

Stabilization design for continuous-time Takagi-Sugeno fuzzy systems based on new relaxed conditions

Pan Juntao Fei Shumin Mu Chaoxu Xue Mingxiang

(Key Laboratory of Measurement and Control of Complex Systems of Engineering of Ministry of Education, Southeast University, Nanjing 210096, China)

Abstract: This paper deals with the problem of stabilization design for a class of continuous-time Takagi-Sugeno(T-S) fuzzy systems. New stabilization conditions are derived based on a relaxed approach in which both fuzzy Lyapunov functions and staircase membership functions are used. Through the staircase membership functions approximating the continuous membership functions of the given fuzzy model, the information of the membership functions can be brought into the stabilization design of the fuzzy systems, thereby significantly reducing the conservativeness in the existing stabilization conditions of the T-S fuzzy systems. Unlike some previous fuzzy Lyapunov function approaches reported in the literature, the proposed stabilization conditions do not depend on the time-derivative of the membership functions that may be the main source of conservatism when considering fuzzy Lyapunov functions analysis. Moreover, conditions for the solvability of the controller design are written in the form of linear matrix inequalities, but not bilinear matrix inequalities, which are easier to be solved by convex optimization techniques. A simulation example is given to demonstrate the validity of the proposed approach.

Key words: Takagi-Sugeno fuzzy model; fuzzy Lyapunov function; staircase membership function; linear matrix inequality

Nonlinear control systems based on the Takagi-Sugeno (T-S) fuzzy model^[1] have received much attention over the last decade. The merit of such fuzzy model-based control methodology is that it offers an effective and exact representation of complex nonlinear systems in a compact set of state variables.

Within the general frameworks of T-S fuzzy model-based control systems, stability analysis and controller synthesis issues are commonly investigated based on searching a quadratic Lyapunov function^[2]. Unfortunately, the results derived by quadratic Lyapunov functions are found to be very conservative^[3]. Many subsequent works^[4-5] are devoted to obtaining less conservative conditions based on the quadratic Lyapunov function. Extra variables may provide advantages to LMI solvers, allowing them to find solutions when the conventional conditions fail due to numerical reasons. In order to reduce the conservativeness, alternative classes of

Lyapunov function candidates have been proposed. It is noted that the piecewise Lyapunov function is an interesting alternative. However, the control design using piecewise Lyapunov functions are generally in terms of bilinear matrix inequalities (BMIs) which are not easy to be solved by convex optimization techniques. Another alternative Lyapunov function candidate was developed for continuous-time T-S fuzzy systems in Ref. [6], and discrete-time T-S fuzzy systems in Ref. [7], namely fuzzy Lyapunov functions. These approaches are employed to reduce the conservativeness given by the quadratic Lyapunov function, though the results in the continuous-time domain are not as powerful as those corresponding to the discrete case. This asymmetry is explained by the fact that most of these Lyapunov functions depend on the same membership functions of the model, hereby taking into account structural information at the price of dealing with the time-derivatives of membership functions. Since the explicit presence of the time-derivative of membership functions in the stability and stabilization conditions, fewer results concerning the fuzzy Lyapunov function in the continuous-time domain have been reported. To overcome this deficiency, a new line-integral fuzzy Lyapunov function approach to analysis and design of T-S fuzzy systems was proposed in Ref. [8]. The derived results do not depend on the time-derivatives of membership functions. Nevertheless, as the stabilization design still relies on BMIs, one must resort to a special algorithm and properly choose some parameters to obtain a fuzzy controller. All the aforementioned studies do not consider the membership functions of the fuzzy model in the analysis and design issues for the T-S fuzzy systems. It is revealed that the information of membership functions plays an important role in the relaxation of the result of stability analysis^[9-10]. In this paper, we present an LMI-based controller design method for continuous-time T-S fuzzy systems using the fuzzy Lyapunov function approach, in which the information of the membership functions are fully utilized.

1 Problem Formulations and Preliminaries

Consider the following T-S fuzzy model with r plant rules as follows:

Plant rule i IF $x_1(t)$ is F_1^i , ..., and $x_n(t)$ is F_n^i , THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t) \quad i = 1, 2, \dots, r \quad (1)$$

where $x(t)$ is the state, and $u(t)$ is the input. $F_j^i (j = 1, 2, \dots, n)$ are the fuzzy sets. r denotes the number of IF-THEN rules. A_i and B_i are constant matrices. The final T-S fuzzy system is inferred as

Received 2010-03-31.

Biographies: Pan Juntao (1983—), male, graduate; Fei Shumin (corresponding author), male, doctor, professor, smfei@seu.edu.cn.

Foundation items: The National Natural Science Foundation of China (No. 60764001, 60835001, 60875035, 61004032), the Postdoctoral Research Fund of Southeast University, the Natural Science Foundation of Jiangsu Province (No. BK2008294).

Citation: Pan Juntao, Fei Shumin, Mu Chaoxu, et al. Stabilization design for continuous-time Takagi-Sugeno fuzzy systems based on new relaxed conditions[J]. Journal of Southeast University (English Edition), 2010, 26 (4): 550 – 553.

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^r h_i(\mathbf{x}(t))(A_i\mathbf{x}(t) + B_i\mathbf{u}(t)) \quad (2)$$

where $h_i(\mathbf{x}(t)) = \frac{\prod_{j=1}^r \mu_{ij}(x_j(t))}{\sum_{i=1}^r \prod_{j=1}^r \mu_{ij}(x_j(t))}$, $\mu_{ij}(x_j(t))$ is the grade of the membership function of $x_j(t)$ in F_{ij}^r , and

$$h_i(\mathbf{x}(t)) \geq 0, \quad \sum_{i=1}^r h_i(\mathbf{x}(t)) = 1 \quad (3)$$

In the following, we will drop the argument of $h_i(\mathbf{x}(t))$ and denote $\{1, 2, \dots, r\}$ by R for simplicity. In this paper, the fuzzy controller is designed using the concept of parallel distributed compensation.

Control rule i IF $x_1(t)$ is F_1^i , ..., and $x_n(t)$ is F_n^i , THEN

$$\mathbf{u}(t) = -\mathbf{K}_i\mathbf{x}(t) \quad i \in R$$

The defuzzified output of the fuzzy controller is proposed as

$$\mathbf{u}(t) = -\sum_{i=1}^r h_i\mathbf{K}_i\mathbf{x}(t) \quad (4)$$

Substituting Eq. (4) into Eq. (2) gives the following closed-loop T-S fuzzy system dynamics:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j (A_i - B_i\mathbf{K}_j)\mathbf{x}(t) \quad (5)$$

For convenience of comparison between this paper and the existing ones, existing stabilization conditions derived by the quadratic Lyapunov function are given as follows.

Theorem 1^[11] Closed-loop T-S fuzzy system (5) is asymptotically stable with the control gain given by $\mathbf{K}_j = \mathbf{M}_j \mathbf{X}^{-1}$, if there exist a symmetric matrix \mathbf{X} and any matrices $\{\mathbf{M}_j\}_{j=1}^r$ such that

$$\left. \begin{array}{l} \theta_{ii} < 0 \quad i \in R \\ \theta_{ij} + \theta_{ji} < 0 \quad i < j \in R \end{array} \right\} \quad (6)$$

where $\theta_{ij} = \mathbf{X}\mathbf{A}_i^T + \mathbf{A}_i\mathbf{X} - \mathbf{M}_j^T\mathbf{B}_i^T - \mathbf{B}_i\mathbf{M}_j$.

Remark 1 The stabilization conditions proposed in theorem 1 have been shown to be very conservative^[7]. Conservativeness comes from two main facts: the first one is using the same simple quadratic Lyapunov function for all the T-S models, and the second fact is that solving Eq. (6) is done independently to form membership functions h_i .

2 Main Results

The following null product will be used to derive new stabilization conditions:

$$2(\mathbf{x}^T(t)\boldsymbol{\Psi} + \dot{\mathbf{x}}^T(t)\boldsymbol{\mu}\boldsymbol{\Psi})\left(\dot{\mathbf{x}}(t) - \sum_{i=1}^r \sum_{j=1}^r h_i h_j (A_i - B_i\mathbf{K}_j)\mathbf{x}(t)\right) = 0 \quad (7)$$

The proposed stabilization conditions are based on the fuzzy Lyapunov function developed in Ref. [8].

$$V(\mathbf{x}(t)) = 2 \int_{\Gamma(0, \mathbf{x})} f(\boldsymbol{\varepsilon}) \cdot d\boldsymbol{\varepsilon} \quad (8)$$

where $\Gamma(0, \mathbf{x})$ is a path from the origin to the present state \mathbf{x} ; (\cdot) stands for the inner product of vectors.

Remark 2 To be a Lyapunov function candidate, Eq. (8) should be independent of $\Gamma(0, \mathbf{x})$ ^[8]. One solution for $V(\mathbf{x}(t))$ to be path independent is given by

$$f(\mathbf{x}) = \sum_{i=1}^r h_i(\mathbf{P}_0 + \mathbf{D}_i)\mathbf{x} = \sum_{i=1}^r h_i\mathbf{P}_i\mathbf{x} \quad (9)$$

where

$$\mathbf{P}_0 = \begin{bmatrix} 0 & p_{12} & \dots & p_{1n} \\ p_{12} & 0 & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1n} & p_{2n} & \dots & 0 \end{bmatrix}, \quad \mathbf{D}_i = \begin{bmatrix} d_{11} & 0 & \dots & 0 \\ 0 & d_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & d_{nn} \end{bmatrix}$$

Theorem 2 Let μ be a given scalar. Closed-loop T-S fuzzy system (5) is asymptotically stable with the control gains given by $\mathbf{K}_j = \mathbf{S}_j^T \mathbf{R}^{-T}$, if there exist predefined scalar λ_{ij} satisfying $h_i h_j - \bar{h}_i \bar{h}_j - \lambda_{ij} \geq 0$ and matrices $\boldsymbol{\Psi}$, $\{\mathbf{T}_i > 0\}_{i=1}^r$, $\{\mathbf{W}_{ij}\}_{i=1}^r$, $\{\mathbf{S}_i\}_{i=1}^r$ and \mathbf{R} , such that

$$\boldsymbol{\Omega}_{ij} + \boldsymbol{\Omega}_{ji} + \mathbf{W}_{ij} + \mathbf{W}_{ji} + 2\mathbf{Y} < 0 \quad (10)$$

$$\sum_{i=1}^r \sum_{j=1}^r ((\bar{h}_i \bar{h}_j + \lambda_{ij})(\boldsymbol{\Omega}_{ij} + \mathbf{W}_{ij}) + \lambda_{ij}\mathbf{Y}) < 0 \quad (11)$$

$$\begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} & \dots & \mathbf{W}_{1r} \\ \mathbf{W}_{12} & \mathbf{W}_{22} & \dots & \mathbf{W}_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{W}_{1r} & \mathbf{W}_{2r} & \dots & \mathbf{W}_{rr} \end{bmatrix} \geq 0 \quad (12)$$

where

$$\boldsymbol{\Omega}_{ij} = \begin{bmatrix} -\mathbf{A}_i\mathbf{R} - \mathbf{R}\mathbf{A}_i^T + \mathbf{B}_i\mathbf{S}_j^T + \mathbf{S}_j\mathbf{B}_i^T & * \\ \mathbf{T}_i - \mu\mathbf{A}_i\mathbf{R}^T + \mu\mathbf{B}_i\mathbf{S}_j^T + \mathbf{R} & \mu(\mathbf{R} + \mathbf{R}^T) \end{bmatrix}$$

Proof Eq. (8) is considered as a Lyapunov function candidate. Taking its time-derivative along the trajectories of system (5) and then applying Eq. (7), it follows that

$$\begin{aligned} \dot{V}(\mathbf{x}(t)) &\leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j (\mathbf{x}^T(t)\mathbf{P}_i\dot{\mathbf{x}}(t) + \dot{\mathbf{x}}^T(t)\mathbf{P}_i\mathbf{x}(t) + \\ &2\mathbf{x}^T(t)\boldsymbol{\Psi}\dot{\mathbf{x}}(t) - 2\mathbf{x}^T(t)\boldsymbol{\Psi}(A_i - B_i\mathbf{K}_j)\mathbf{x}(t) + \\ &2\dot{\mathbf{x}}^T(t)\boldsymbol{\mu}\boldsymbol{\Psi}\dot{\mathbf{x}}(t) - 2\dot{\mathbf{x}}^T(t)\boldsymbol{\mu}\boldsymbol{\Psi}(A_i - B_i\mathbf{K}_j)\mathbf{x}(t)) = \\ &\sum_{i=1}^r \sum_{j=1}^r h_i h_j [\mathbf{x}^T(t) \quad \dot{\mathbf{x}}^T(t)] \times \\ &\begin{bmatrix} -\boldsymbol{\Psi}(A_i - B_i\mathbf{K}_j) - (A_i - B_i\mathbf{K}_j)^T\boldsymbol{\Psi}^T & * \\ \mathbf{P}_i - \mu\boldsymbol{\Psi}(A_i - B_i\mathbf{K}_j) + \boldsymbol{\Psi}^T & \mu(\boldsymbol{\Psi} + \boldsymbol{\Psi}^T) \end{bmatrix} \times \\ &\begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} \end{aligned} \quad (13)$$

Denote $\boldsymbol{\theta}(t) = \text{diag}(\boldsymbol{\Psi}^T, \boldsymbol{\Psi}^T)\boldsymbol{\xi}(t)$, $\mathbf{R} = \boldsymbol{\Psi}^{-1}$, $\mathbf{T}_i = \mathbf{R}\mathbf{P}_i\mathbf{R}^T$, and $\mathbf{S}_j = \mathbf{R}\mathbf{K}_j^T$. From Eq. (13), we have

$$\begin{aligned} \dot{V}(\mathbf{x}(t)) &= \sum_{i=1}^r \sum_{j=1}^r \bar{h}_i \bar{h}_j \boldsymbol{\theta}^T(t)\boldsymbol{\Omega}_{ij}\boldsymbol{\theta}(t) + \\ &\sum_{i=1}^r \sum_{j=1}^r (h_i h_j - \bar{h}_i \bar{h}_j)\boldsymbol{\theta}^T(t)\boldsymbol{\Omega}_{ij}\boldsymbol{\theta}(t) \end{aligned} \quad (14)$$

where \bar{h}_i are the staircase membership functions, which consist of a finite number of discrete values to approximate the

continuous membership functions h_i . To facilitate the stabilization design for T-S fuzzy system (5), the staircase membership functions \bar{h}_i proposed here satisfy the properties of membership functions in (3), namely $\sum_{i=1}^r \sum_{j=1}^r \bar{h}_i \bar{h}_j = 1$.

Considering $\sum_{i=1}^r \sum_{j=1}^r \bar{h}_i \bar{h}_j \mathbf{W}_{ij} \geq 0$ and $h_i h_j - \bar{h}_i \bar{h}_j - \lambda_{ij} \geq 0$, we proceed from Eq. (14) with $\mathbf{Y} = \mathbf{Y}^T$.

$$\begin{aligned} \dot{V}(\mathbf{x}(t)) &= \sum_{i=1}^r \sum_{j=1}^r \bar{h}_i \bar{h}_j \boldsymbol{\theta}^T(t) \boldsymbol{\Omega}_{ij} \boldsymbol{\theta}(t) + \\ &\quad \sum_{i=1}^r \sum_{j=1}^r (h_i h_j - \bar{h}_i \bar{h}_j) \boldsymbol{\theta}^T(t) \boldsymbol{\Omega}_{ij} \boldsymbol{\theta}(t) + \\ &\quad \sum_{i=1}^r \sum_{j=1}^r h_i h_j \boldsymbol{\theta}^T(t) \mathbf{W}_{ij} \boldsymbol{\theta}(t) + \\ &\quad \sum_{i=1}^r \sum_{j=1}^r (h_i h_j - \bar{h}_i \bar{h}_j) \boldsymbol{\theta}^T(t) \mathbf{Y} \boldsymbol{\theta}(t) = \\ &\quad \sum_{i=1}^r \sum_{j=1}^r \boldsymbol{\theta}^T(t) ((\bar{h}_i \bar{h}_j + \lambda_{ij})(\boldsymbol{\Omega}_{ij} + \mathbf{W}_{ij}) + \lambda_{ij} \mathbf{Y}) \boldsymbol{\theta}(t) + \\ &\quad \frac{1}{2} \sum_{i=1}^r \sum_{j=1}^r (h_i h_j - \bar{h}_i \bar{h}_j - \lambda_{ij}) \boldsymbol{\theta}^T(t) (\boldsymbol{\Omega}_{ij} + \boldsymbol{\Omega}_{ij} + \\ &\quad \mathbf{W}_{ij} + \mathbf{W}_{ji} + 2\mathbf{Y}) \boldsymbol{\theta}(t) \end{aligned} \quad (15)$$

From Eqs. (10) and (11), we have $\dot{V}(\mathbf{x}(t)) < 0$. Then the asymptotic stability of the T-S fuzzy system is guaranteed.

3 Numerical Example

Consider a T-S fuzzy model in the form of Eq. (2) with the following parameters:

$$\mathbf{A}_1 = \begin{bmatrix} 1.59 & -7.29 \\ 0.01 & 0 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 0.02 & -4.64 \\ 0.35 & 0.21 \end{bmatrix}$$

$$\mathbf{A}_3 = \begin{bmatrix} -a & -4.33 \\ 0 & 0.05 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 8 \\ 0 \end{bmatrix}, \mathbf{B}_3 = \begin{bmatrix} 6-b \\ -1 \end{bmatrix}$$

The membership functions of the considered system are defined as

$$h_1(x_1(t)) = \begin{cases} 1 & x_1(t) < -10 \\ -x_1(t) + 2/12 & -10 \leq x_1(t) \leq 2 \\ 0 & x_1(t) > 2 \end{cases}$$

$$h_2(x_1(t)) = 1 - h_1(x_1(t)) - h_3(x_1(t))$$

$$h_3(x_1(t)) = \begin{cases} 0 & x_1(t) < -2 \\ x_1(t) + 2/12 & -2 \leq x_1(t) \leq 10 \\ 1 & x_1(t) > 10 \end{cases}$$

As in Ref. [10], the staircase membership functions employed in this example are chosen as $\bar{h}_i = h_i(\alpha\beta)$, ($\alpha - 0.5$) $\beta < x_1(t) < (\alpha + 0.5)\beta$. Considering $\beta = 0.05$, it can be found that $\lambda_{11} = -0.004145$, $\lambda_{22} = -0.002773$, $\lambda_{33} = -0.004162$, $\lambda_{12} = \lambda_{21} = -0.002070$, $\lambda_{13} = \lambda_{31} = -0.000681$, $\lambda_{23} = \lambda_{32} = -0.002018$, which satisfy the inequality $h_i h_j - \bar{h}_i \bar{h}_j - \lambda_{ij} \geq 0$. Theorem 2 is applied to verify the stabilization design of fuzzy systems for several values of parameters, comprising $a \in [2, 9]$ and $b \in [-10, 35]$. For comparison purposes, the stability regions given by different stabilization conditions are shown in Fig. 1 and

Fig. 2. It can be seen that the proposed relaxed stabilization conditions are able to produce a larger stability region.

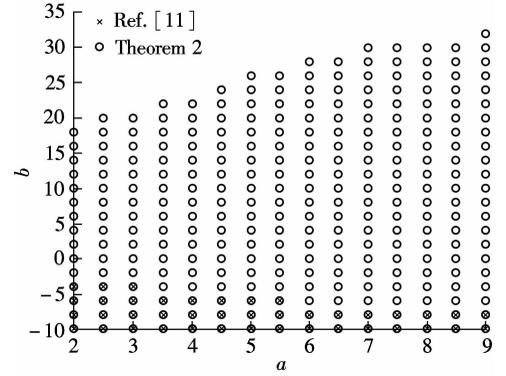


Fig. 1 Stability regions given by Ref. [11] and theorem 2

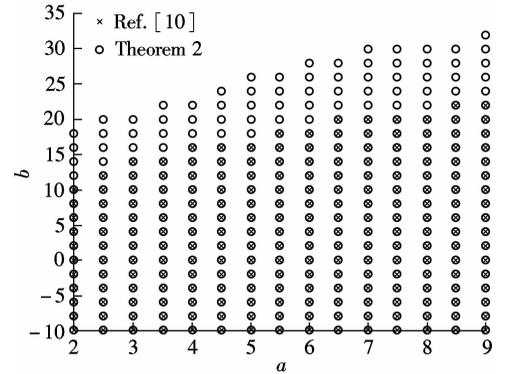


Fig. 2 Stability regions given by Ref. [10] and theorem 2

4 Conclusion

A new approach to the stabilization design for a class of continuous-time T-S fuzzy systems is presented in this paper. The new stabilization conditions are derived based on a relaxed approach in which both fuzzy Lyapunov functions and staircase membership functions are used. Larger stability regions can be guaranteed compared with previous results. A simulation example is given to illustrate the validity of the proposed approach.

References

- [1] Takagi T, Sugeno M. Fuzzy identification of systems and its application to modeling and control [J]. *IEEE Transactions on System, Man and Cybernetics*, 1985, **15**(1): 116–132.
- [2] Tanaka K, Wang H O. *Fuzzy control systems design and analysis: a linear matrix inequality approach* [M]. New York, USA: John Wiley and Sons, 2001.
- [3] Tanaka K, Hori T, Wang H O. A multiple Lyapunov function approach to stabilization of fuzzy control systems [J]. *IEEE Transactions on Fuzzy Systems*, 2003, **11**(4): 582–589.
- [4] Kim E, Lee H. New approaches to relaxed quadratic stability condition of fuzzy control systems [J]. *IEEE Transactions on Fuzzy Systems*, 2000, **8**(5): 523–534.
- [5] Liu X D, Zhang Q L. New approaches to H_∞ controller designs based on fuzzy observers for T-S fuzzy systems via LMI [J]. *Automatic*, 2003, **39**(9): 31571–31582.
- [6] Tanaka K, Hori T, Wang H O. A fuzzy Lyapunov approach to fuzzy control systems design [C]//*American Control*

- Conference. Arlington, USA, 2001: 4790 – 4795.
- [7] Guerra T M, Vermeiren L. LMI-based relaxed non-quadratic stabilization conditions for nonlinear systems in Takagi-Sugeno's form [J]. *Automatic*, 2004, **40**(5): 823 – 829.
- [8] Rhee B J, Won S. A new fuzzy Lyapunov function approach for a Takagi-Sugeno fuzzy control system design [J]. *Fuzzy Sets and Systems*, 2006, **157**(9): 1211 – 1228.
- [9] Sala A, Arino C. Relaxed stability and performance LMI conditions for Takagi-Sugeno systems with polynomial constraints on membership function shapes [J]. *IEEE Transactions on Fuzzy Systems*, 2008, **16**(5): 1328 – 1336.
- [10] Lam H K, Narimani M. Quadratic-stability analysis of fuzzy-model-based control systems using staircase membership functions [J]. *IEEE Transactions on Fuzzy Systems*, 2010, **18**(1): 125 – 137.
- [11] Tanaka K, Sugeno M. Stability analysis and design of fuzzy control systems [J]. *Fuzzy Sets and Systems*, 1992, **45**: 135 – 156.

基于新松弛条件的连续时间 Takagi-Sugeno 模糊系统镇定设计

潘俊涛 费树岷 穆朝絮 薛明香

(东南大学复杂工程系统测量与控制教育部重点实验室, 南京 210096)

摘要: 研究了一类连续时间 Takagi-Sugeno 模糊系统的镇定控制器设计问题, 提出了一种新的基于模糊 Lyapunov 函数和阶梯隶属度函数的系统镇定条件. 通过阶梯隶属度函数逼近原系统的隶属度函数, 系统隶属度函数信息被引入到其镇定设计中, 从而大大降低了现有 T-S 模糊控制系统镇定设计的保守性. 与已有基于模糊 Lyapunov 函数的连续 T-S 模糊系统的镇定设计不同, 所提出的镇定条件不依赖于隶属度函数的导数, 因此具有较少保守性. 此外, 所得镇定条件可表示为一组线性矩阵不等式, 而不是双线性矩阵不等式, 因而可以利用凸优化算法方便地进行求解. 仿真实例表明了所提方法的有效性.

关键词: Takagi-Sugeno 模糊模型; 模糊 Lyapunov 函数; 阶梯隶属度函数; 线性矩阵不等式

中图分类号: TP183