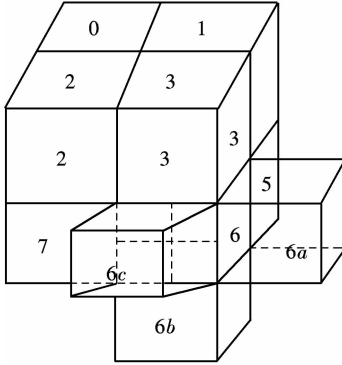
**Case 1**  $n - 1 > 0$ ,  $V = V + V(T_i)$ , where  $V(T_i)$  represents the volume of node  $T_i$ . Set  $F_t$  of all the levels of child nodes of node  $T_i$  to be true to make sure that all these nodes have already been counted in, then trace back to level  $l_{n-2}$  and check the three adjacent outer nodes  $T_{i_{va}}$ ,  $T_{i_{vb}}$  and  $T_{i_{vc}}$  of the eight child nodes  $T_{i_v}$  ( $0 \leq v \leq 7$ ) (see Fig. 2), and determine if the traversal flag  $F_t$  is false or not. The three adjacent outer nodes are at the same level  $l_{n-2}$ . If  $F_t$  is true, terminate the calculation of this node branch; if  $F_t$  is false, check whether its flag  $F_p$  is true or not. If  $F_p$  is true it denotes that this node meets the criteria “This node belongs to a pore and it has never been traversed”, so go back to step 3; if  $F_p$  is false, it means that not all the eight children of this node belong to a pore, go to step 6.

**Case 2**  $n - 1 = 0$ ,  $V = V + V(T_i)$ . Set the flag  $F_t$  of node  $T_i$  to be true. Unlike case 1, there are no child nodes in node  $T_i$  of level  $l_0$ , so  $T_i$  has six adjacent outer nodes  $T_{i_a}$ ,  $T_{i_b}$ ,  $T_{i_c}$ ,  $T_{i_d}$ ,  $T_{i_e}$  and  $T_{i_f}$ . Check marks  $F_t$  and  $F_p$  respectively. If  $F_t$  is false and  $F_p$  is true, it means that this node meets the criteria “This node belongs to a pore and it has never been traversed”, go back to step 3. Otherwise, if none of the six adjacent outer nodes  $T_{i_a}$ ,  $T_{i_b}$ ,  $T_{i_c}$ ,  $T_{i_d}$ ,  $T_{i_e}$  and  $T_{i_f}$  meets the condition “ $F_t$  is false and  $F_p$  is true”, it means that the adjacent outer nodes either have been traversed or are the boundary, and then terminate the calculation of this node branch.

**Step 6** As shown in Fig. 2, the cube marked with numbers 0 to 7 can be regarded as  $T_{i_v}$  ( $0 \leq v \leq 7$ ). The convex node 6c can be regarded as another node  $T_{j_u}$  ( $u = 5$ ) and now we assume its flag  $F_t$  is false and  $F_p$  is false, so we know that not all of its children nodes meet the criteria “This node belongs to a pore and it has never been traversed”. If it cannot be divided, it needs to be terminated and returned; if it does not belong to level  $l_0$ , it needs to be decomposed as follows (see Fig. 2). Divide this node into eight child nodes  $T_{j_{uc}}$  ( $u = 5, 0 \leq c \leq 7$ ), choose the four child nodes  $T_{j_{uc}}$  ( $c \in \{0, 1, 4, 5\}$ ) which are adjacent to  $T_{i_v}$  ( $v = 6$ ) and then check their flags  $F_p$  and  $F_t$  respectively. If

the flags of node  $T_{j_{uc}}(c \in \{0, 1, 4, 5\})$  meet “ $F_t$  is false and  $F_p$  is true”, go to step 3; otherwise, divide node  $T_{j_{uc}}(c \in \{0, 1, 4, 5\})$  and try this step again.

**Step 7** After all the procedures above, a pore can be calculated and marked. Now return to step 1 to go on calculating the next pore.



**Fig. 2** Volume decomposition and combination model

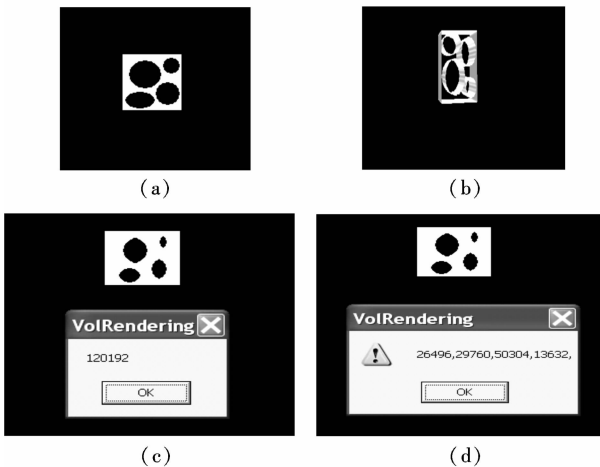
### 1.6 Completeness of traversal algorithm

Can this method traverse all the nodes of the 0th level? In the following, we will prove it with its contradiction. First assume that the location of the first traversing node is  $(x, y, z)$  and a certain node  $(x', y', z')$  cannot be traversed. This means that the nodes  $(x' \pm 1, y' \pm 1, z' \pm 1)$  have also not been traversed, and it can be inferred that the node  $(x, y, z)$  has not been traversed either, which is contradictory to the assumption, thus all the nodes should be traversed.

## 2 Results and Discussion

Sixty-five slices of the image in Fig. 3(a) stack a volume dataset as shown in Fig. 3(b), which contains four columnar pores. By calculating the volume of these irregular column pores respectively, the proposed algorithm is validated.

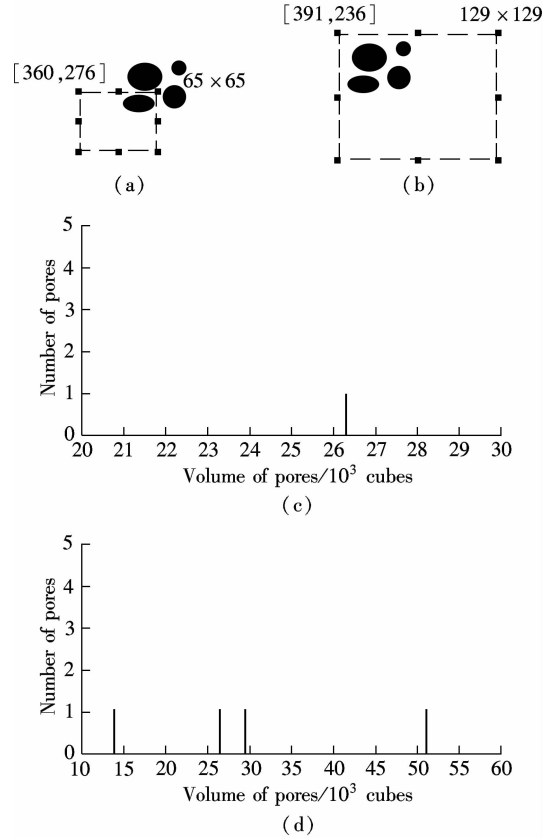
In Fig. 3(c), using the regular image segmentation method, the total volume of voxels of the four columnar pores is counted to the number of 120 192. In Fig. 3(d), the four volume numbers 26 496, 29 760, 50 304 and 13 632 represent



**Fig. 3** Algorithm validation by the simulated image data. (a) Simulated slice composing volume data; (b) Visualization effect of volume data; (c) Volume of all pores; (d) Volumes of four pores

the volumes of the four columnar pores achieved by our algorithm, respectively. As a result, the sum of the four volumes is equal to the total sum shown in Fig. 3(c) and the size difference of the four columns is consistent with our intuitive sense.

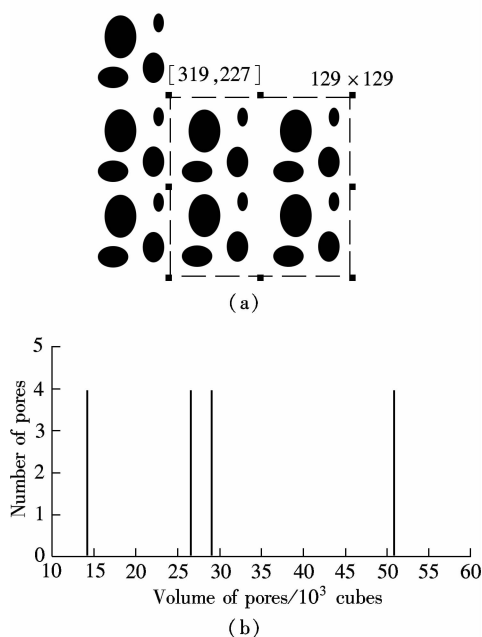
For further validation, a cubic ROI with the size of  $65 \times 65 \times 65$  is chosen, which only contains a columnar pore (see Fig. 4(a)), and its volume distribution is shown in Fig. 4(c). Another  $65 \times 65 \times 65$  cubic region which contains four columnar pores are shown in Fig. 4(b) and its ROI volume distribution is shown in Fig. 4(d). A  $129 \times 129 \times 129$  cubic region (see Fig. 5(a)) which contains 16 columnar pores copies four times from the original image Fig. 3(a) and its ROI volume distribution is shown in Fig. 5(b). From Fig. 4(d), it can be clearly seen that there is one column which is the same as the one in Fig. 4(c). This means that the method is feasible. Comparing Fig. 5(b) with Fig. 4(d), it is obvious that the region selected in Fig. 5(b) contains the same columns with the region selected in Fig. 4(d). The sum of different columns is four times greater than that in Fig. 4(d). The result corresponds to the setting of the original data, so this method is correct.



**Fig. 4** Volume distribution with different ROI. (a) One columnar pore in ROI; (b) Four columnar pores in ROI containing the same one in (a); (c) Volume distribution of ROI of (a); (d) Volume distribution of ROI of (b)

## 3 Conclusion

This paper focuses on calculating the volume of the pores by using the octree algorithm. This method depends on graphic analysis to get close to the real size of the pores. If all the locations of the voxels composing of the pores are



**Fig. 5** Volume distribution diagram. (a) Selected region contains 16 column pores copied from original four column pores; (b) Volume distribution of ROI

achieved, it is more convenient for the visualization and detail analysis of the pores. The results show that the proposed algorithm can make correct calculations for all the simulating pores. In the future, we will put this algorithm into the

real pores data from micro CT to validate the effectiveness of the algorithm.

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## 基于八叉树算法的多孔材料特征提取

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**摘要:**通过对八叉树数据结构的分析,将闭孔材料的体数据转换成具有八叉树分层结点的体数据,详述了基于八叉树结点的孔泡逐层搜索算法的6步体积增长遍历和一步体积分解遍历步骤,给出了算法继续遍历的条件和终止遍历的3种可能,并从理论上证明了该遍历算法的完备性.在算法调用过程中,确定了孔泡基于体素为单位的体积和分布等参数的计算方法.利用仿真的柱状孔泡图像体数据,对八叉树遍历算法的提取效果进行了验证.实验结果表明,基于八叉树结构的遍历算法能准确提取闭孔材料中孔泡的体积和分布信息,为更有效地进行多孔材料性能分析奠定了基础.

**关键词:**八叉树;闭孔材料;孔体积;遍历算法

**中图分类号:**TP301.6