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System reliability assessment based on Wiener process and competing failure analysis

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Abstract: Considering the dependence and competitive relationship between traumatic failure and degradation, the reliability assessment of products based on competing failure analysis is studied. The hazard rate of traumatic failure is regarded as a Weibull distribution of the degradation performance, and the Wiener process is used to describe the degradation process. The parameters are estimated with the maximum likelihood estimation (MLE) method. A reliability model based on competing failure analysis is proposed. A case study of the GaAs lasers is given to validate the effectiveness of the model and its solving method. The results indicate that if only the degradation failure is considered, the estimated result will be comparably optimistic. Meanwhile, the correlation between the degradation and traumatic failure has a great influence on the accuracy of reliability assessment.

Key words: degradation data; Wiener process; competing failure; reliability assessment

Based on the failure mechanisms, product failure can be classified into either traumatic failure or degradation failure, in which traumatic failure means that products will suddenly fail during the working process, and degradation failure means that products will gradually lose their function and performance until the performance reaches a certain level. For most products, the two kinds of failure mode exist simultaneously. So the failure of a product usually is the result of the competition between degradation failure and traumatic failure. We call it competing failure[1].

The system reliability analysis of the single failure mode has been intensively studied for some decades, but the research work on the interaction between degradation and traumatic failure is yet not enough[2].

In recent years, competing failure has caught much attention from academia and industry. Yang and Xue[3] extended the binary state reliability model to the continuous state model. The normal stochastic process was used to describe the degradation of products. Both the degradation and the traumatic failure could be analyzed simultaneously. Zuo and Jiang[4] summarized three kinds of reliability analysis methods. By supposing that the degradation and the traumatic failure are independent of each other, a mixed model was presented. In Ref. [5], the traumatic failure and the degradation failure were supposed to be independent of each other and they both obeyed the Weibull distribution. A reliability model considering competing failure was presented, and it could be used for electronic devices. Bocchetti et al. [6] used the non-homogenous Poisson process to describe degradation failure, and the Weibull distribution was used to describe traumatic failure. A competing failure model was presented to analyze the reliability of a cylinder engine. Deng et al[7] extended the traditional reliability method, and investigated the reliability and derived the hazard function under the competing failure condition. Zhao et al. [8] considered the interaction between degradation and traumatic failure and proposed a competing failure model based on the known distribution of degradation. Wu et al. [9] extended the hazard proportion model, and a competing failure model based on hazard proportion analysis was presented. Peng et al. [10] proposed a comprehensive reliability assessment method, which considered both degradation and failure data.

The above references discussed the competing failure problem from different views, most of which used the continuous distribution function to describe the degradation of products. However, for some electronic devices the degradation indices are non-monotonic[11]. The existing methods cannot describe this phenomenon properly. The Wiener process is a kind of process with independent increment, and it can be used to describe non-monotonic properties, so it is more suitable for electronic devices. In this paper, the Wiener process is used to describe the degradation, and a reliability evaluation model considering a competing failure model between degradation failure and traumatic failure is presented.

1 Competing Failure Model Based on Wiener Process

1.1 Analysis of degradation process

Denote X_t as the degradation performance at time t, and D is the failure threshold. When the degradation performance X_t exceeds D for the first time, the product is regarded as having failed.

Assume that $X_t - X_0 = \mu t + \sigma B(t)$, where μ is the drift parameter; σ is the variance parameter, and $B(t)$ is the standard Brownian motion. The initial degradation performance is defined as zero, which means when time $t=0$, the degradation performance $X_0 = 0$. So the degradation performance at time t is normally distributed, denoted as $X_t \sim N(\mu t, \sigma^2 t)$. The probability density function of degradation performance can be defined as

$$f(x, t) = \frac{1}{\sqrt{2\pi\sigma^2 t}} \exp\left[-\frac{(x-\mu t)^2}{2\sigma^2 t} \right] \tag{1}$$

The failure time is defined as the first time that the degra-

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dation performance exceeds the failure threshold D, which is denoted as

$$T = \inf\{t \mid X_t \geqslant D, t \geqslant 0\}$$

As we know that the first passage time of the Wiener process has an inverse Gaussian distribution with the following pdf[11]:

$$g(t) = \sqrt{\frac{D^2}{2\pi\sigma^2 t^3}} \exp\left[-\frac{(D-\mu t)^2}{2\sigma^2 t}\right] \tag{2}$$

1.2 Competing failure model

The failure of products is related to both degradation failure and traumatic failure. Traditionally, the traumatic hazard rate is regarded as the function of time t. However, the probability of traumatic failure is not always only related to time in practice; sometimes, it is also related to the degradation performance x at time t.

Assuming that the traumatic failure rate is related to degradation performance and there are p kinds of traumatic failure modes, we denote T_h^l as the failure time of the l-th failure mode and $\lambda^l(x, t)$ as the hazard rate function of failure time T_h^l. Then the reliability function of the l-th failure mode can be obtained by

$$R_h^l(X(t)) = P(T_h^l \geqslant t \mid X(t)) = \exp\left(-\int_0^t \lambda^l(x, t)\,dt\right)$$

The reliability function of the traumatic failure is

$$R_h(X(t)) = \prod_{l=1}^{p} R_h^l(X(t)) = \prod_{l=1}^{p} P(T_h^l \geqslant t \mid X(t)) = \exp\left(-\sum_{l=1}^{p}\int_0^t \lambda^l(x, t)\,dt\right) \tag{3}$$

Thus the reliability function of the competing failure is

$$R_c(t) = P(T_h \geqslant t, T_s \geqslant t) = \int_0^D R_h(X(t)) f(x, t)\,dx = \int_0^D \exp\left(-\sum_{l=1}^{p}\int_0^t \lambda^l(x, t)\,dt\right)\frac{1}{\sqrt{2\pi\sigma^2 t}} \cdot \exp\left[-\frac{(x-\mu t)^2}{2\sigma^2 t}\right]dx \tag{4}$$

If the degradation and traumatic failure are independent, the competing failure model can be defined as

$$R_c(t) = P(T_h \geqslant t, T_s \geqslant t) = R_h(t) R_s(t) = \exp\left(-\sum_{l=1}^{p}\int_0^t \lambda^l(x, t)\,dt\right)\Phi\left(\frac{D-\mu t}{\sqrt{\sigma^2 t}}\right)$$

1.3 Parameters estimation

1.3.1 Parameters estimation of degradation failure

It is supposed that there are $M+p$ units in the test, where degradation will occur in M units and traumatic failure will occur in p units. The degradation data and failure times are all known.

According to Eq. (1), the probability density function of degradation performance of the k-th unit at the i-th observation time can be written as

$$f(\Delta X_{kt_i}; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2 \Delta t_{ki}}} \exp\left[-\frac{(\Delta x_{kt_i} - \mu\Delta t_{ki})^2}{2\sigma^2 \Delta t_{ki}}\right]$$

So the likelihood function of the units is

$$L(\Delta X_{1t_1}, \Delta X_{1t_2}, \dots, \Delta X_{Mt_N}; \mu, \sigma) = \prod_{k=1}^{M}\prod_{i=1}^{N} f(\Delta X_{kt_i}; \mu, \sigma) = \prod_{k=1}^{M}\prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2 \Delta t_{ki}}} \exp\left[-\frac{(\Delta x_{kt_i} - \mu\Delta t_{ki})^2}{2\sigma^2 \Delta t_{ki}}\right] \tag{5}$$

Taking the logarithm on both sides of the function, and letting both the partial derivatives of μ and σ^2 be equal to zero, the estimates of μ and σ^2 can be obtained as

$$\hat{\mu} = \frac{\sum_{k=1}^{M}\sum_{i=1}^{N} \Delta x_{kt_i}}{\sum_{k=1}^{M}\sum_{i=1}^{N} \Delta t_{ki}} \tag{6}$$

$$\hat{\sigma}^2 = \frac{1}{MN}\sum_{k=1}^{M}\sum_{i=1}^{N}\left[\frac{(\Delta x_{kt_i} - \mu\Delta t_{ki})^2}{\Delta t_{ki}}\right] \tag{7}$$

1.3.2 Parameters estimation of traumatic failure

1) Judge the dependency of degradation and traumatic failure based on the testing data;

2) If they are independent, the traumatic failure data will be fitted with reasonable distribution, and the parameters are estimated;

3) If the degradation and traumatic failure are dependent, the degradation performance will be recorded as $X_l (l = 1, 2, \dots, p)$ until failure occurs. The data will be arranged in an ascending order, and it is denoted as the set $\{X_i\}$.

The estimation of the reliability can be written as $\hat{R}_h^i = ((M+p) - i)/(M+p)$, which means that the proportion of surviving units is within all the units. So the reliability function can be defined as

$$R_h^i(X(t)) = \exp\left(-\int_0^t \lambda^i(x, t)\,dt\right) = \exp\left(-\int_0^t \lambda^i(x, t, \dot{\boldsymbol{\pi}})\,dt\right)$$

where $\dot{\boldsymbol{\pi}}$ is the parameters vector. The value of the parameters vector can be estimated by the least squares estimation method.

2 Case Study

The GaAs laser is a kind of highly reliable and long-lifetime product. The operating current of the laser increases with the increase of the working time. In fact, it is an important kind of degradation phenomenon of the product. Usually, when the operating current is 10% greater than the initial value, it will be regarded as having failed. It means that the threshold of the degradation failure is 10%.

Twenty-two GaAs lasers were tested at 80 ℃; the degradation failure and traumatic failure data were measured every 250 h, until 4 000 h. The concrete test data can be found in Ref. [12]. Among those samples, the degradation failure occurred in 15 samples and the traumatic failure occurred in 7 samples. Fig. 1 shows the current data of the GaAs lasers at temperature 80 ℃.

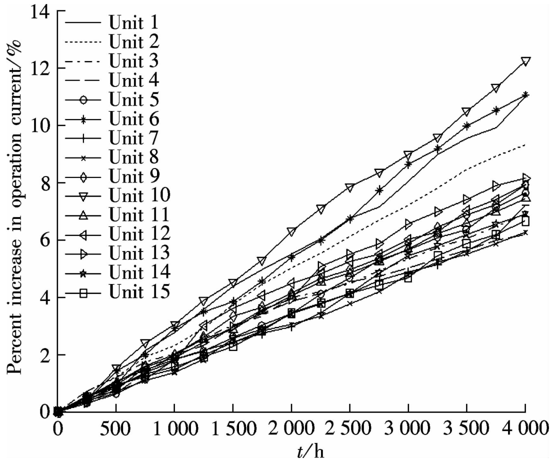

Fig. 1 Degradation paths of GaAs lasers

Fig. 2 shows the reliability curves using different estimation methods with the test data, and details can be seen in Ref. [13]. But the competing failures are not considered in these methods. From Fig. 2, we can see that using different estimation methods all the reliability curves are nearly the same before 4 250 h, while after 4 250 h, the estimated reliability values have a little difference.

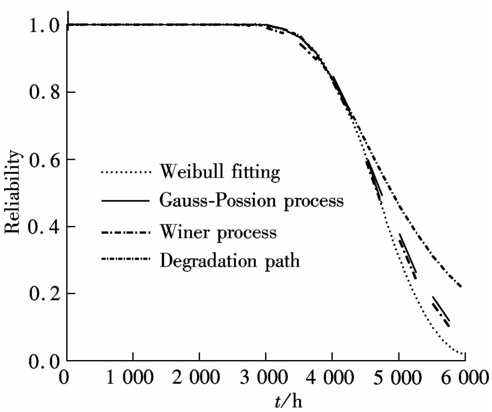

Fig. 2 Degradation reliability curves using different methods

2.1 Parameters estimation of degradation failure model

There are 15 data in every inspection time. According to the Ryan-Joiner test, the degradation data can be regarded as normally distributed. Owing to the independent property of every current increment, the Wiener process is used to model the degradation process.

On the basis of the degradation data at every inspection time, the degradation increments of every time interval are obtained. According to Eq. (6) and Eq. (7), the maximum likelihood estimations of μ and σ^2 are obtained: $\hat{\mu} = 0.002\,04$, $\hat{\sigma}^2 = 0.000\,160$.

2.2 Parameters estimation of traumatic failure model

It is supposed that the reliability of traumatic failure of the laser is the function of degradation performance X_t:

$$R_h(X(t)) = \exp\left(-\int_0^t \lambda(x,t)\,\mathrm{d}t\right) = \exp\left(-\int_0^x h(x)\,\mathrm{d}x\right)$$

Denote $h(x) = \beta/\eta(x/\eta)^{\beta-1}$, which means that the reliability of the traumatic failure is a Weibull function of degradation performance. Failure will occur when the degradation performances are 1.81, 3.97, 6.33, 7.05, 8.02, 8.35 and 9.62, respectively. The correspondent reliabilities of the traumatic failure are 21/22, 20/22, 19/22, 18/22, 17/22, 16/22, 15/22. Denote $m = \log[-\log R_h(x)]$ and $n = \log x$, and we can obtain $m = \beta n + c$, where $c = -\beta\log\eta$. The estimation of η and β can be obtained by the least squares estimation method: $\hat{\eta} = 24.059$, $\hat{\beta} = 1.237\,3$. The fitting plot of traumatic failure data is shown in Fig. 3.

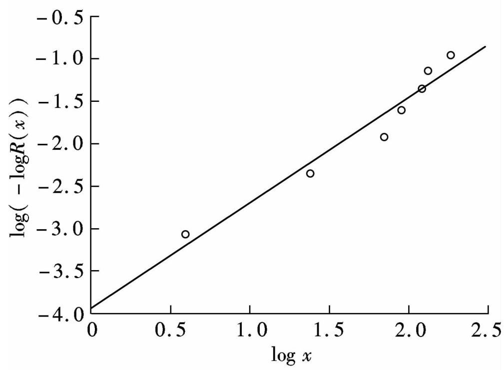

Fig. 3 Fitting result of traumatic failure data

2.3 Discussion about competing failure analysis

Based on the estimation of the above parameters and the failure threshold, the formula of the reliability under competing failure can be obtained, and the reliability can be calculated. The plot of reliability under competing failure is shown in Fig. 4. In which, the upper curve is the reliability under degradation failure and the lower curve is the reliability under competing failure.

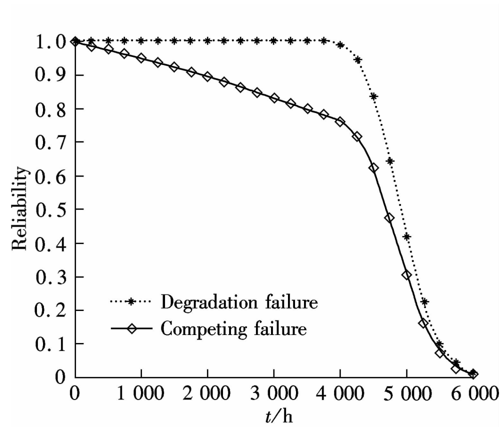

Fig. 4 Reliability curves considering competing failure

From Fig. 2 and Fig. 4, we can see that there are great differences between the curves when considering competing failure or not. If we only consider the degradation failure, the estimated results will be comparably optimistic, which will lead one to draw wrong conclusions and take wrong actions.

3 Conclusion

In this paper, the Wiener process is used to describe the degradation process, and the traumatic failure is considered to be the Weibull distribution of the degradation performance. A reliability evaluation model based on competing failure analysis is proposed. The study shows that compared with the non-competing failure estimation methods, a more accurate reliability assessment result can be obtained if we consider the competition between degradation failure and traumatic failure.

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基于 Wiener 过程和竞争失效分析的系统可靠性评估

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摘要:考虑突发失效和性能退化之间的相关性和竞争关系,研究了基于竞争失效分析的产品可靠性评估问题.将突发失效的发生率视为性能退化量的函数,并采用 Weibull 分布加以描述;利用 Wiener 过程描述性能退化过程,采用极大似然估计法估计模型参数;提出了一种基于竞争失效分析的可靠性评估模型.以一种 GaAs 激光器性能退化和失效数据为例,验证模型及其求解方法的有效性.结果表明:若仅考虑退化失效将使得评估结果偏向乐观,突发失效与退化失效的相关性对可靠性评估的准确性有很大影响.

关键词:退化数据;Wiener 过程;竞争失效;可靠性评估

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